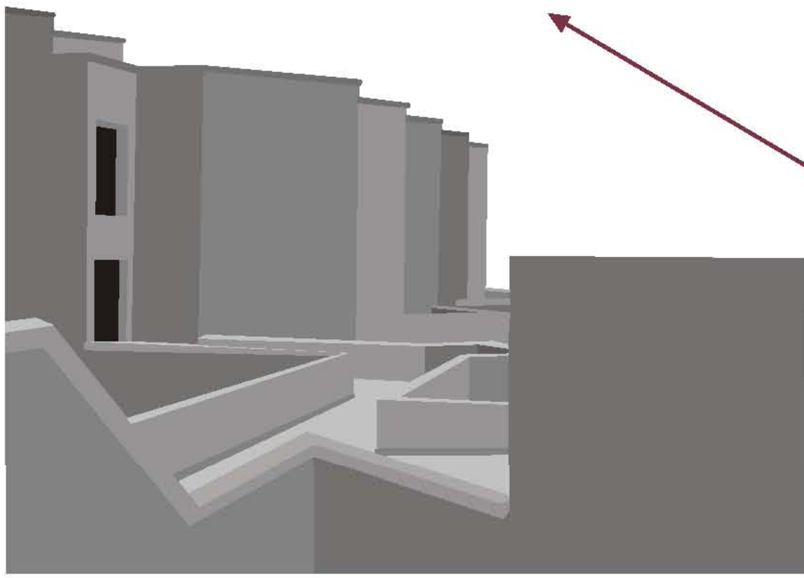

Comunicaciones del CIMAT

Strong Local Linear Approximations for Stochastic Differential Equations Driven by Semimartingales

R. Biscay, J. C. Jimenez and L. A. Salomón

Comunicación del CIMAT No
I-16-02/06-06-2016 (PE/CIMAT)

CIMAT

Comunicaciones del CIMAT

Strong Local Linear approximations for stochastic differential equations driven by semimartingales

R. Biscay [*], J.C. Jimenez [†], L.A. Salomón [‡]

Abstract

In previous works, the Local Linearization method has been expanded for the simulation of a specific type of stochastic differential equations driven by semimartingales, namely equations with additive noise. The effectiveness of the resulting Local Linear approximation was shown by means of simulations, and a strong convergence result was also obtained for such approximation. In this note, the Local Linearization method is extended to a notably wider class of equations, and the resulting Local Linear approximations are proved to be bounded and convergent uniformly on compacts in probability.

Key words and phrases: Stochastic Differential Equation, Semimartingale, Local Linearization.

MSC 2010: 60J75, 60J60, 60H35

1 Introduction

There exist a wide variety of methods for approximating the trajectories of stochastic differential equations (SDEs) driven by Brownian motions (see, e.g., [2],[9]). In contrast, for SDEs driven by Lévy processes or general semimartingales, the Euler method has been the main approach for designing strong numerical approximations (see, e.g., [1]; [3]; [4]; [5]). Alternatively, in [12] the known Local Linearization method for diffusion or jump-diffusions processes was expanded for the simulation of a specific type of SDEs driven by semimartingales. In that work, a comparison with the Euler method from the viewpoint of stability of the simulated paths was emphasized. In such simulation study, carried out with a number of moderate stiff equations driven by $\alpha-$stable Levy processes, the explosive trajectories of Euler method contrast with the stable paths of the Local Linearization method. In [11], the strong convergence of the approximation considered in [12] was also proved. This desirable performance of the Local Linearization approach motives the further developments that we report in what follows.

*Centro de Investigación en Matemáticas, Guanajuato, Mexico. email: rolando.biscay@cimat.mx

†Instituto de Cibernética, Matemática y Física. La Habana, Cuba. email: jcarlos@icimaf.cu

‡Universidad EAFIT, Medellin, Colombia. email: algoritmo.cu@gmail.com

First, in this note, the Local Linearization method is extended to a notably wider class of SDEs driven by semimartingales. Furthermore, the resulting Local Linear approximations are proved to be bounded and convergent uniformly on compacts in probability.

2 Local Linearization method

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space, $(\mathcal{F}_t)_{t\geq 0}$ be an increasing right continuous family of complete sub σ-algebras of $\mathcal{F}$, and $\mathcal{D}^d$ be the space of $\mathbb{R}^d$-valued, adapted processes with càdlàg paths. Consider the stochastic differential equation

$$\mathbf{X}_t = \mathbf{X}_0 + \int_0^t \mathbf{f}\left(\cdot, \mathbf{X}_\cdot\right)_{s-} ds + \int_0^t \mathbf{g}\left(\cdot, \mathbf{X}_\cdot\right)_{s-} d\mathbf{Z}_s + \int_0^t \mathbf{h}(\cdot)_{s-} d\mathbf{Z}_s, \tag{1}$$

where $(\mathbf{Z}_t)_{t\geq 0}$ is an $\mathbb{R}^d$-valued semimartingale with respect to $(\mathcal{F}_t)_{t\geq 0}$, $\mathbf{Z}_0 = \mathbf{0}$, the random vector $\mathbf{X}_0 \in \mathbb{R}^d$ is $\mathcal{F}_0$-measurable, and $\mathbf{f} : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}^d$, $\mathbf{g} : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}^{d\times m}$, $\mathbf{h} : \mathbb{R}_+ \to \mathbb{R}^{d\times m}$ are functions with continuous first derivatives. Suppose that (1) has a unique strong solution $(\mathbf{X}_t)_{t\geq 0}$ on $\mathcal{D}^d$ as stated, e.g., in [10] (Theorem 7, pp. 197).

In order to construct a numerical approximation to the solution $\mathbf{X}$, let Π^N $(N = 1, 2, ...)$ be a sequence of random partitions $0 = T_0^N \leq T_1^N \leq ... \leq T_{k_N}^N$ formed by finite stopping times T_n^N such that $\lim_N T_{K_N}^N = \infty$ almost surely (a.s.) and $\lim_N \max\left\{T_{n+1}^N - T_n^N : n = 0, 1, ..., k_N - 1\right\} = 0$ a.s. The convention $T_{k_N+1}^N = \infty$ is adopted. For simplicity, T_n^N will eventually be denoted by T_n.

The Local Linearization method is a general strategy for designing numerical integrators for differential equations based on a local (piecewise) linearization of the given equation on consecutive time intervals. The numerical integrators are then iteratively defined as the solution of the resulting piecewise linear equation at each consecutive interval. Specifically, by applying this strategy to the SDE (1) the following approximation results:

$$\mathbf{Y}_t = \mathbf{Y}_{T_n} + \int_{]T_n,t]} \left(\mathbf{f}_n + \mathbf{J}_n\left(\mathbf{Y}_{s-} - \mathbf{Y}_{T_n}\right) + \mathbf{d}_n\left(s - T_n\right)\right) ds + \int_{]T_n,t]} \mathbf{g}\left(\cdot, \mathbf{Y}_\cdot\right)_{s-} d\mathbf{Z}_s + \int_{]T_n,t]} \mathbf{h}(\cdot)_{s-} d\mathbf{Z}_s \tag{2}$$

for $t \in]T_n, T_{n+1}]$, where $\mathbf{f}_n = \mathbf{f}\left(T_n, \mathbf{Y}_{T_n}\right)$, $\mathbf{J}_n = \mathbf{f}_{\mathbf{X}}\left(T_n, \mathbf{Y}_{T_n}\right)$, and $\mathbf{d}_n = \mathbf{f}_t\left(T_n, \mathbf{Y}_{T_n}\right)$, being $\mathbf{f}_{\mathbf{X}}$ and $\mathbf{f}_t$ the derivatives of $\mathbf{f}(\mathbf{x},t)$ with respect to $\mathbf{x}$ and t, respectively. By following the ideas of previous works on Local Linearization methods for diffusion processes, jump diffusion processes and SDEs driven by driven by $\alpha-$stable Levy processes (see, e.g., [7],[13],[6],[12]), a number of Local Linearization schemes can be derived by approximating the integrals in the right hand side of (2). For instance, we define the numerical schemes given by

$$\mathbf{Y}_{T_{n+1}} = \mathbf{Y}_{T_n} + \int_{]T_n,T_{n+1}]} e^{(T_{n+1}-s)\mathbf{J}_n}\left(\mathbf{f}_n + \mathbf{d}_n\left(s - T_n\right)\right) ds + \mathbf{g}_n\left(\mathbf{Z}_{T_{n+1}} - \mathbf{Z}_{T_n}\right) \tag{3}$$

for equations with pure multiplicative noise (i.e., $\mathbf{h} \equiv \mathbf{0}$), and the scheme

$$\mathbf{Y}_{T_{n+1}} = \mathbf{Y}_{T_n} + \int_{]T_n,T_{n+1}]} e^{(T_{n+1}-s)\mathbf{J}_n}\left(\mathbf{f}_n + \mathbf{d}_n\left(s - T_n\right)\right) ds + \mathbf{L}_{]T_n,T_{n+1}]} \tag{4}$$

for equations with pure additive noise (i.e., $\mathbf{g} \equiv \mathbf{0}$). Here, $\mathbf{g}_n = \mathbf{g}\left(T_n, \mathbf{Y}_{T_n}\right)$ and $\mathbf{L}_{]T_n,T_{n+1}]}$ is an approximation to the integral $\int_{]T_n,T_{n+1}]} \mathbf{h}(\cdot)_{s-} d\mathbf{Z}_s$. In general, for equations with both additive and multiplicative noise, we consider the scheme

$$\mathbf{Y}_{T_{n+1}} = \mathbf{Y}_{T_n} + \int_{]T_n,T_{n+1}]} e^{(T_{n+1}-s)\mathbf{J}_n} \left(\mathbf{f}_n + \mathbf{d}_n \left(s - T_n\right)\right) ds + \mathbf{g}_n \left(\mathbf{Z}_{T_{n+1}} - \mathbf{Z}_{T_n}\right) + \mathbf{L}_{]T_n,T_{n+1}]}. \quad (5)$$

A number of previous works (e.g., [7], [6] and [12]) can be consulted for different ways to compute the integral involving matrix exponential in (3), (4) and (5), and also for designing approximations $\mathbf{L}_{]T_n,T_{n+1}]}$ for particular types of processes $\mathbf{Z}$.

It is worth to mention that the wide family of schemes just defined notably extends the earlier works [11] and [12], which deal with an specific approximation of the form (4) for the SDE (1) with pure additive noise ($\mathbf{g} \equiv \mathbf{0}$).

3 Bound and convergence

In what follows, $|\cdot|$ denotes the Frobenious norm for vectors and matrices, $\wedge$ the minimum of two numbers, and η the process defined by

$$n_t = \max\left\{n : T_n \leq t, 0 \leq n \leq k_N\right\}.$$

All the Local Linearization schemes introduced in the previous section can be written as solutions of integral equations of the general form

$$\mathbf{Y}_t^N = \mathbf{X}_0 + \int_0^t \mathbf{F}^N\left(t, \mathbf{Y}_\cdot^N\right)_{s-} ds + \int_0^t \mathbf{G}^N\left(t, \mathbf{Y}_\cdot^N\right)_{s-} d\mathbf{Z}_s + \mathbf{L}_t^N, \quad t \geq 0, \qquad (6)$$

evaluated at the time instants $t \in \Pi^N$, where

$$\begin{aligned}
\mathbf{F}^N\left(t, \mathbf{Y}_\cdot^N\right)_s &= e^{(t \wedge T_{n_s+1}-s)\mathbf{J}_{n_s}} \left(\mathbf{f}_{n_s} + \mathbf{d}_{n_s}\left(s - T_{n_s}\right)\right), \\
\mathbf{G}^N\left(t, \mathbf{Y}_\cdot^N\right)_s &= \mathbf{g}_{n_s},
\end{aligned}$$

and $\mathbf{L}_t^N$ is a semimartingale that approximates the integral

$$\mathbf{L}_t = \int_0^t \mathbf{h}(\cdot)_{s-} d\mathbf{Z}_s$$

on the time partition Π^N, with $\mathbf{L}_{t=0}^N = 0$.

For later reference, let us state the following standard conditions on the coefficients of the equation (1).

Hypothesis 1 *Suppose that*
i) $\mathbf{f} \in C^2\left(\mathbb{R}_+ \times \mathbb{R}^d, \mathbb{R}^d\right)$, $\mathbf{g} \in C^2\left(\mathbb{R}_+ \times \mathbb{R}^d, \mathbb{R}^{d\times m}\right)$ *and* $\mathbf{h} \in C^2\left(\mathbb{R}_+, \mathbb{R}^{d\times m}\right)$;
ii) $|\mathbf{f}(t, \mathbf{x})| \leq K_1 (1 + |\mathbf{x}|)$ *and* $|\mathbf{g}(t, \mathbf{x})| \leq K_1 (1 + |\mathbf{x}|)$;
iii) $\left|\frac{\partial}{\partial \mathbf{x}} \mathbf{f}(t, \mathbf{x})\right| \leq K_2$ *and* $\left|\frac{\partial}{\partial \mathbf{x}} \mathbf{g}(t, \mathbf{x})\right| \leq K_2$
for all $t \in \mathbb{R}_+$, $\mathbf{x} \in \mathbb{R}^d$, *where* K_1 *and* K_2 *are positive constants.*

The next theorem shows that the Local Linear approximation $\mathbf{Y}_t^N$ defined in (6) is uniformly bounded on compacts in probability.

Theorem 2 *Under the Hypothesis 1, for any* $s_0 > 0$, $\sup_{0\leq t\leq s_0}\left|\mathbf{Y}_t^N\right|$ *is bounded in probability uniformly in* N.

Proof. For simplicity, in what follows, write $\mathbf{Y}_t = \mathbf{Y}_t^N$. From (6) it follows that, for all $t \geq 0$,

$$\begin{aligned}
|\mathbf{Y}_t| &= \left|\mathbf{X}_0 + \int_{]0,t]} \mathbf{F}^N(t,\mathbf{Y})_{s-}\, ds + \int_{]0,t]} \mathbf{G}^N(t,\mathbf{Y})_{s-}\, d\mathbf{Z}_s + \int_{]0,t]} d\mathbf{L}_s^N\right| \\
&\leq |\mathbf{X}_0| + \left|\int_{]0,t]} \mathbf{F}^N(t,\mathbf{Y})_{s-}\, ds\right| + \left|\int_{]0,t]} \mathbf{G}^N(t,\mathbf{Y})_{s-}\, d\mathbf{Z}_s\right| + \left|\int_{]0,t]} d\mathbf{L}_s^N\right|.
\end{aligned}$$

Let $M > 0$ be an arbitrary constant. Define

$$\eta_M = 1_{\{|\mathbf{X}_0|\leq M\}}, \quad \widetilde{\mathbf{Y}}_t = \mathbf{Y}_t\eta_M, \quad \text{and} \quad \varphi_t = \sup_{0\leq s\leq t}\left|\widetilde{\mathbf{Y}}_s\right|^2.$$

Then,

$$\left|\widetilde{\mathbf{Y}}_t\right| \leq M + \left|\int_{]0,t]} \mathbf{F}^N(t,\mathbf{Y})_{s-}\,\eta_M ds\right| + \left|\int_{]0,t]} \mathbf{G}^N(t,\mathbf{Y})_{s-}\,\eta_M d\mathbf{Z}_s\right| + \left|\int_{]0,t]} \eta_M d\mathbf{L}_s^N\right|.$$

This and the elementary inequality $(a_1 + ... + a_q)^2 \leq 2^{q-1}\left(a_1^2 + ... + a_q^2\right)$ imply that

$$\begin{aligned}
E\left(\varphi_{\tau-}\right) \leq 8M^2 &+ 8E\sup_{t<\tau}\left|\int_{]0,t]} \mathbf{F}^N(t,\mathbf{Y})_{s-}\,\eta_M ds\right|^2 + 8E\sup_{t<\tau}\left|\int_{]0,t]} \mathbf{G}^N(t,\mathbf{Y})_{s-}\,\eta_M d\mathbf{Z}_s\right|^2 \\
&+ 8E\sup_{t<\tau}\left|\int_{]0,t]} \eta_M d\mathbf{L}_s^N\right|^2 \qquad (7)
\end{aligned}$$

for any stopping time τ.

Let A be a control processes for the semimartingales $\mathbf{Z}$, $\mathbf{L}_t^N$ and $(t)_{t\geq 0}$ (see, e.g., [8]). Define the stopping times

$$\tau_j = \inf\{t \in \mathbb{R}_+ : A_t > j\} \qquad \text{for } j \in \mathbb{N}.$$

From (7), $A_{\tau_j -} \leq j$ and Theorem 24.4 in [8] it follows that, for any stopping time $\tau \leq \tau_j$,

$$\begin{aligned}
E\left(\varphi_{\tau-}\right) \leq 8M^2 &+ j8E\left\{\int_{]0,\tau[} \left|\mathbf{F}^N(t,\mathbf{Y})_{s-}\,\eta_M\right|^2 dA_s\right\} + j8E\left\{\int_{]0,\tau[} \left|\mathbf{G}^N(t,\mathbf{Y})_{s-}\,\eta_M\right|^2 dA_s\right\} \\
&+ j8E\left\{\int_{]0,\tau[} \eta_M dA_s\right\}.
\end{aligned}$$

Fix an arbitrary $t_0 > s_0$, and set $\tau_{j0} = \tau_j \wedge t_0$. Let τ be any stopping time $\tau \leq \tau_{j0}$. By Hypothesis 1, for all $s \leq t \leq \tau$,

$$\left|\mathbf{F}^N(t, \mathbf{Y})_{s-}\right| \leq K(1 + \left|\mathbf{Y}_{T_{n_{s-}}}\right|),$$
$$\left|\mathbf{G}^N(t, \mathbf{Y})_{s-}\right| \leq K(1 + \left|\mathbf{Y}_{T_{n_{s-}}}\right|),$$

where K is a positive constant. Thus,

$$\begin{aligned} E\left(\varphi_{\tau-}\right) &\leq 8M^2 + j16K^2 E\left\{\int_{]0,\tau[} \left(\eta_M + \left|\mathbf{Y}_{T_{n_{s-}}}\eta_M\right|^2\right) dA_s\right\} \\ &\quad + j16K^2 E\left\{\int_{]0,\tau[} \left(\eta_M + \left|\mathbf{Y}_{T_{n_{s-}}}\eta_M\right|^2\right) dA_s\right\} + j8E\left\{\int_{]0,\tau[} \eta_M dA_s\right\} \\ &\leq 8M^2 + j32K^2 E\left\{\int_{]0,\tau[} \left(\eta_M + \varphi_{s-}\right) dA_s\right\} + j8E\left\{\int_{]0,\tau[} \eta_M dA_s\right\}. \end{aligned}$$

Therefore, there exist positive constants C_0 and C_1 (depending on M and j but not on N) such that

$$E\left(\varphi_{\tau-}\right) \leq C_0 + C_1 E\left\{\int_{]0,\tau[} \varphi_{s-} dA_s\right\}.$$

Since here the stopping time $\tau \leq \tau_{j0}$ is arbitrary, Lemma 29.1 in [8] implies that

$$E\left(\varphi_{\tau_{j0}-}\right) \leq 2C_0 \sum_{i=0}^{[2jC_1]} \left(2jC_1\right)^i, \tag{8}$$

where $[\cdot]$ denotes the integer part of a real number.

Furthermore, for any constant $R > 0$,

$$\begin{aligned} P\left(\sup_{0\leq t\leq s_0} |\mathbf{Y}_t| > R\right) &\leq P\left(\sup_{0\leq t< t_0} |\mathbf{Y}_t| > R\right) \\ &\leq P\left(\varphi_{t_0-}^{1/2} > R\right) + P\left(|\mathbf{X}_0| > M\right) \\ &\leq P\left(\varphi_{\tau_{j0}-}^{1/2} > R\right) + P\left(\tau_{j0} < t_0\right) + P\left(|\mathbf{X}_0| > M\right) \\ &\leq \frac{E\left(\varphi_{\tau_{j0}-}\right)}{R^2} + P\left(\tau_{j0} < t_0\right) + P\left(|\mathbf{X}_0| > M\right), \end{aligned} \tag{9}$$

where the second inequality follows from the definition of φ_t, the third one from basic rules of the calculus of probabilities of events, and the last one from the Markov's inequality.

The assumption that $\mathbf{X}_0$ is a finite random vector implies that $P\left(|\mathbf{X}_0| > M\right) \to 0$ as $M \to \infty$. Since A has finite left and right limits a.s., $\lim_j \tau_j = \infty$ a.s., and so $\lim_j P\left(\tau_{j0} < t_0\right) = 0$.

Furthermore, (8) implies that for each fixed j, $E\left(\varphi_{\tau_{j0-}}\right)/R^2 \to 0$ uniformly in N as $R \to \infty$. From these facts and (9) it follows that that, $P\left(\sup_{0\le t\le s_0}|\mathbf{Y}_t| > R\right) \to 0$ uniformly in N as $R \to \infty$. ■

In what follows, the Local Linear approximation (6) is studied in the sense of convergence uniformly on compacts in probability (*ucp*) as it is defined, for instance, in [10].

Lemma 3 *Under the Hypothesis 1 it holds that*
i) $\mathbf{F}^N\left(t,\mathbf{X}_\cdot^N\right)_s$ *converges to* $\mathbf{f}\left(\cdot,\mathbf{X}_\cdot\right)_s$ *in ucp;*
ii) $\mathbf{G}^N\left(t,\mathbf{X}_\cdot^N\right)_s$ *converges to* $\mathbf{g}\left(\cdot,\mathbf{X}_\cdot\right)_s$ *in ucp.*

Proof. Consider $t,s\ge 0$ with $s\le t$. Since $\mathbf{f}\in\mathcal{C}^1$ and $\mathbf{f}$ has linear growth,

$$\begin{aligned}\left|\mathbf{F}^N\left(t,\mathbf{X}_\cdot^N\right)_s - \mathbf{f}\left(\cdot,\mathbf{X}_\cdot\right)_s\right| &= \left|e^{(t\wedge T_{n_s+1}-s)\mathbf{J}_{n_s}}\left(\mathbf{f}_{n_s}+\mathbf{d}_{n_s}\left(s-T_{n_s}\right)\right)-\mathbf{f}\left(s,\mathbf{X}_s\right)\right| \\ &\le \left|e^{(t\wedge T_{n_s+1}-s)\mathbf{J}_{n_s}}\mathbf{f}_{n_s}-\mathbf{f}\left(s,\mathbf{X}_s\right)\right| + \left|e^{(t\wedge T_{n_s+1}-s)\mathbf{J}_{n_s}}\mathbf{d}_{n_s}\left(s-T_{n_s}\right)\right| \\ &\le \left|e^{(t\wedge T_{n_s+1}-s)\mathbf{J}_{n_s}}\left(\mathbf{f}\left(T_{n_s},\mathbf{X}_{\tau_{n_s}}\right)-\mathbf{f}\left(s,\mathbf{X}_s\right)\right)\right| \\ &\quad + \left|\left(e^{(t\wedge T_{n_s+1}-s)\mathbf{J}_{n_s}}-\mathbf{I}\right)\mathbf{f}\left(s,\mathbf{X}_s\right)\right| + \left|e^{(t\wedge T_{n_s+1}-s)\mathbf{J}_{n_s}}\mathbf{d}_{n_s}\left(s-T_{n_s}\right)\right| \\ &\le K_1\left|e^{(t\wedge T_{n_s+1}-s)\mathbf{J}_{n_s}}\right|\left|\mathbf{X}_{T_{n_s}}-\mathbf{X}_s\right| + K_2\left|e^{(t\wedge T_{n_s+1}-s)\mathbf{J}_{n_s}}-\mathbf{I}\right|\left|\mathbf{X}_s\right| \\ &\quad + \left|e^{(t\wedge T_{n_s+1}-s)\mathbf{J}_{n_s}}\mathbf{d}_{n_s}\right|\left(s-T_{n_s}\right),\end{aligned}$$

where $\mathbf{I}$ denotes the $d-$dimensional identity matrix. Denote $\delta_N = \max_n\left|T_{n+1}^N - T_n^N\right|$. It can be assumed that $T_{k_N}^N \ge t$ by taking N large enough. Then, $|s-T_{n_s}|\le\delta_N$ and $|t\wedge T_{n_s+1}-s|\le\delta_N$. Since $\sup_{0\le s\le T}|\mathbf{X}_s|<\infty$ and $\delta_N\to 0$ as N goes to ∞,

$$P\left(\sup_{0\le s\le T}\left|\mathbf{F}^N\left(t,\mathbf{X}_\cdot^N\right)_s - \mathbf{f}\left(\cdot,\mathbf{X}_\cdot\right)_s\right| > r\right)\to 0 \text{ as } N\to\infty$$

for any constant $r>0$. Similarly,

$$P\left(\sup_{0\le s\le T}\left|\mathbf{G}^N\left(t,\mathbf{X}_\cdot^N\right)_s - \mathbf{g}\left(\cdot,\mathbf{X}_\cdot\right)_s\right| > r\right)\to 0 \text{ as } N\to\infty,$$

which concludes the proof. ■

Theorem 4 *Suppose that the approximation* $\mathbf{L}_t^N$ *is a semimartingale that converges to* $\mathbf{L}_t$ *in ucp. Then, under the Hypothesis 1, the Local Linear approximation* $\mathbf{Y}_t^N$ *defined in (6) converges to the solution* $\mathbf{X}_t$ *of (1) in ucp.*

Proof. The thesis is a directly consequence of Lemma 3 and Theorem 15, pp. 209, in [10]. ■

Acknowledgments. The second author thanks the financial support of CIMAT to this research and to Prof. C. Mora (Universidad de Concepción, Chile) for useful comments on issues related to the present work.

References

[1] Karandikar, R. L. (1991). An almost sure convergence of modified Euler-Peano approximation of solution to an SDE driven by semimartingales. In *Seminaire de Probabilites XXV*, eds. J. Azema et al., Springer-Verlag: Berlin, pp. 113-120.

[2] Kloeden, P. E. and Platen, E. (1995). *Numerical Solution of Stochastic Differential Equations*, Springer-Verlag, Berlin, 2nd. Edition.

[3] Kohatsu-Higa, A. and Protter, P. (1991). The Euler scheme for SDE's driven by semimartingales. In *Stochastic Analysis on Infinite Dimensional Spaces*, ed. H. H. Kuo, Longman Scientific and Technical: England, pp. 141-151.

[4] Kurtz, T. G and Protter, P. (1991). Weak limit theorems for stochastic integrals and stochastic differential equations. *Ann. Prob.* **19**, 1035-1070.

[5] Janicki, A., Podgórski, K., and Weron, A. (1993). Computer simulation of $\alpha-$stable Ornstein-Uhlenbeck process. In *Stochastic Processes. A Festschrift in Honour of Gopinath Kallianpur*, eds. S. Cambanis et al., Springer-Verlag: Berlin, pp. 161-170.

[6] Jimenez, J. C. and Carbonell, F. (2006). Local Linear approximations for jump diffusion processes. *J. Appl. Prob.* **43**, 185-194.

[7] Jimenez, J. C. and de la Cruz, H. (2012). Convergence rate of strong Local Linearization schemes for stochastic differential equations with additive noise. *BIT Numer. Math.* **52**, 357-382.

[8] Métivier, M. (1982). *Semimartingales. A Course on Stochastic Processes.* Walter de Gruyter: Berlin-New York.

[9] Milstein, G. N. and Tretyakov, M. V. (2004). *Stochastic Numerics for Mathematical Physics*, Springer.

[10] Protter, P. (1990). *Stochastic Integration and Differential Equations. A New Approach.* Springer-Verlag.

[11] Salomon, L. A. and Biscay, R. J. (2005-2006). Método de linealizacion local para la solución numérica de ecuaciones diferenciales estocásticas con ruido de Lévy estables. *Rev. Ciencias Matemáticas* **23**, 45-53.

[12] Salomon, L. A. and Biscay R. J. (2007). A simulation study of the Local Linearization method for the numerical (strong) solution of stochastic differential equations driven by alpha-stable Lévy motions. *Rev. Invest. Oper.* **28**, 51-60.

[13] Shoji, I. (2011). A note on convergence rate of a linearization method for the discretization of stochastic differential equations. *Commun. Nonlinear Sci. Numer. Simulat.* **16**, 2667-2671.