Basis Functions for Estimating Intravoxel Structure in DW-MRI.

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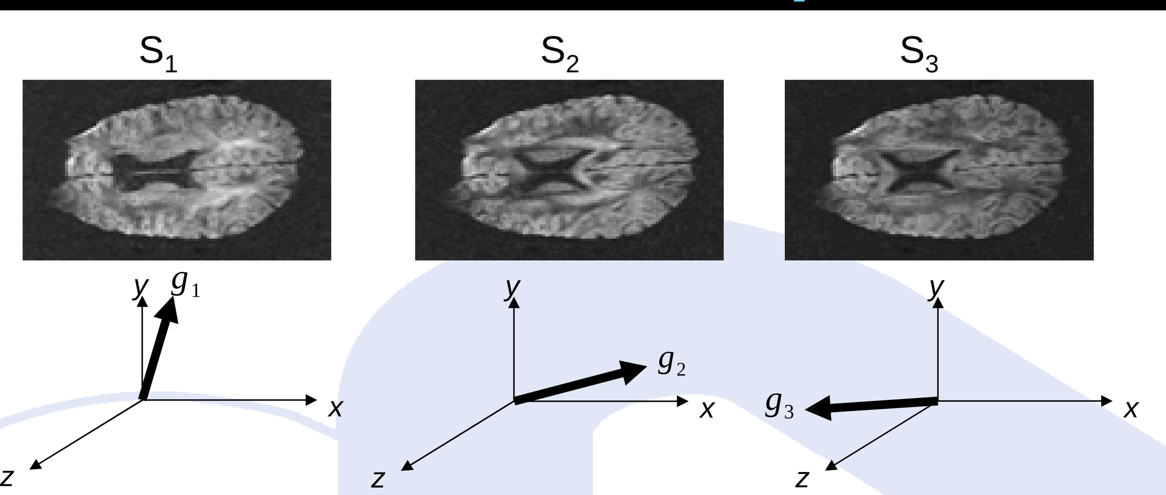
1 – Abstract

- We present a new method for estimating and recovering the intra-voxel information of brain water diffusion.
- We use Diffusion Weighted Magnetic Resonance Images, DW-MRI.
- Recovers the intra--voxel information at voxels that contain axon fiber crossings or bifurcations by means of a mixture of a fixed diffusion basis functions.
- The required number of images of our formulation is small (12).
- The solution schema is simple.
- The solution is numerically stable for more than two diffusion directions.
- We apply a spatial regularization to the recovered multi-tensor field in order to eliminate the noise.
- The regularization uses the prior information about the piecewise smoothness of nerve bundle orientation.

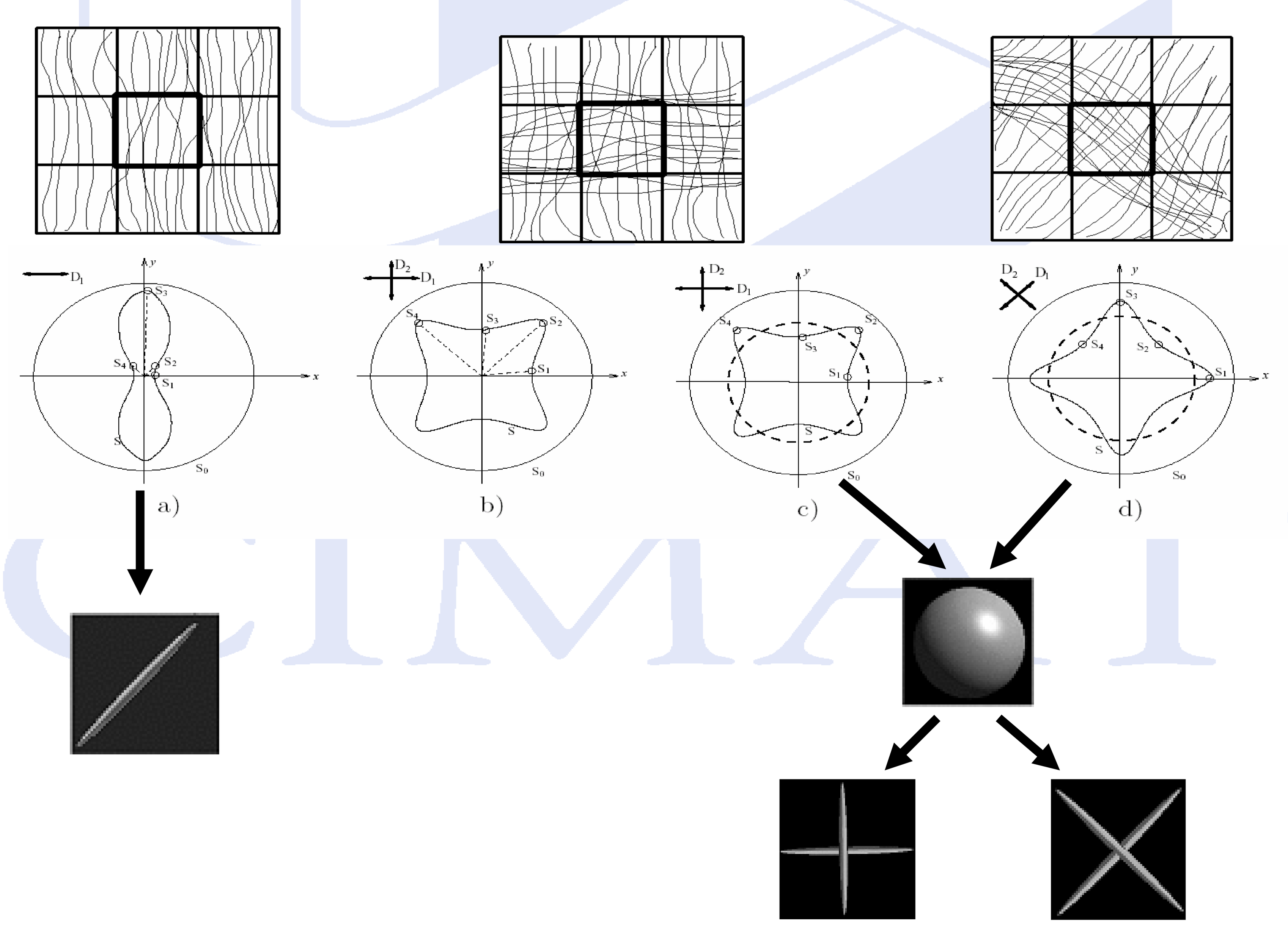
2- Statement of the problem



Standard Diffusion Tensor, obtained according to the Stejskal-Tanner equation.

$$S_i = S_0 \exp\left(-b\, g_i^T D g_i\right) \longrightarrow D = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{bmatrix}$$

Partial volume problem at fiber crosses and bifurcations.



3 - The multi-tensor approach and HARDI.

Tuch et al. proposed this model (requires 128 measures, Alexander et al. use 54 measures):

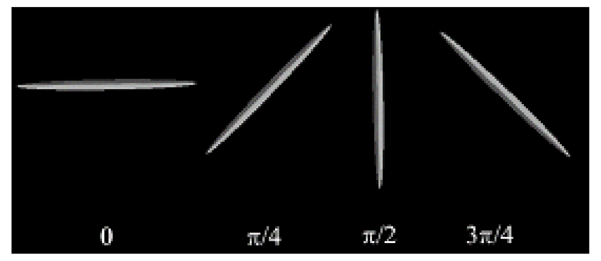
$$S_i = S_0 \sum_{j=1}^{2} f_j \exp\left(-b g_i^T \left[R_j^T \Lambda_j R_j\right] g_i\right)$$

$$R_j^T \Lambda_j R_j = D_j = \begin{bmatrix} D_{xx,j} & D_{xy,j} & D_{xz,j} \\ D_{yx,j} & D_{yy,j} & D_{yz,j} \\ D_{zx,j} & D_{zy,j} & D_{zz,j} \end{bmatrix}$$

4 – Our proposal, Diffusion Basis Functions (DBF).

$$\Phi_{ij} = \exp\left(-b\, g_i^T\, T_j\, g_i\right)$$

Schema of a 2D tensor basis:



$$\longrightarrow \quad T = [T_1, T_2, \ldots, T_N]$$

The observation model as a finite mixture of DBF's:

$$S_{i,r} = S_{0,r} \sum_{j=1}^{N} a_{j,r} \Phi_{ij} + h_{ir}$$

$$a_{j,r} \geq 0$$

DBF's can be precomputed.

In this model, the only unknown is the a vector field

Each $a_{j,r}$ denotes the contribution of the *j*-th base tensor at the *r* position.

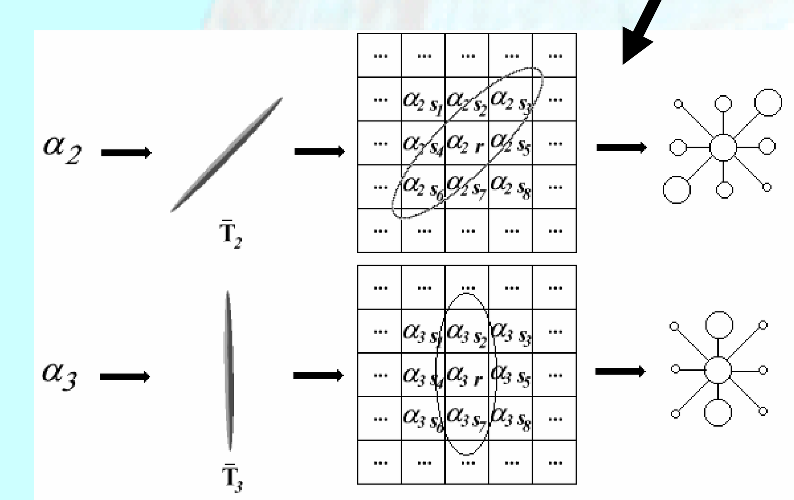
5 – Recovering the unknown vector field a

Cost function to be minimized. The data term is defined as:

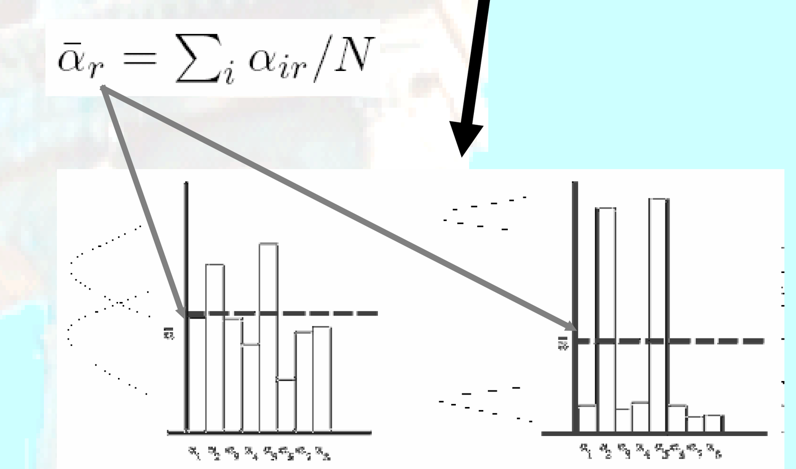
$$U(a,r) = \sum_{i=1}^{M} \left(S_{i,r} - S_{0,r} \sum_{j=1}^{N} a_{jr} \Phi_{ij}\right)^2$$

For dealing with noise, we apply a spatial regularization on a.

$$U(a,r) = \sum_{i=1}^{M} \left(S_{i,r} - S_{0,r} \sum_{j=1}^{N} a_{jr} \Phi_{ij}\right)^2 + \lambda_s \sum_{s:s\in N_r} w_{irs}\left(a_{ir} - a_{is}\right)^2 - \lambda_c \sum_{j=1}^{N} \left(a_{ir} - \bar{a}_r\right)^2$$



The spatial regularization is performed along the associated diffusion direction.



This regularization term, promotes a high contrast in the a vector.

Implementation details:

-The constraint of positivity on α is satisfied by projecting to zero the negative values in each iteration.

-According to our experiments, it is important to set the parameter $\lambda_c = 0$, then, once the algorithm has converged (with a low contrast solution) one refines the solution with the adequate λ_c value.