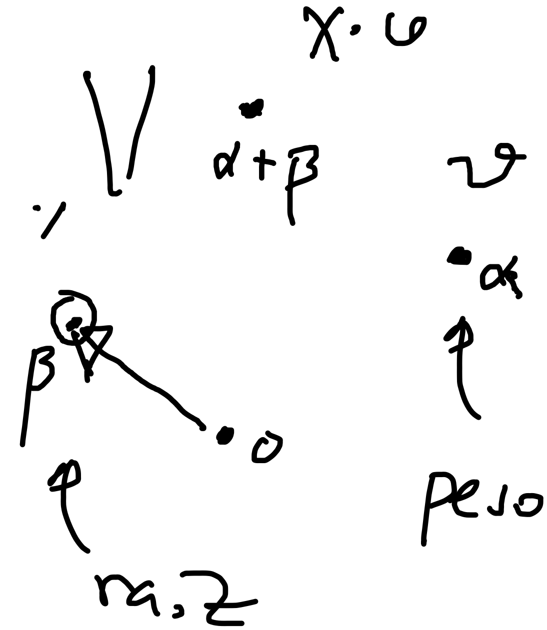
ej 9 (notas #4)

rep'n de SU_3 ∴ V

$\alpha \in \Omega(\rho)$

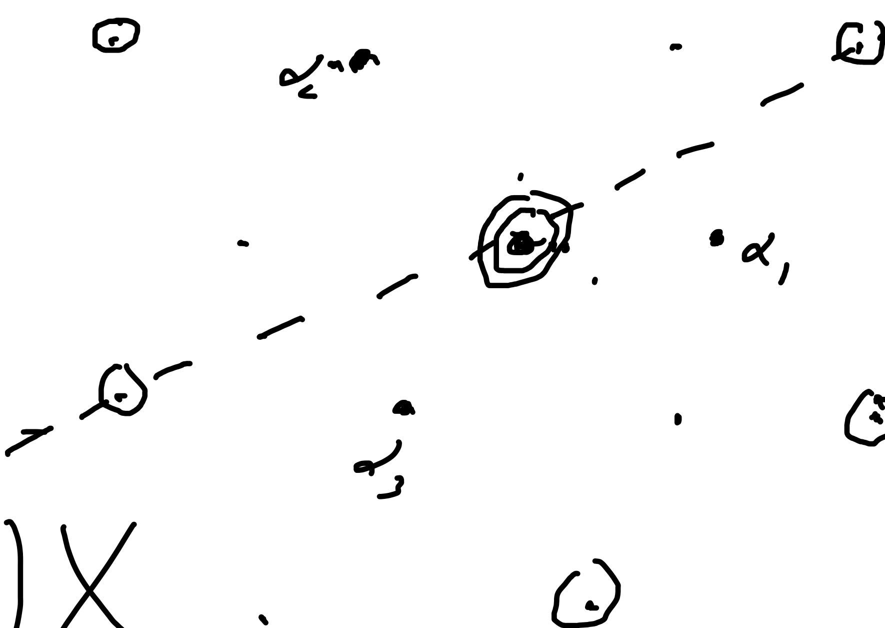


$X \in (su_3)^{\otimes\mathbb{C}}_\beta$

$0 \neq v \in V_\alpha$

⇕

$Y \in \mathfrak{h}$



$Y \cdot v = i\alpha(Y)\, v$

$[Y, X] = i\beta(Y)\, X$

$\Rightarrow Y \cdot (X \cdot v) =$ $\in V_{\alpha+\beta}$ $\neq 0$ / $= 0$

ej

O sea, para $\forall\, Y \in \mathfrak{h}$, $v \in V_\alpha$

$X \in (su_3 \otimes \mathbb{C})_\beta \qquad \alpha \in \Omega(\rho)$

$$X \cdot v \in V_{\alpha+\beta} \quad \text{ó} \quad X \cdot v = 0$$

$$Y(Xv) \overset{?}{=} \underbrace{i(\alpha+\beta)(Y)}_{\text{eigenvalor}}(Xv)$$

$$YXv = YXv - XYv + XYv$$

$$= [Y,X]v + XYv =$$

$$= i\beta(Y)Xv + X[i\alpha(Y)v] =$$

$$= [i(\alpha+\beta)Y]\,Xv. \quad \blacksquare$$

Ej: Sea $\alpha \in \Omega(\rho) \subset \mathfrak{h}^*$ ← 2-dim

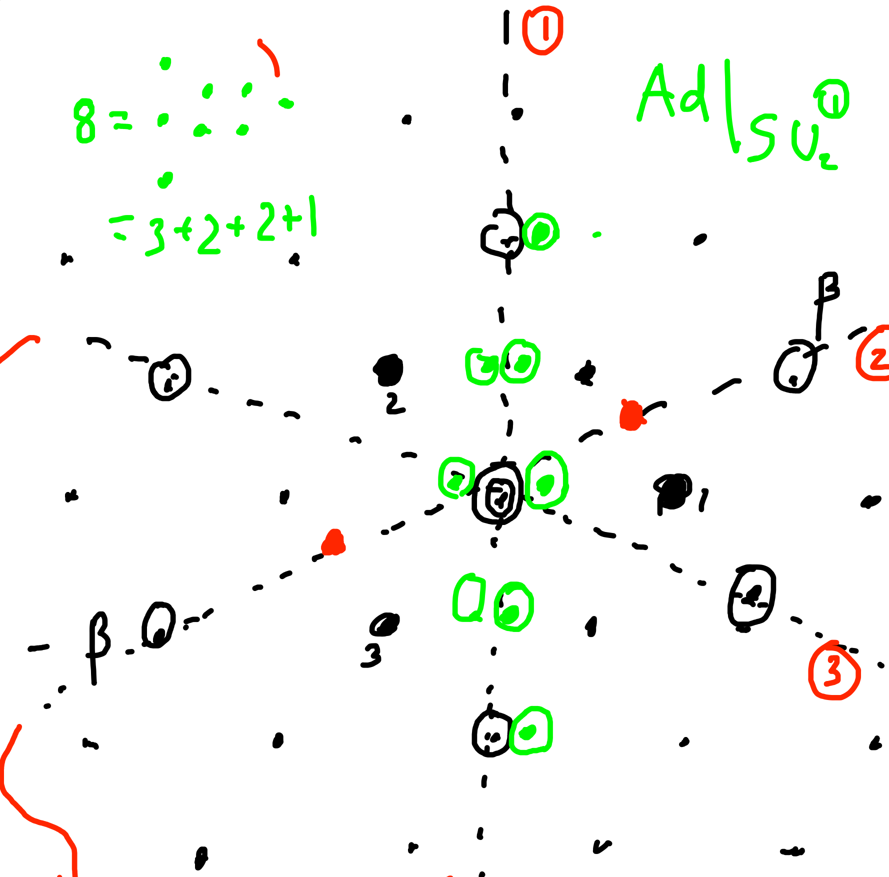
ρ ← rep'n de SU_3.

copia de SU_2 → Sea $G \subset SU_3$ un subgrupo

$$\begin{array}{ccc} G & \subset & SU_3 \\ \cup & & \cup \\ H_1 & \subset & H \end{array}$$

copia de U_1 → H_1

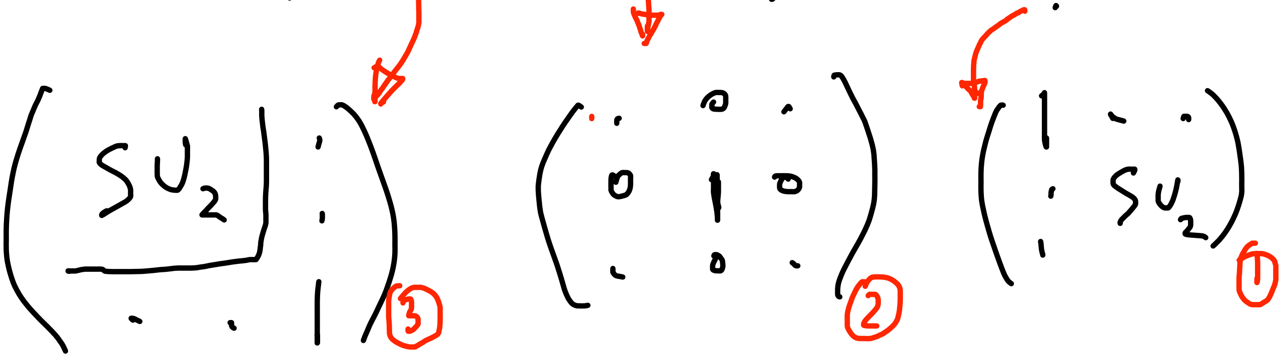
$\Omega(\rho|_G) \subset \mathfrak{h}_1^*$ ← 1-dim.



$[X_\beta, X_{-\beta}] = Y_\beta \in \mathfrak{h}$

$X_\beta, X_{-\beta}, Y_\beta$

genera un subalg. (por def).



3 copias de SU_2 en SU_3.

Sea $\alpha \in \Omega(\rho) \subset \mathfrak{h}^*$, $V = \bigoplus_{\alpha} V_\alpha$

$\mathfrak{h}$ eigen

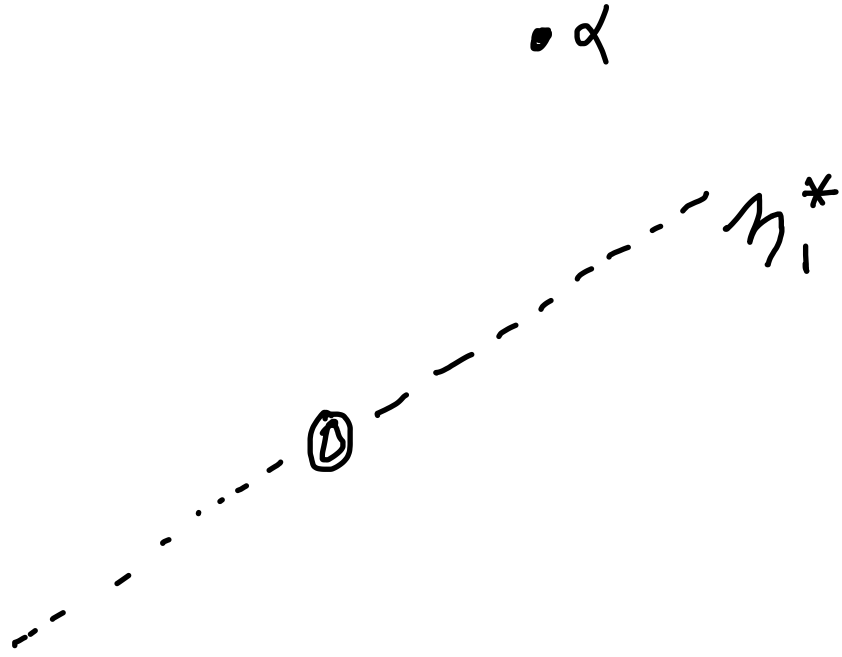
$\Omega(\rho|_G)$

$H_1 \subset H$

$v \in V_\alpha \iff \forall Y \in \mathfrak{h}, \; Y \cdot v = i\alpha(Y) v$

$\mathfrak{h} \supset \mathfrak{h}_1$

$\therefore \quad \Omega(\rho|_G) = \{ \alpha|_{\mathfrak{h}_1} \mid \alpha \in \Omega(\rho) \}$



ej: $\alpha|_{\mathfrak{h}_1}$ es la proy. ortog de α sobre $\mathfrak{h}_1^*$