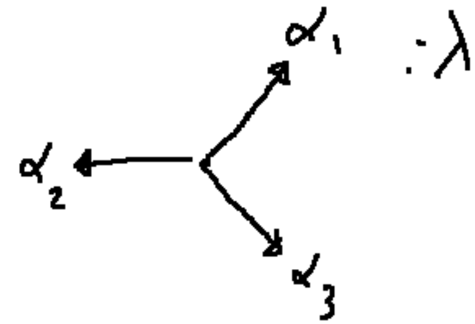
Representaciones de SU_3. "Toro max"

- $H = \left\{ \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix} \middle| \sum \alpha_i \equiv 0 \pmod{2\pi} \right\}$.

- $\mathfrak{h} \approx \mathbb{R}^2 \rightsquigarrow \mathfrak{h}^*$

$$\mathfrak{h} \ni X = \begin{pmatrix} i\alpha_1(X) & & \\ & i\alpha_2(X) & \\ & & i\alpha_3(X) \end{pmatrix}$$



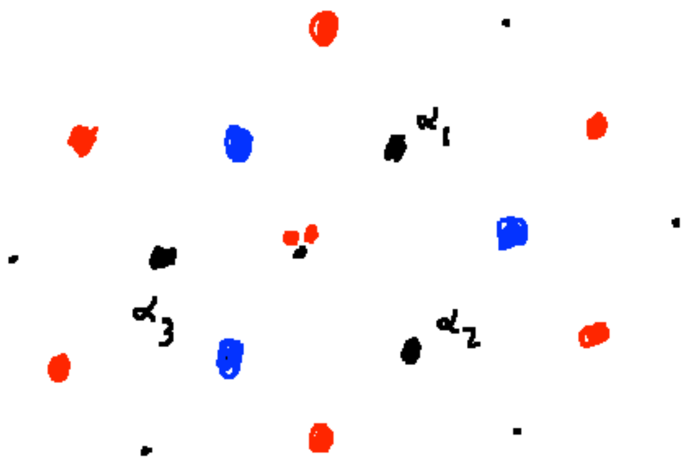
- $G \xrightarrow{\rho} GL(V) \xRightarrow{\rho|_H} V = \bigoplus_{\alpha \in \Omega(\rho)} V_\alpha$, V = rep / $\mathbb{C}$ 1-dim de H

rep 1-dim de $H \rightsquigarrow$ rep 1-dim $\mathfrak{h} \rightsquigarrow$ func lin $\alpha \in \mathfrak{h}^*$

$X \cdot v = i\alpha(X)\, v$ $\qquad \rho'|_{\mathfrak{h}} : \mathfrak{h} \to End(V) = \mathbb{C}$

$v \in V_\alpha \quad \alpha \in \mathfrak{h}^*$

La retícula de los pesos



La rep'n estandar de SU_3 en $\mathbb{C}^3$

sistema de pesos?

Rel a H (ver arriba)

es $\Omega(\rho)=\{\alpha_1,\alpha_2,\alpha_3\}$

Pi: si el peso de una rep 1-dim V es $\alpha\in\mathfrak{h}^*$
cual es el peso de la rep dual en V^*?

Resp: $-\alpha$.

$$X=\begin{pmatrix} ix_1 & & \\ & ix_2 & \\ & & ix_3 \end{pmatrix}\in\mathfrak{h}. \quad x_i=\alpha_i(X),\ \sum x_i=0$$

$$e^X\in H$$

$$\bar{X}=\begin{pmatrix} -ix_1 & \\ & \ddots \end{pmatrix}$$

$$\left[(e^X)^t\right]^{-1}=e^{\bar{X}}$$

La rep adjunta (complexificada).

$[X, Y] = \alpha(X)Y$, $\quad \mathfrak{g}\otimes\mathbb{C} = \bigoplus \mathfrak{g}_\alpha$

$X \in \mathfrak{h}$, $\quad Y \in \mathfrak{g}\otimes\mathbb{C}$

· pesos de la rep'n adjunta se llaman "raíces".

Hecho: los pesos no nulos de una rep. real complexificada de $H = S^1\times S^1$ vienen en pares antípodas $\alpha, -\alpha$

$SO_2 \circlearrowright \mathbb{R}^2$ $\quad \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$ $\quad \begin{pmatrix} e^{i\theta} & \\ & e^{-i\theta}\end{pmatrix}$

Ej.

$\alpha_2-\alpha_1 \qquad \alpha_1-\alpha_2 \qquad \alpha_1-\alpha_3 \qquad \alpha_2-\alpha_3$

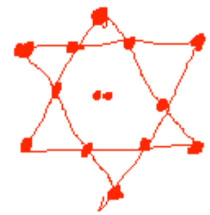
$$Y = \begin{pmatrix} \cdot & \boxed{\cdot} & \boxed{\cdot} \\ \cdot & \cdot & \boxed{\cdot} \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$Y \in su_3\otimes\mathbb{C} = sl_3(\mathbb{C})$

$E_{ij} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot\, 1 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$ 1 en lugar ij, resto 0.

$$\left[\begin{pmatrix} h_1 & & \\ & h_2 & \\ & & h_3\end{pmatrix}, E_{12}\right] =$$

Diagramas de Raíces de grupos de Lie compactos simples de rango 2 = dim toro max



SU_3 Sp_4 SO_5 $G_2 \subset SO_7$

8   10   14

Otra rep de SU_3 $\mathbb{C}^3$, $\overline{\mathbb{C}}^3$

$$\mathbb{C}^3 \otimes \mathbb{C}^3 = \bigoplus_{i,j} V_{\alpha_i+\alpha_j}$$

$$\|$$

$$(V_{\alpha_1} \oplus V_{\alpha_2} \oplus V_{\alpha_3}) \otimes (\qquad) = \bigoplus \underbrace{\overset{v \in}{V_{\alpha_i}} \otimes \overset{w \in}{V_{\alpha_j}}}$$

$X \cdot v = \alpha(X) v$

$X \cdot w = \beta(X) w$

$X \cdot (v \otimes w)$

$= Xv \otimes w + v \otimes Xw = (\alpha+\beta)(X) v \otimes w$