Repns de SU_3, $\mathfrak{su}_3$, $H=\left\{\begin{pmatrix} * & & 0\\ & * & \\ 0 & & *\end{pmatrix}\right\} \subset SU_3$

- $\forall g \in SU_3$ es conjugado a H
- toda rep'n de SU_3 está determinada por su caracter

$\Rightarrow$ basta encontrar las restricciones de rep $\rho|_H$ -

Esta restricción está dada por $\Omega(\rho) \subset \mathfrak{h}^*$ $\leftarrow$ 2-dim

$\uparrow$ sistema de pesos de ρ.

(ρ, V) rep. compleja de G

$\Rightarrow \rho|_H$, $V = \bigoplus_\alpha V_\alpha$, $\dim V_\alpha = 1$, V_α = vectores de peso α.

$\underset{2}{\mathfrak{h}} \subset \underset{8}{\mathfrak{su}_3}$ $\leftarrow$ tiene un prod. int. natural, la forma Killing

$\langle X,Y\rangle = tr(ad\, X \circ ad\, Y)$ es no-deg., pos def., Ad-inv

el único tal prod. int (mod const.) $= tr(XY)$

Forma de Killing rest. a $\mathfrak{h}$, $\|X\|^2 = a_1^2+a_2^2+a_3^2$

$$\begin{pmatrix} ia_1 & & \\ & ia_2 & \\ & & ia_3\end{pmatrix} \xmapsto{\alpha_i} a_i$$

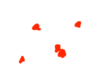
- Que pesos ocurren?

forman una retícula

(Ej 1 para jueves)

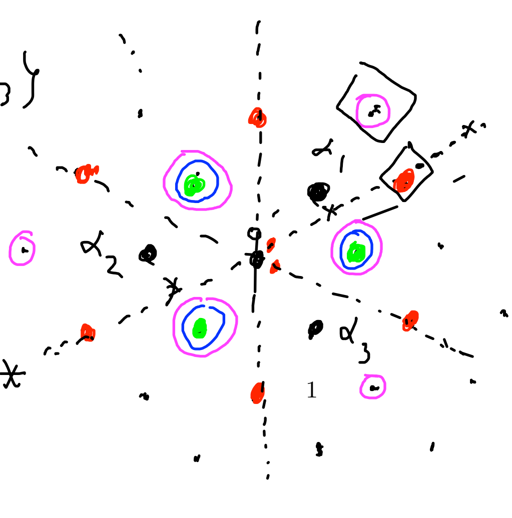
$\Omega(\mathbb{C}^3) = \{\alpha_1, \alpha_2, \alpha_3\}$

$\Omega((\mathbb{C}^3)^*) = \{-\alpha_1, -\alpha_2, -\alpha_3\}$

$\Omega(Ad_{SU_3} \otimes \mathbb{C}) =$ 

$\Omega(\mathbb{C}^3 \otimes \mathbb{C}^3) =$

$= \Omega(S^2\mathbb{C}^3) \sqcup \Omega(\Lambda^2\mathbb{C}^3)$



$Ad_{SU_3} \otimes \mathbb{C} = \mathfrak{sl}_3(\mathbb{C}) \subset End(\mathbb{C}^3) = \mathfrak{sl}_3(\mathbb{C}) \oplus \mathbb{C}I = \mathbb{C}^3 \otimes (\mathbb{C}^3)^*$

Parece: $\Lambda^2\mathbb{C}^3 \simeq (\mathbb{C}^3)^*$

$\uparrow$ rep'n de SU_3

$v \in V_\alpha$ $\qquad X \cdot v = \alpha(X)\, v$, $\quad X \cdot_{\rho'} (v \otimes w) = \frac{d}{dt}\Big|_{t=0} \rho(h(t))[v \otimes w]$

$w \in V_\beta$ $\qquad X \in \mathfrak{g}$

$$= \frac{d}{dt}\Big|_{t=0} [h(t)v \otimes h(t)w]$$

$X \in \mathfrak{g}$, $h(t)$, $h(0) = I \in H$, $\dot{h}(0) = X$

$$= (X \cdot v) \otimes w + v \otimes Xw$$

$$\rho'(X) = \frac{d}{dt}\Big|_{t=0} \rho(h(t)) : \quad h(t)\; v \otimes w$$

$$= (\alpha(X) + \beta(X))\, v \otimes w$$

$$= (\alpha + \beta)(X)\, v \otimes w$$

<u>Resumen</u> $\Omega(\rho_1 \otimes \rho_2) = \Omega(\rho_1) + \Omega(\rho_2) = \{ \alpha + \beta \mid \alpha \in \Omega(\rho_1),\ \beta \in \Omega(\rho_2) \}$.

$$\mathbb{C}^3 \otimes \mathbb{C}^3 = \underbrace{S^2(\mathbb{C}^3)}_{6} \oplus \underbrace{\Lambda^2(\mathbb{C}^3)}_{3}$$

$\uparrow$

rep'n estandar de SU_3 $\underbrace{\qquad}_{9}$

$V \otimes V = S^2(V) \oplus \Lambda^2(V)$

$\uparrow$

rep. de $GL(V)$

$\sigma(v_1 \otimes v_2) = v_2 \otimes v_1$

$g(v_1 \otimes v_2) = gv_1 \otimes g v_2$

$g\sigma = \sigma g$.

$v \otimes w + w \otimes v = v \odot w + v \wedge w$

$$\Omega(V \otimes V) = \Omega(V) + \Omega(V)$$

$$= \Omega(S^2(V)) \sqcup \Omega(\Lambda^2(V))$$

$$\Omega(V \oplus W) = \Omega(V) \sqcup \Omega(W)$$

$\Omega(S^2(V)) = \{\alpha_i + \alpha_j \mid i \leq j\}$ $\qquad v_i \odot v_j = (v_i \otimes v_j + v_j \otimes v_i)/2$

$\Omega(V) = \{\alpha_1, \alpha_2, \dots\}$

$\Omega(\Lambda^2(V)) = \{\alpha_i + \alpha_j \mid i < j\}$ $\qquad v_i \wedge v_j = (\cdots - \cdots)/2$

$\Omega(\Lambda^2(\mathbb{C}^3)) \doteq \{\alpha_1 + \alpha_2, \alpha_2 + \alpha_3, \alpha_1 + \alpha_3\}$

$\Omega(S^2(\mathbb{C}^3)) = \{ \qquad \} \cup \{2\alpha_1, 2\alpha_2, 2\alpha_3\}$

$\Lambda^2(\mathbb{C}^3) \simeq (\mathbb{C}^3)^* \iff \Lambda^2(\mathbb{C}^3) \times \mathbb{C}^3 \to \mathbb{C}$

$vol \in \Lambda^3((\mathbb{C}^3)^*)$ $\qquad (v_1 \wedge v_2, v_3) \longrightarrow \underline{\underline{vol}}(v_1 \wedge v_2 \wedge v_3)$

$\quad \| $

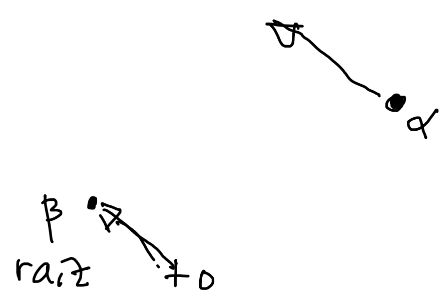
$(\Lambda^3(\mathbb{C}))^*$

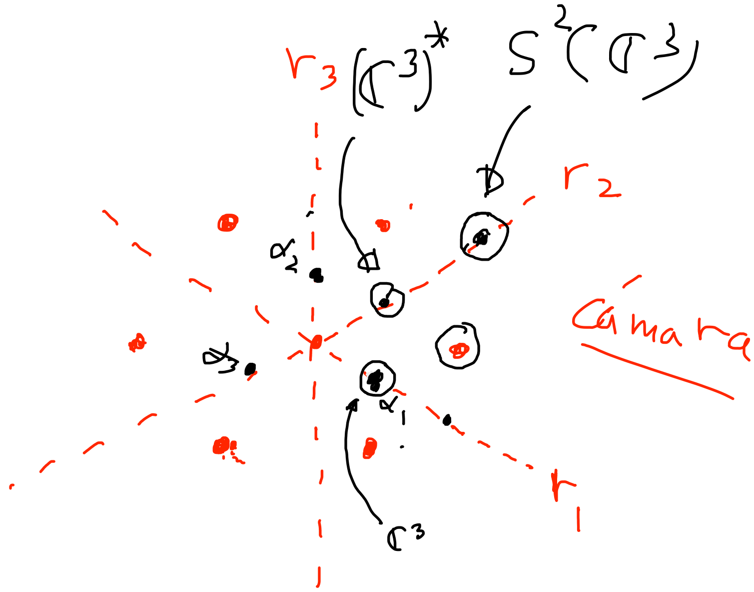
$$SL_3(\mathbb{C}) \supset SU_3$$

Hecho (ejr.) $v \in V_\alpha$

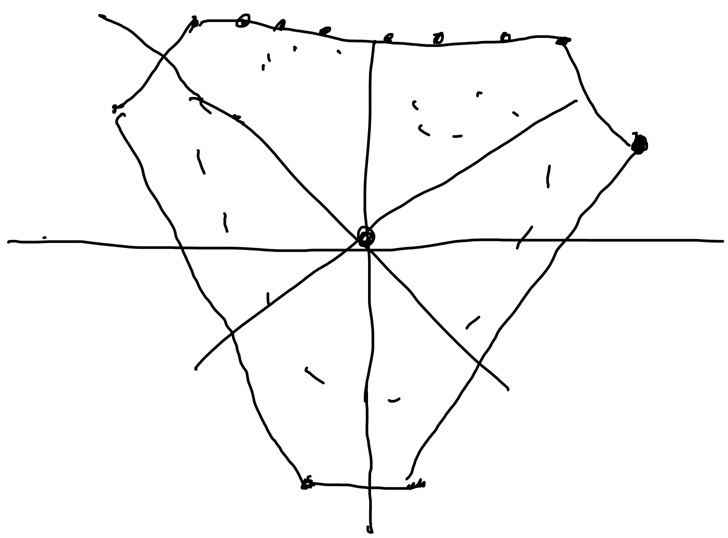
$X \in \mathfrak{su}_3$ de raíz β

$\Rightarrow X \cdot v = 0$ ó $\neq 0$ de peso $\alpha + \beta$





$$W = \langle r_1, r_2, r_3 \rangle \subset O_2$$



La retícula de pesos de $H \cong S^1 \times S^1$

Empezamos con $\underline{\underline{S^1}}$

$$\rho : S^1 \to GL(\mathbb{C})$$

$$\rho_n = (\rho_1)^{\otimes n}$$

$$\rho : S^1 \times S^1 \to GL(\mathbb{C}), \quad \boxed{\rho_{m,n} = \rho_n \boxtimes \rho_m}$$

$$\rho' : \mathbb{R}^2 \to \mathbb{C}$$

$$\rho_{m,n}(e^{i\theta_1}, e^{i\theta_2}) = e^{i(m\theta_1 + n\theta_2)}$$

$$\rho'(X) \rightsquigarrow \alpha(x,y) = mx + ny$$