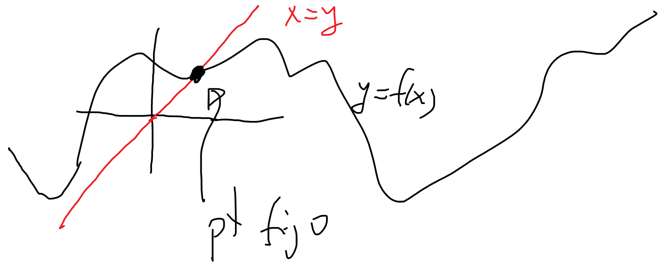
Top. dif.

Friday, September 18, 2020 12:34 PM

10. Let $f: X \to X$ be a map with fixed point x; that is, $f(x) = x$. If $+1$ is not an eigenvalue of $df_x: T_x(X) \to T_x(X)$, then x is called a *Lefschetz fixed point* of f. f is called a *Lefschetz map* if all its fixed points are Lefschetz. Prove that if X is compact and f is Lefschetz, then f has only finitely many fixed points.



p.f. x es lefschetz $df_x : T_xX \to T_xX$ si

$\Delta(X) = \{(x,x) \mid x \in X\} \subset X \times X = \text{graph}(id_X)$.

1 no es valor propio de df_x.

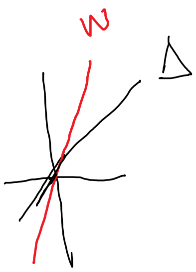
$\Updownarrow$?

$\text{graph}(f) \pitchfork \Delta(X)$

(9) es esto ? para f lineal. $A: V^n \to V$, $\underset{W}{\underset{\shortparallel}{\text{graph}(A)}} \pitchfork \Delta \iff 1$ no es e.v. de A.

$\underbrace{(v,Av)}_{q} \in W \cap \Delta \ni 0$, $v = Av$,

$\underset{\underset{n}{W}}{\underset{\shortparallel}{T_qW}} + \underset{\underset{n}{\Delta}}{\underset{\shortparallel}{T_q\Delta}} = V \oplus V \iff q$ es un pto de int $\pitchfork$



$W \cap \Delta = \{0\}$

$q \neq 0 \Rightarrow \mathbb{R}q \subset W \cap \Delta$

$X \xrightarrow{f} Y$

$\text{graph}(f) \pitchfork X$

$V_1, V_2 \subset V$ $\quad V_1 \pitchfork V_2$?

nec: $\dim V_1 + \dim V_2 \geq \dim V$.

si f: $\dim V_1 + \dim V_2 = \dim V$

y $V_1 \cap V_2 = \{0\}$

(8)

si $q \in V_1 \cap V_2$, $q \neq 0$

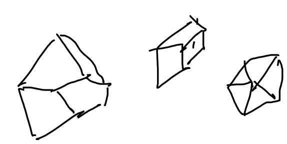
$\mathbb{R}q \subset V_1 \cap V_2$

$\Rightarrow \dim V_1 + V_2 \leq \dim V_1 + \dim V_2 - 1$

$\#(C_1 \cup C_2) = \#C_1 + \#C_2 - \#(C_1 \cap C_2)$

$\dim(V_1 + V_2) = \dim V_1 + \dim V_2 - \dim(V_1 \cap V_2)$

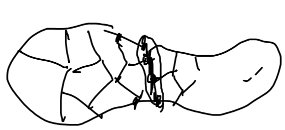
$\chi(X \cup Y) = \chi(X) + \chi(Y) - \chi(X \cap Y)$

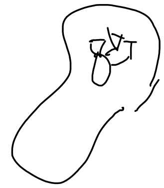


$5 - 10 + 6 = 1$

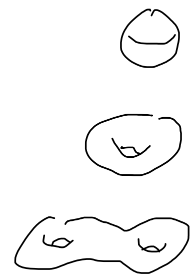
$\chi = C - A + V$

χ





plano Y



1

2

0

−2

X cpct $\#\big(\overbrace{\text{graph}(X) \cap \Delta}^{I}\big) < \infty$

int. transv.

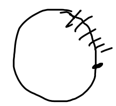
• TFI $\Rightarrow$ I subvar. de $X \times X$ de dim = 0

$\mathbb{Z} \subset \mathbb{R}$

dim = 0 y $\# \mathbb{Z} < \infty$

~~Prop: X compacto $Y \subset X$ de dim 0 $\Rightarrow$ Y es finito.~~

$\left\{ \frac{1}{n} \,\middle|\, \cancel{n = 1, 2, 3, \dots} \right\} \subset$



La intersección $\text{graph}(f) \cap \Delta$ es finita

si no, hay punto de acumulación en ?

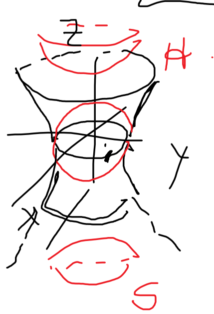
$f: X \to X$

$\Delta \subset X \times X$

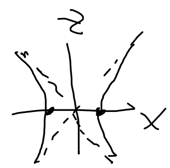
$\boxed{I \text{ es compacta}}$, var. de dim 0, $\Rightarrow$ pts aislados,

de la hoja 6H,

8. For which values of a does the hyperboloid defined by $x^2 + y^2 - z^2 = 1$ intersect the sphere $x^2 + y^2 + z^2 = a$ transversally? What does the intersection look like for different values of a?



$y^2 - z^2 = 1$



$a < 0$ $S = \emptyset$.

$a = 0$ $S = \{0\}$ $I = \emptyset$

$a < 1$ $\emptyset$

$a = 1$, I = círculo $= \{x^2 + y^2 = 1, z = 0\}$

$X, Y \subset Z$,

$z \in X \cap Y$,

X y Y son tang. en z

si $T_z X \subset T_z Y$ o v.v.

si dim X = dim Y

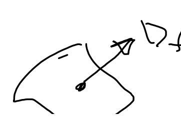
tang en z $\Leftrightarrow$ $T_z X = T_z Y$

$X = f^{-1}(0)$, $Y = g^{-1}(0)$, $f, g: \mathbb{R}^3 \to \mathbb{R}$

$z \in X \cap Y$ $T_z X = T_z Y$

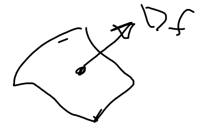
ssi $df_z = \lambda \, dg_z$, para un $\lambda \neq 0$.

$T_z X = (df_z)^0$



ssi $df_z = \lambda\, dg_z$, para um $\lambda \neq 0$. $T_z X = (df_z)^\circ$

$T_z Y = (dg_z)^\circ$



$f = x^2 + y^2 - z^2 - 1$, $df = 2x\,dx + 2y\,dy - 2z\,dz$

$g = x^2 + y^2 + z^2 - a$, $dg = 2x\,dx + 2y\,dy + 2z\,dz$

Next: Teo. de Sard