Top. dif

Wednesday, September 23, 2020 12:34 PM

prob 8,

$$\underset{\text{cpct}}{X} \xrightarrow[f]{} \underset{\text{cpct}}{Y} \qquad f = \text{difeo}$$

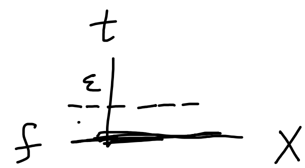
$F: X \times [0,1] \to Y$ suave, $F(x,0) = f(x)$

$\cap$

$\mathbb{R}^m \times \mathbb{R} = \mathbb{R}^{m+1}$

$\;\;\psi \quad\;\; \psi$

$\;\;x \quad\;\; t$



P.D.: $\exists \varepsilon > 0$ t.q. $\forall\, t \in [0, \varepsilon]$

$f_t = F(\cdot, t) : X \to Y$ es difeo.

paso 1 X conexo. $\Rightarrow Y$ conexo

$\exists \varepsilon > 0$ t q $f_t : X \to Y$ es difeo loc $\forall t \in [0, \varepsilon]$.

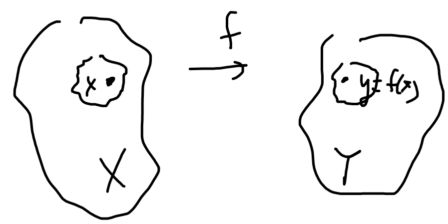
paso 2: f_t es iny + sobre

- $f_t(X)$ es ab. en Y.
- X cpct, f_t cont $\Rightarrow f_t(X)$ es cpct $\Rightarrow$ cerrado en Y. $K \subset Y$

$\therefore$ $f_t(X)$ es ab + cerr. en Y, Y conex. $\Rightarrow f_t(X) = Y$.

Ej §3.10

TFI



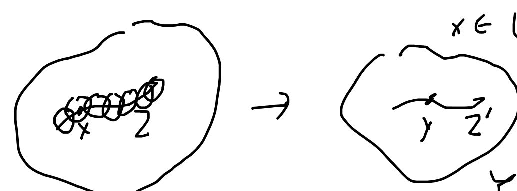
$df_x : T_x X \to T_y Y$ (iso)

$\Downarrow$

$f|_U : U \to V$ difeo

$x \in U$, $y \in V$

TFIG ?



$df_x : T_x X \to T_{f(x)} Y$ $\forall x \in Z$.

$X = \text{cpct}?$

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① $X = \sqcup X$ ... $Y = \sqcup Y$ ... $f: X \to Y$ ...

$X = \text{cpct}!$ $\quad X \quad \quad Y \quad \forall x \in Z.$

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(1) $X = \coprod_{\alpha \in A} X_\alpha \qquad Y = \coprod Y_\alpha$

$A = X/\sim$

$f: X \to Y$ homeo.

$\Rightarrow f(\text{comp. conexa}) = \text{comp. conexa}$

$\Rightarrow \tilde{f}: X/\sim \xrightarrow{\text{biy}} Y/\sim$

(2) sea $f: X \times [0,1] \to Y$ cont.

$X_1 \subset X$ comp. $\overset{\text{arco}}{\text{conexa}}$ (← def?)

$f_0(X_1) \subset Y_1 =$ comp. $\overset{\text{arco}}{\text{conexa}}$. de Y.

$\Rightarrow f_t(X_1) \subset Y_1$.

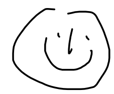
$X_1 \bigcirc \xrightarrow{\sim} \bigcirc Y_1$

$X_2 \bigcirc \xrightarrow{\sim} \bigcirc Y_2$

arc. con. $\qquad$ no-con. $\qquad$ ac $\Rightarrow$ con.

no pasa en ver.

5 MNTS → 1142.



$A: S^{2n} \to S^{2n}$, $\quad A(x) = -x$

no es homótopica a la id.

Porqué?

$F: S^{2n} \times I \xrightarrow{\text{cont}} S^{2n}$

$F_0 = A, \quad F_1 = id.$

grado(f)

$\boxed{\pi_1(S^1) = \mathbb{Z}}$ $\qquad \forall K$

$\pi_1(X)$

$S^1 \to X$

- $\pi_k(X) =$ clases homot. $S^k \to X$.

$\pi_2, \pi_3, \ldots$

$X \leadsto \pi_k(X)$

$f \sim g$

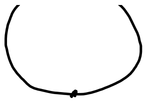
$f: X \to Y \qquad f_*: \pi_k(X) \to \pi_k(Y)$

$g_* = f_*: \pi_1(X) \to \pi_1(Y)$

$S^n \qquad n > 1$

$A_*: \pi_n(S^n) \to \pi_n(S^n)$

$\quad \parallel \qquad \qquad \parallel$



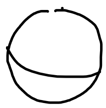
$A_*: H_n(\ ) \to H_n(\ )$

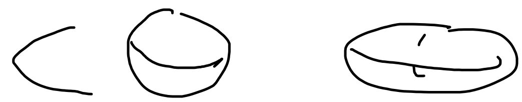
$\qquad\qquad\ \ \| \qquad\qquad \|$

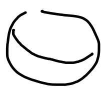
$\qquad\qquad\ \ \mathbb{Z} \qquad\qquad \mathbb{Z}$

$S^{2n+1} \subset \mathbb{R}^{2n} = \mathbb{R}^2 \oplus \mathbb{R}^2$ | $A_* = (-1)^{n+1}$

La inversion de S^2 segun Smale

 $\xrightarrow{i_0} \mathbb{R}^3$



 $\xrightarrow{i_1} \mathbb{R}^3$ "alreves"

$i_0 \overset{\text{isotopico}}{\sim} i_1$? Si