***5.** Prove that intersection theory is vacuous in contractable manifolds: if Y is contractable and $\dim Y > 0$, then $I_2(f, Z) = 0$ for every $f: X \to Y$, X compact and Z closed, $\dim X + \dim Z = \dim Y$. (No dimension-zero anomalies here.) In particular, intersection theory is vacuous in Euclidean space.

***6.** Prove that no compact manifold—other than the one-point space—is contractable. [HINT: Apply Exercise 5 to the identity map.]

***7.** Prove that S^1 is not simply connected. [HINT: Consider the identity map.]

Si S^1 es 1-conexo

$\boxed{id_{S^1} \sim 1}$

1-conexo

$Z = \{1\} \subset S^1$.

$X = Y = S^1$.

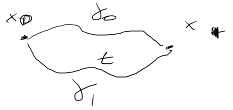
$f: S^1 \xrightarrow{id} S^1$

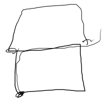
1-conexo (esp. top.):

- arco-conexo (para var $\iff$ conexo)

- $\gamma_0, \gamma_1 : [0,1] \to X$

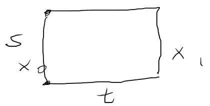
  $\gamma_0(i) = \gamma_1(i) = x_i$, $i = 0,1$

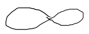




$\implies \exists\, F : \underset{s}{[0,1]} \times \underset{t}{[0,1]}$

$F(i, \cdot) = \gamma_i$

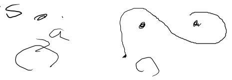


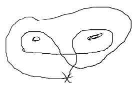


En particular: si γ_0, γ_1 son cerrados, i.e. $x_0 = x_1$, $F(s, \cdot) = \gamma_s$ es cerrado

y $\gamma_s(0) = \gamma_s(1) = x_0$, $\forall s$.

Hecho: homotopía "libre" $\not\sim$ homot. con punto base





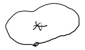
$\mathbb{Z} * \mathbb{Z}$

$ab = ba$

$aba^{-1}b^{-1}$

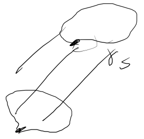
Pero: para el círculo es lo mismo.

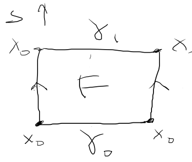
$S^1, S^3 < \mathbb{H}$

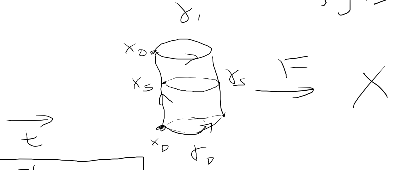


$\gamma_0 \underset{\text{libre}}{\sim} \gamma_1 \implies \gamma_0 \sim \gamma_1$ con puntos bases fijos

cerrados







$\boxed{\tilde{F}(s,t) = (x_s)^{-1} F(s,t)}$

$\gamma_s(0) = x_s$

Conclusión (ej 7): S^1 no es 1-conex.

tenemos 2 def. de "1-conexo"

- G&P
- la estándar (Wiki)

. Según G&P: si fuera 1-conexo $\implies id_{S^1} \overset{lib}{\sim} 1$

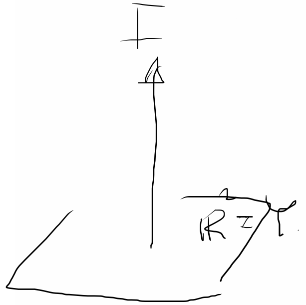
pero S^1 es comp. $\overset{(6)}{\implies}$ imposible.

. según la def estándar, si fuera 1-conexo $\implies id_{S^1} \overset{\text{base fijo}}{\sim} 1$

~~esa homot. es def.~~ existe homot. suave que hace lo mismo $\overset{(6)}{\implies}$ imposible.

(def estudiar $X \sqcup Z = \partial W$)

***14.** Two compact submanifolds X and Z in Y are *cobordant* if there exists a compact manifold with boundary, W, in $Y \times I$ such that $\partial W = X \times \{0\} \cup Z \times \{1\}$. Show that if X may be deformed into Z, then X and Z are cobordant. However, the "trousers example" in Figure 2-17 shows that the converse is false.



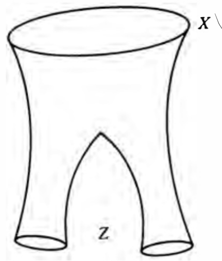


Figure 2-17

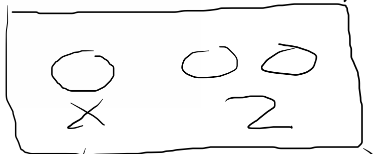
$Y = \mathbb{R}^2$

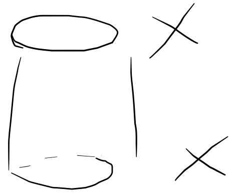
$X \overset{cob.}{\sim} Z$

$X \sim 0$?

$X = \partial W$

$X + X \sim 0$





$[X] + [Y] = [X \sqcup Y]$

Thom: calculó el grupo (Milnor) de cobordismo (wikipedia!)

Def: X se puede deformar en Z

$X \xhookrightarrow{z_0} Y$

$z_s : X \xrightarrow{enc.} Y$

$z_1(X) = Z$

$X \overset{difeo.}{\approx} Z$

$z : X \times I \to Y$.

$W \subset Y \times I$

X

$X \times I \longrightarrow Y \times I$

$(x, s) \longmapsto (z(x, s), s)$

$W = \{(z(x,s), s) \mid x \in X,\ s \in [0,1]\} \subset Y \times I$

P.D. • $W \subset Y \times I$ es una subvar. (con ∂)

• $\partial W = X \times \{0\} \cup Z \times \{1\}$.

$\boxed{\partial(X \times Y) = X \times \partial Y}$

8

$$\begin{tikzcd} \mathbb{R} \arrow[r] \arrow[d, "\exp"'] \arrow[dr, phantom, "\circlearrowright"] & \mathbb{R} \arrow[d] \\ S^1 \arrow[r] & S^1 \end{tikzcd}
\qquad
\begin{tikzcd} & \mathbb{R} \arrow[d, "\exp"] \\ X \arrow[r] \arrow[ur, dashed, "\exists !"] & S^1 \end{tikzcd}$$

Massey

Singer Thorpe

$$\exp(t) = e^{2\pi i t}$$

Buscar esto "levantamiento"

"lifting" exist + unicidad

Sesión extra (ej 8)

Lunes 1pm, 30 nov.