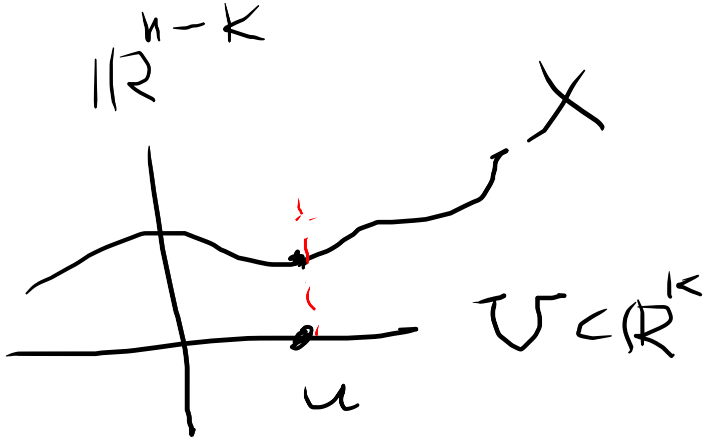
Fuente (mayor) de variedades

= gráficas de funciones, $f : U \to \mathbb{R}^{n-k}$ abierto

$U \subset \mathbb{R}^k$

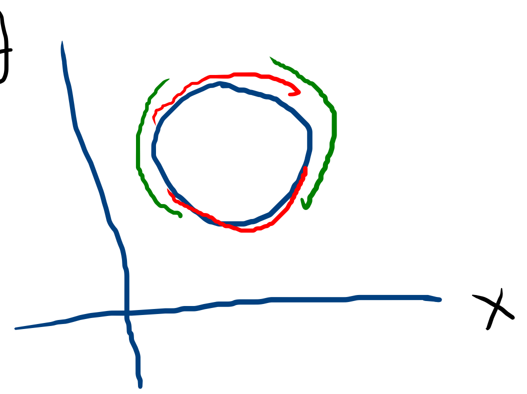
$X = \mathrm{graph}(f) \subset \mathbb{R}^k \times \mathbb{R}^{n-k} = \mathbb{R}^n$



$U \underset{\text{difeo}}{\approx} X$ ✓ $\qquad u \mapsto (u, f(u))$

- No toda variedad se produce así
- lo que sí es cierto es que toda variedad es loc. difeo. a tal variedad

ej.



$$\begin{array}{ccc} x & & y = f(x) \\ \cap & & \cap \\ f : X & \longrightarrow & Y \\ \cap & & \cap \\ \mathbb{R}^n & & \mathbb{R}^m \end{array} \quad \rightsquigarrow \quad df_x : T_xX \longrightarrow T_yY$$

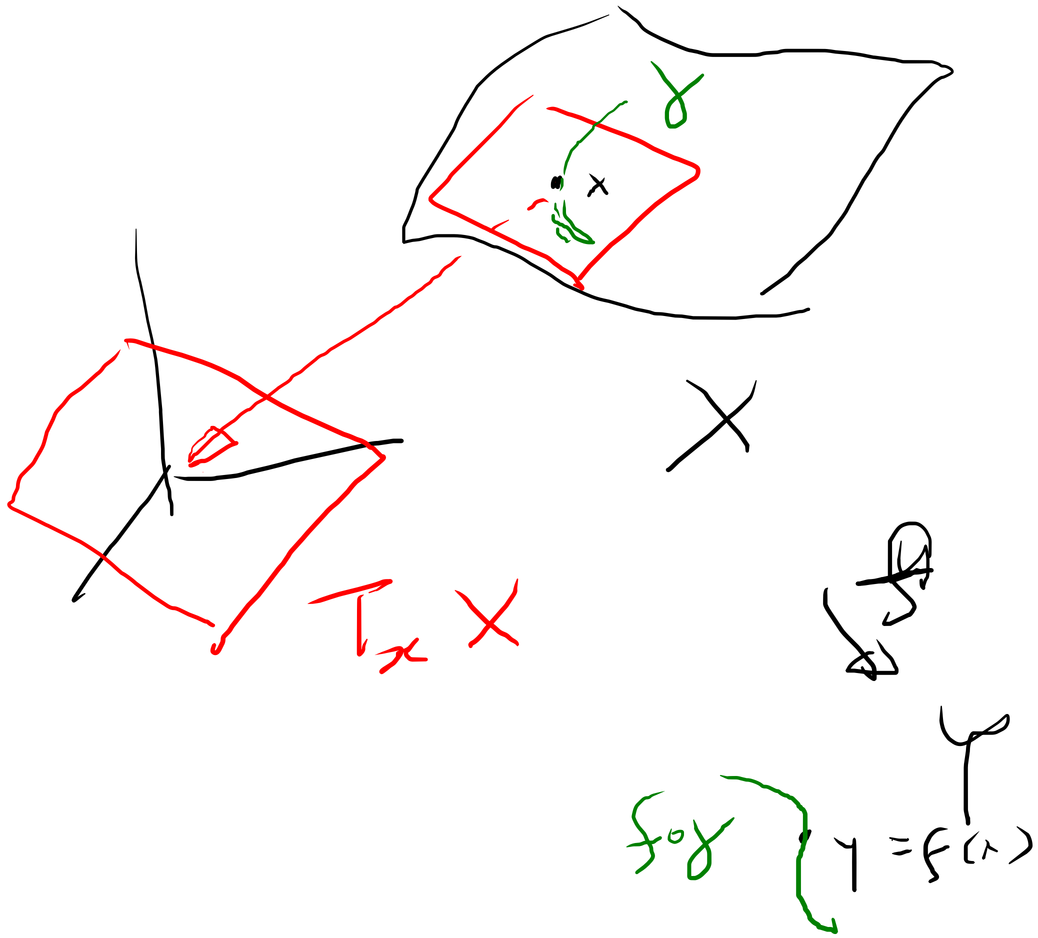
trans f. lin.

$$\boxed{df_x\, v = \dot{(f\circ\gamma)}(0)}$$

$$v \in T_xX$$
$$\shortparallel$$
$$\dot\gamma(0)$$

$$\gamma : (-\varepsilon, \varepsilon) \longrightarrow X \subset \mathbb{R}^n$$

$$f\circ\gamma : (-\varepsilon, \varepsilon) \longrightarrow Y$$



Regla de la cadena "la deriv. de la comp. es la comp. de las derivadas"

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[rr, bend left, "g\circ f"] & Y \arrow[r, "g"] & Z \\ x \arrow[u, phantom, "\in" rotate=90] & y=f(x) \arrow[u, phantom, "\in" rotate=90] & z=g(y) \arrow[u, phantom, "\in" rotate=90] \end{tikzcd}$$

$$\boxed{d(g\circ f)_x = (dg)_y \circ (df)_x}$$

comp

comp.

Aplicación:

- $\boxed{f : X \to Y \text{ difeo.} \implies \dim X = \dim Y}$

$\Uparrow$ Reg. cad

D:

$\boxed{\text{Alg. lin } V \underset{\text{iso. lin}}{\cong} W \iff \dim V = \dim W}$

d: feo

$$\begin{tikzcd} X \arrow[r, bend left, "f"] & Y \arrow[l, bend left, "g = f^{-1}"] \end{tikzcd}$$

$$\begin{cases} g \circ f = id_X \\ f \circ g = id_Y \end{cases}$$

RC

$$df_x \circ dg_y = id_{T_yY}$$

$$dg_y \circ df_x = d(id_X)_x = id_{T_xX}$$

$id_X : X \to X$

$(id_X)_x : T_xX \to T_xX$

$$\boxed{\begin{tikzcd} T_xX \arrow[r, bend left, "df_x"] & T_yY \arrow[l, bend left, "dg_y"] \end{tikzcd}}$$

d

$$d(id_X)_x v = (id_X \circ \gamma)^{\cdot}(0) = \dot{\gamma}(0) = v$$

$\dot{\gamma}(0) = v$

$||$ id_{T_xX}

El TFI (teo. func. implícita/inversa)

→ inmersiones + sumersión (¿)

(immersion) (submersion)

$$\boxed{\begin{array}{c} X \xrightarrow{f} Y \\ x \qquad y = f(x) \\ \text{es difeo} \end{array}} \overset{RC}{\Longrightarrow} \boxed{\begin{array}{c} T_x X \xrightarrow{df_x} T_y Y \\ \forall x \text{ es isom. lin} \end{array}}$$

$$\overset{?}{\Longleftarrow}$$

<u>TFInv</u>: loc. es cierto: df_x es iso. lin. para algun $x \in X$

$\Rightarrow \exists U \ni x$ t.q $f|_U : U \to f(U)$ es difeo.

<u>Note</u>: $\{x \mid df_x \text{ es iso}\}$ es abierto.

$= \{\det \neq 0\}$

Def: difeo loc $f : X \to Y$

$$\forall x \in X,\ \exists U \ni x,\ f|_U : U \to f(U) \underset{}{\overset{ab}{\subset}} Y$$

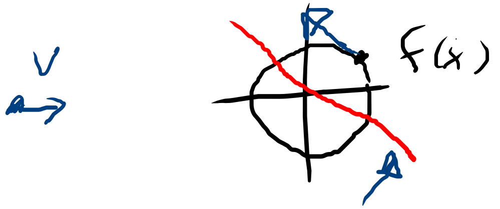
difeo

$$\text{TFI} \quad \Updownarrow$$

$$\forall x \in X \quad df_x \text{ es iso.}$$

Ej: de un difeo local que no es global

$$\mathbb{R} \xrightarrow{f} S^1, \quad f(x) = (\cos x, \operatorname{sen} x)$$



$$f'(x) = (-\operatorname{sen} x, \cos x)$$

$$\gamma(t) = x + tv$$

$$df_x v = (f(x + tv))'\Big|_{t=0} = f'(x)\, v$$

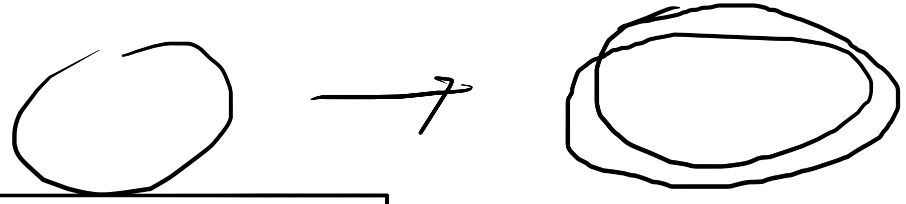
$$\|f'(x)\| = 1$$

$$df_x : T_x\mathbb{R} \to T_{f(x)}\mathbb{R}^2$$

$$v \longmapsto f'(x)\, v$$

• otro ejemplo $S^1 \to S^1$

$z \to z^n$, $z \in S^1 \subset \mathbb{C}$.



Defi:

imersión	df_x	iny.	$\forall x$
Submersión	~	supra.	$\forall x$

• $X^k \subset \mathbb{R}^n$

• $\mathbb{R}^n \to \mathbb{R}^k$ $\quad k \le n$

TF Imp $\xrightarrow[\text{variedad}]{}$ $\{x \mid f(x)=y\} = f^{-1}(y)$