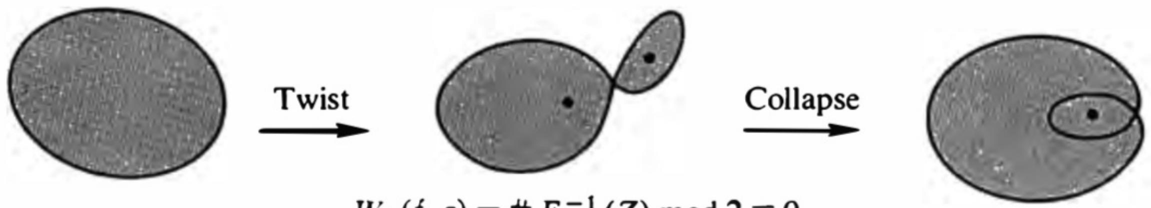
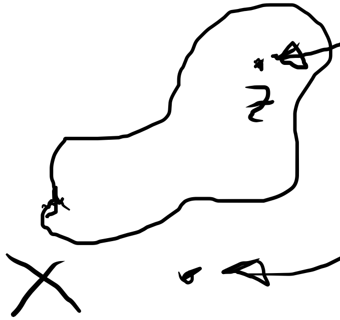
cerrad. de adentro(?)

Theorem. Suppose that X is the boundary of D, a compact manifold with boundary, and let $F: D \longrightarrow \mathbf{R}^n$ be a smooth map extending f; that is, $\partial F = f$. Suppose that z is a regular value of F that does not belong to the image of f. Then $F^{-1}(z)$ is a finite set, and $W_2(f, z) = \# F^{-1}(z) \bmod 2$. That is, f winds X around z as often as F hits z, mod 2. (See Figure 2-21.)



$W_2(f, z) = \# F^{-1}(Z) \bmod 2 = 0$

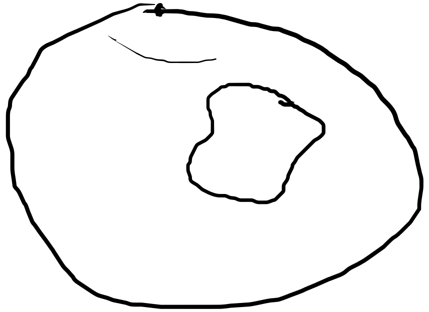
Figure 2-21



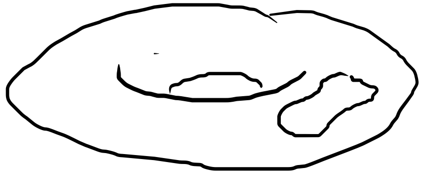
"Adentro" $= \{ z \mid W_2(z, X) = 1 \}$ a. conexo acotado

$z \notin X$

"Afuera" $= \{ \text{— — — —} = 0 \}$ a. conex. no acotado.

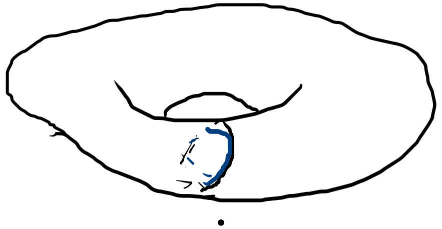


(0, 0)



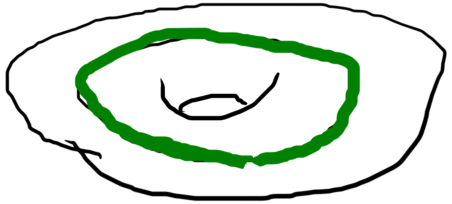
sep.

(1, 0)

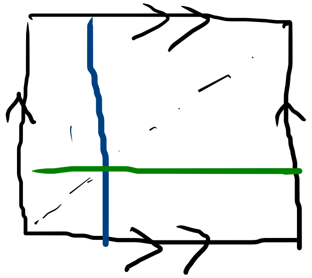


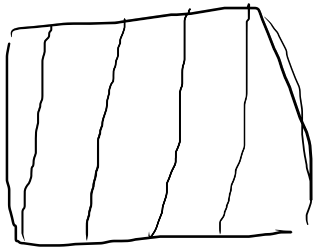
no sep.

(0, 1)

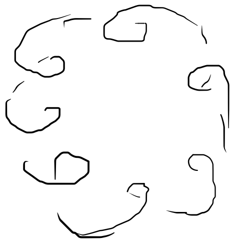


?





(4, 1)



(7,)

Cor del teo de la pag. 87.

Sea $D = \overline{U} \subset \mathbb{R}^n$, U ab. acotado, $\partial D = X$ var. comp. de dim $n-1$.

$\overset{\text{Teo}}{\Rightarrow} \forall z \in D \setminus X$, $W_2(z, X) = 1$

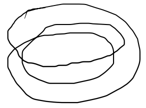
z val. reg de $F : D \to \mathbb{R}^n$

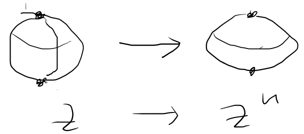
$\#F^{-1}(z) = \#\{z\} = 1 \underset{\text{teo.}}{\equiv_2} W_2(z, X)$.

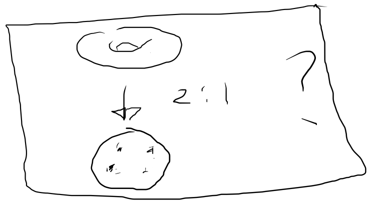
Si $z \notin F(D) \Rightarrow$
$u : X \to S^{n-1}$ se ext. a
$u : D \to S^{n-1}$
$\Rightarrow$ ~~W_2~~ ~~gra~~ $W_2(z, X) = \text{grado}(u) \equiv_2 0$

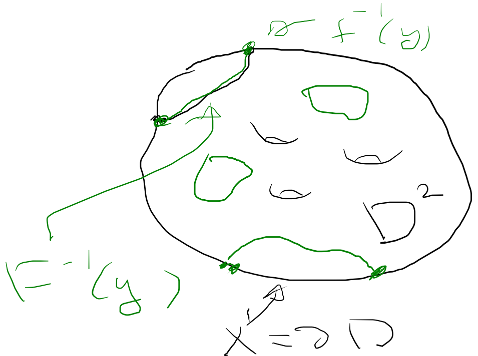
Teo de ~~extensión~~ frontera

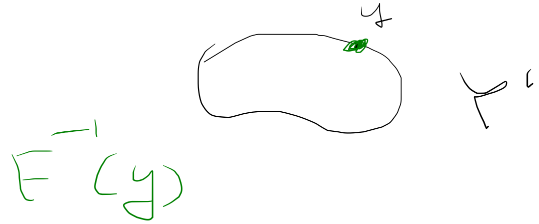
$X^{n-1} \xrightarrow{f} Y^{n-1} \ni y$, $f \Rightarrow F$
$X^{n-1} = \partial D^n$, F
$\Rightarrow \text{grado}_2(f) = 0$.











2. Suppose that $F^{-1}(z) = \{y_1, \ldots, y_l\}$, and around each point y_i let B_i be a ball, (That is, B_i is the image of a ball in $\mathbf{R}^n$ via some local parametrization of D.) Demand that the balls be disjoint from one another and from $X = \partial D$. Let $f_i : \partial B_i \longrightarrow \mathbf{R}^n$ be the restriction of F, and prove that

$$W_2(f, z) = W_2(f_1, z) + \cdots + W_2(f_l, z) \quad \text{mod } 2.$$

(See Figure 2-22.)

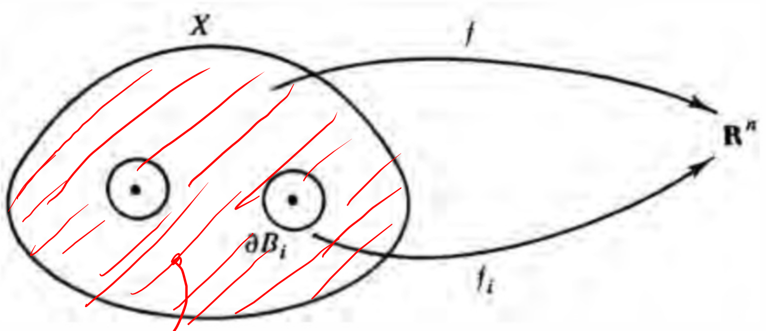


Figure 2-22

$X = X_1 \sqcup X_2$

$\text{grado}(X \to Y) =$
$= \text{grado}(X_1 \to Y)$
$+ \text{grado}(X_2 \to Y).$

$D', \ \partial D' = \underset{\substack{\parallel \\ X}}{\partial D} \cup \left(\bigcup \partial B_i \right), \ F|_{D'}$

$\Longrightarrow 0 = W_2(\partial D', z) = \text{grado}(\partial D' \xrightarrow{u} S^{n-1}) = \text{grado}(X \to S^{n-1})$
$\underset{1}{}$
$+ \sum_i \underbrace{\text{grado}(\partial B_i^n \to S^{n-1})}_{1}$ difeo

$\Longrightarrow \text{grado}(X \to S^{n-1}) \equiv_2 \# F^{-1}(z).$

4. Let $z \in \mathbf{R}^n - X$. Prove that if x is any point of X and U any neighborhood of x in $\mathbf{R}^n$, then there exists a point of U that may be joined to z by a curve not intersecting X (Figure 2-23).

Pista

Teo. de ε-vecindad

$X^\varepsilon \setminus X$ tiene a lo mas 2 comp.

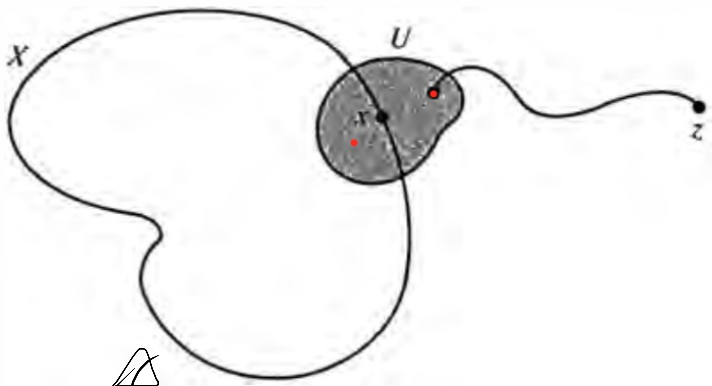
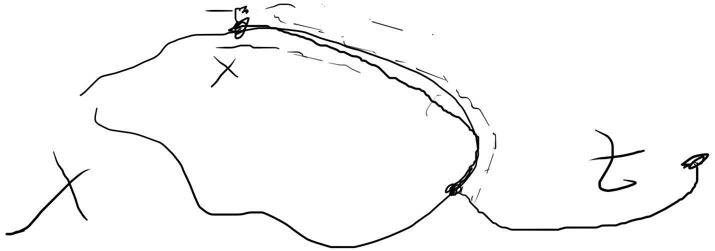
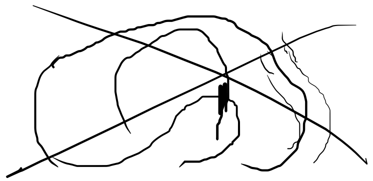


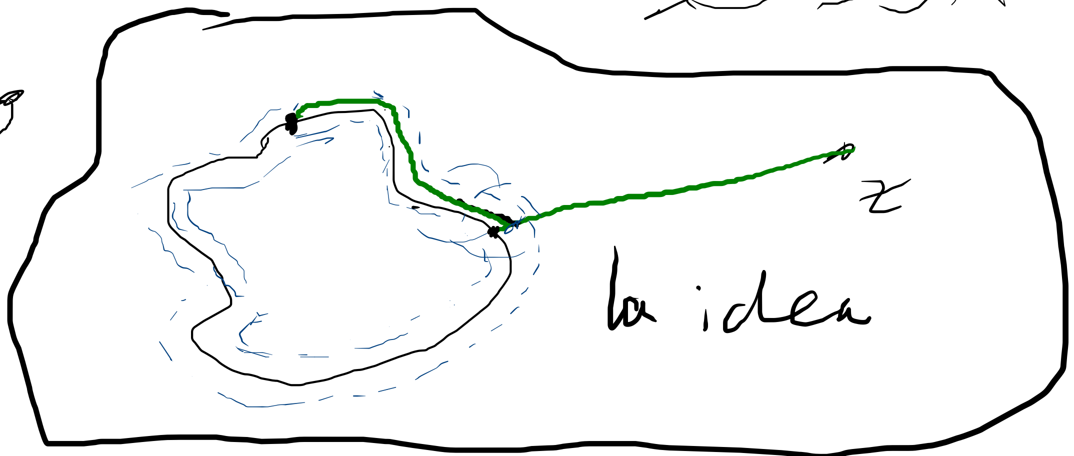
Figure 2-23

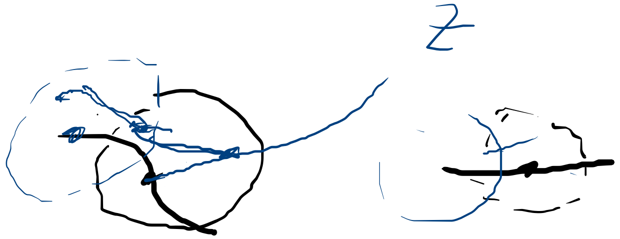
compacta conexa $\quad X \subset X^\varepsilon \quad \pi : X^\varepsilon \to X$

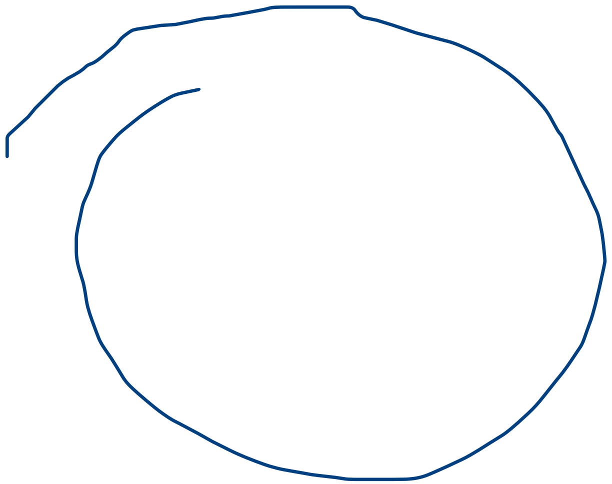
$X \subset \mathbb{R}^n$

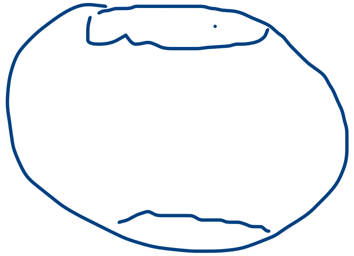






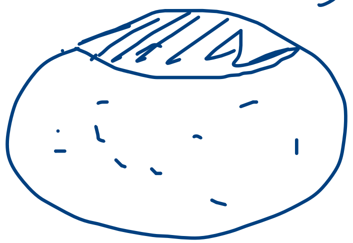




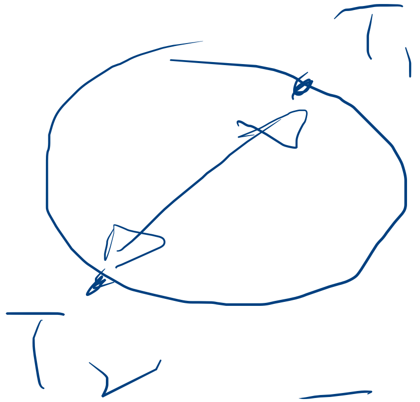


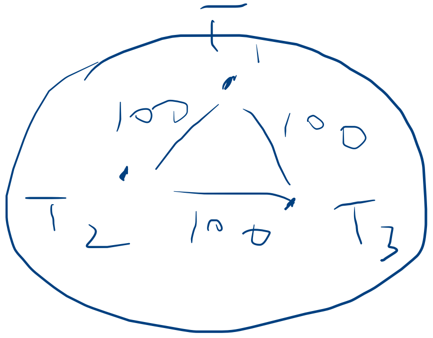
slice cap
"tapa"

area < 1/3



≥ 4 moscas





$T_1 = T_2 = T_3$

ej. $T_1 < T_2$

ej*