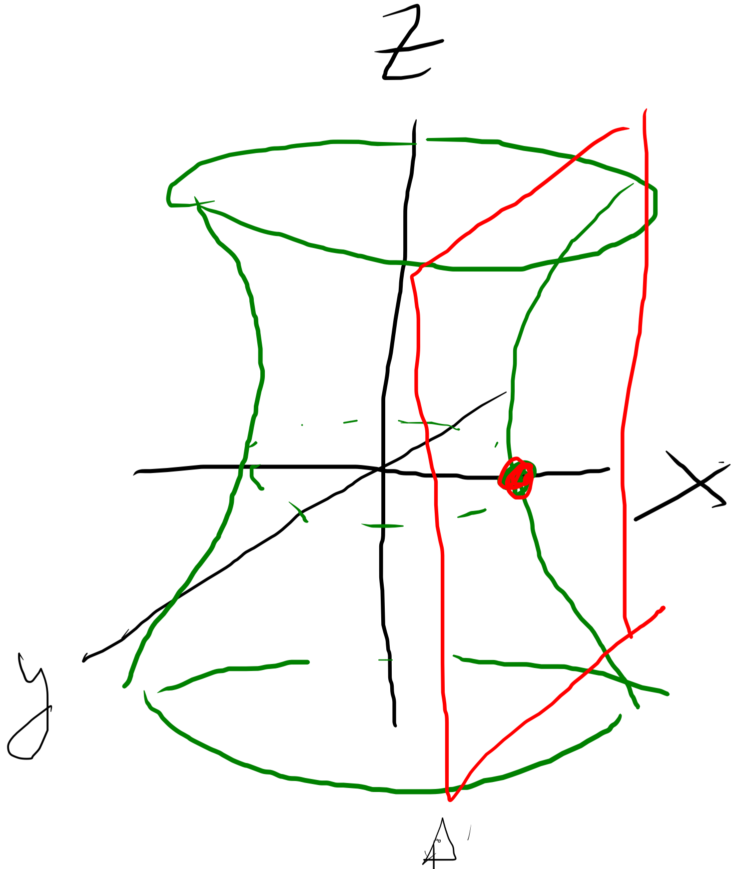
8. What is the tangent space to the paraboloid defined by $x^2 + y^2 - z^2 = a$ at $(\sqrt{a}, 0, 0)$, where $(a > 0)$?

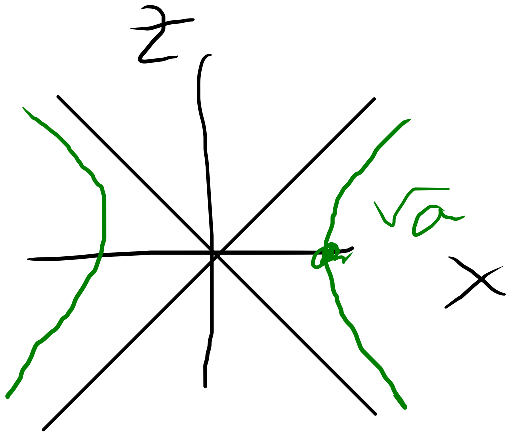
$x^2+y^2=a+z^2$

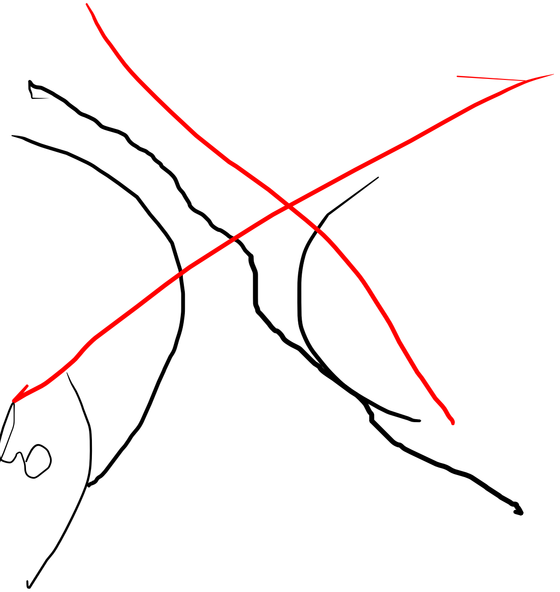
$x^2-z^2=a \quad / \div a$

$\left(\frac{x}{\sqrt{a}}\right)^2-\left(\frac{z}{\sqrt{a}}\right)^2=1$

$\left(\frac{x}{\sqrt{a}}\right)^2=\left(\frac{z}{\sqrt{a}}\right)^2 \qquad z=\pm x$



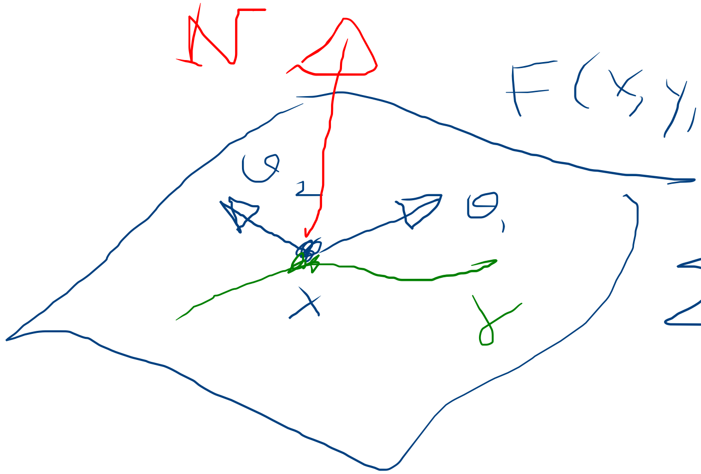




eqn (translado ao origen)

Resp. $\longrightarrow \boxed{\{x=0\}}$

$T_x\Sigma = \text{span}\{v_1, v_2\}$



$F(x,y,z) = x^2 + y^2 - z^2 = a$

$\Sigma = F^{-1}(a)$

$N = (T_x\Sigma)^{\perp}$

$T_x\Sigma = N^{\perp}$

$N = \nabla F$

<u>P.D.</u> $N \perp T_x\Sigma$

$\Leftrightarrow \langle N, \dot\gamma(0)\rangle$, γ tray. en Σ por x.

$F(\gamma(t)) = a \quad \forall t$

$\langle \nabla F, \dot\gamma(0)\rangle = 0$

$\mathbb{R}^l \subset \mathbb{R}^k$ $\quad l \leq k$

$\shortparallel$

$\{(x_1, \dots, x_l, 0, \dots, 0)\} = \{x_{l+1} = \dots = x_k\}$

$\{x_1, \dots, x_k\}$ coord en $\mathbb{R}^k$.

Para toda $M^l \subset N^k$

$\ni$

x

$\exists$ coord $x_1, \dots, x_k$ t.q. $\quad M \cap U = \{ \quad \}$

en $U \subset N^k$, $x \in U$

<u>TFI</u>: ¿cómo "se ve" el espacio de soluciones de un sistema de ecuaciones?

Resp. (muy parcial): localmente, si las ecuaciones "no son feas" se ve "bonito".

sumersión

subvar = la gráfica de una función.

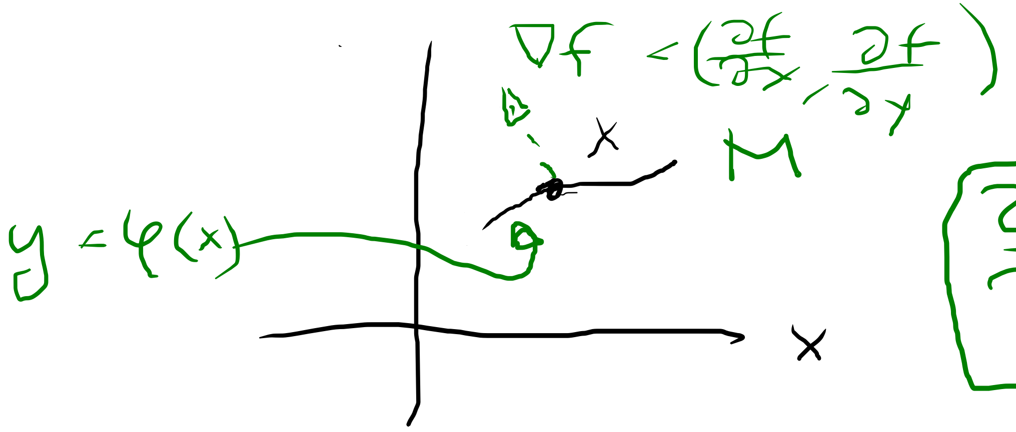
<u>ecns</u>: n incógnitas $x_1, \ldots, x_n$, $n-k$ ecuaciones.

$$f_1(x_1, \ldots, x_n) = c_1$$

$$\vdots$$

$$f_{n-k}(x_1, \ldots, x_n) = c_{n-k}$$

$$f = (f_1, \ldots, f_{n-k}) : \underset{\mathbb{R}^n}{\overset{\cap}{U}} \to \mathbb{R}^{n-k}$$



$$\boxed{\frac{\partial f}{\partial y} \neq 0}$$

$n = 2$

$K = 1$

$M = \{ f(x_1, x_2) = c \}$

TFI: si

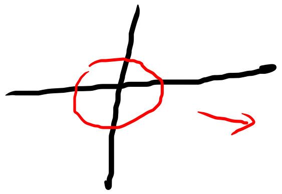
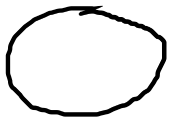
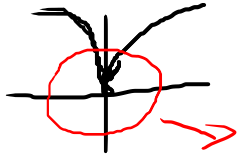
$$Df = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)(x,y) \neq 0$$

ent. M en una vec. de x es una subvar. (curva)

(2) Más preciso: si $\frac{\partial f}{\partial y}(x,y) \neq 0$

ent $M \cap U = \text{Graph}(y = \varphi(x))$

U una vec. de (x,y)

ecn.	M	
ej: $x^2 + y^2 = -1$	$\emptyset$	
$xy = 0$		→ TFI falla
$x^2 + y^2 = 1$		← TFI loc
$y = 0$		← funciona glob
$x^2 = y^3$		→ TFI falla

En general $M=\{f(x,y)=c\}$

$\det\left(\frac{\partial f}{\partial y}\right)\neq 0$

$x=(x_1,\dots,x_k)\in\mathbb{R}^k$

$y=(y_1,\dots,y_{n-k})$

$c = \mathbb{R}^{n-k}$

$\Rightarrow M\cap U = \text{graph}\{y=\varphi(x)\}$

$$df=\begin{pmatrix}\frac{\partial f_1}{\partial x_1} & \\ \cdots & \\ \boxed{\frac{\partial f_1}{\partial y_{n-k}}} & \end{pmatrix}\Big\}\ n-k \text{ filas l.i}$$

df_x es suprayectiva

sumersión $\Leftrightarrow$ df_x es suprayectiva $\forall x$.

Def: x es regular si df_x es sobre

$y\in Y$ es regular si $f^{-1}(y)$ todos son puntos regulares.

TFI (versión Var. dif.): $f: X^n \to Y^{n-k}$ y si df_x es suprayectiva $\forall x\in X \Rightarrow f^{-1}(y)$ es subvar. de dim k, $\forall y$.

Ej: $M = \{ A \mid AA^T = I \} \subset Mat_{n\times n}(\mathbb{R})$

es una variedad, porque $I \in Sim_{n\times n}(\mathbb{R})$

es un $\boxed{\text{valor regular}}$ de la función

$$Mat_{n\times n}(\mathbb{R}) \longrightarrow Sim_{n\times n}(\mathbb{R})$$

$$A \longmapsto AA^T$$

ejc. verificar esto