

3. Encuentra los focos y asíntotas de la hipérbolas (a)  $x^2 - 2y^2 = 3$ ; (b)  $xy = 3$ ; (c)  $(x-1)y = 3$ .

Hipérbola en "forma estándar"

$$a, c \Rightarrow \begin{cases} |d_1 - d_2| = 2a = \text{const.} \\ (d_1 - d_2)^2 = 4a^2 \end{cases}$$

$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1 \xrightarrow{\text{asínt.}} \left(\frac{\pm}{a}\right)^2 = \left(\frac{\pm}{b}\right)^2$$

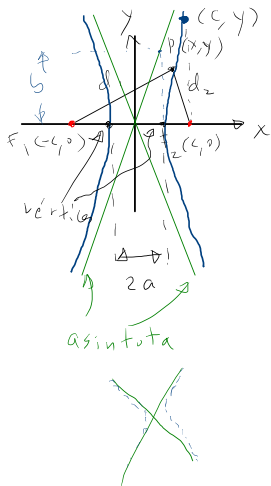
$$b^2 = ?$$

$$\frac{x}{a} = \pm \frac{y}{b}$$

$$\text{pend} = \pm \frac{b}{a}$$

a)  $x^2 - 2y^2 = 3 \quad / : 3$

$$\frac{x^2}{3} - \frac{2}{3}y^2 = 1$$



$$\sqrt{4c^2 + y^2} - y = 2a \Rightarrow 4c^2 + y^2 = (2a + y)^2$$

$$4c^2 = 4a^2 + 4ay$$

$$c^2 = a^2 + ay$$

$$\left(\frac{c}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$$

$$\left(\frac{c}{a}\right)^2 - \left(\frac{c^2 - a^2}{ab}\right)^2 = 1 / a^2 b^2 \quad y = \frac{c^2 - a^2}{a}$$

$$c^2 b^2 - (c^2 - a^2)^2 = a^2 b^2$$

$$b^2 (c^2 - a^2) = (c^2 - a^2)^2$$

$$b^2 = c^2 - a^2$$

$c^2 = a^2 + b^2$   
¿dónde está el triáng. rect.?!?

$$Y^2 = \frac{(X+Y)^2}{2} + \frac{(X-Y)^2}{2}$$

$$= \frac{2X^2 + 2Y^2}{2} = X^2 + Y^2$$

$$\left(\frac{X}{\sqrt{3}}\right)^2 - \left(\frac{Y}{\sqrt{2}}\right)^2 = 1$$

$$a = \sqrt{3}, b = \sqrt{3/2} \Rightarrow c = \sqrt{a^2 + b^2} = \sqrt{3 + 3/2} = \sqrt{9/2} = \frac{3}{\sqrt{2}}$$

$$\Rightarrow \text{asint: } Y = \pm \frac{\sqrt{3/2}}{\sqrt{3}} X = \pm \frac{1}{\sqrt{2}} X$$

$$\frac{3}{\sqrt{2}} > \sqrt{3}$$

$$\frac{9}{2} > 3 \quad \checkmark$$

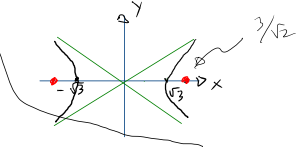
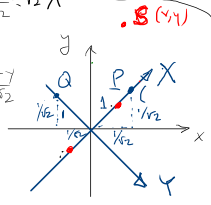
b)  $XY = 3$

$$\begin{cases} X = (X+Y)/\sqrt{2} \\ Y = (X-Y)/\sqrt{2} \end{cases} \Rightarrow \begin{cases} X+Y = \sqrt{2} X \\ X-Y = \sqrt{2} Y \end{cases}$$

$$XY = \frac{(X+Y)(X-Y)}{2} = 3$$

$$\left(\frac{X}{\sqrt{6}}\right)^2 - \left(\frac{Y}{\sqrt{6}}\right)^2 = 1$$

$a = b = \sqrt{6}$   
 $c = \sqrt{a^2 + b^2} = 2\sqrt{3}$



$$(A+B)(A-B) = A^2 - B^2$$

P:  $X=1, Y=0, x=1/\sqrt{2}, y=1/\sqrt{2} \checkmark$   
Q:  $X=0, Y=1, x=-1/\sqrt{2}, y=1/\sqrt{2}$

Focos:  $X = \pm c = \pm 2\sqrt{3}$   
 $Y = 0$

$$\Rightarrow x = \frac{X+Y}{\sqrt{2}} = \pm \frac{2\sqrt{3}}{\sqrt{2}} = \pm \sqrt{6}$$

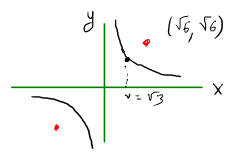
$$y = \pm \sqrt{6}$$

$(\sqrt{6}, \sqrt{6}), (-\sqrt{6}, -\sqrt{6})$  focos en coord xy

asint:  $\left(\frac{X}{\sqrt{6}}\right)^2 - \left(\frac{Y}{\sqrt{6}}\right)^2 = 0 \Rightarrow X^2 = Y^2 \Rightarrow \begin{cases} X+Y=0 \Leftrightarrow x=0 \text{ (eje de } y) \\ X-Y=0 \Leftrightarrow y=0 \text{ (eje de } x) \end{cases}$

$$v^2 = 3 \Rightarrow v = \sqrt{3}$$

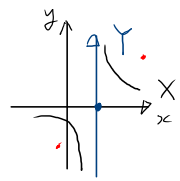
$$XY = 3$$



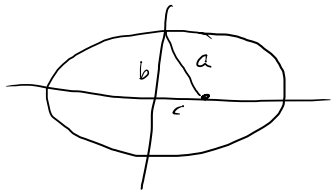
c)  $\underbrace{(X-1)}_X \underbrace{Y}_Y = 3$   
 $\begin{cases} X = x-1 \\ Y = y \end{cases} \Rightarrow \begin{cases} x = X+1 \\ y = Y \end{cases}$

$$XY = 3$$

asint:  $(X-1)Y = 0 \Rightarrow \begin{cases} Y = 0 \\ X-1 = 0 \Leftrightarrow X = 1 \end{cases}$



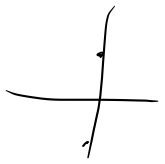
1b



$$a^2 = b^2 + c^2$$

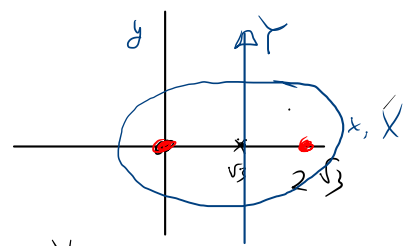
$$1^2 + 1^2 = 2$$

$$a = \sqrt{2}$$



$$\left(\frac{X}{\sqrt{2}}\right)^2 + Y^2 = 1$$

$$X^2 + \left(\frac{Y}{\sqrt{2}}\right)^2 = 1$$



$$a^2 = c^2 + b^2$$

$$c^2 = a^2 - b^2$$

$$a = 2$$

$$b = 1$$

$$c = \sqrt{3}$$

$$x = X + \sqrt{3} \Rightarrow X = x - \sqrt{3}$$

$$y = Y \Rightarrow Y = y$$

$$\left(\frac{X}{2}\right)^2 + Y^2 = 1$$

$$\left(\frac{x - \sqrt{3}}{2}\right)^2 + y^2 = 1$$