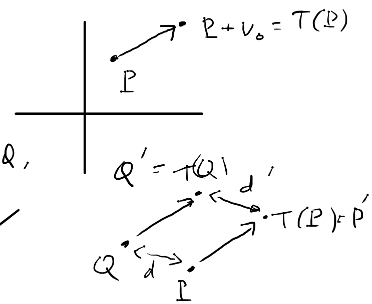
2. *a*) Toda traslación es una isometría y es invertible. Mismo para rotación por el origen y la reflexión por el eje de x.

Traslación: $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(P) = P + v_0$



P.D. Es una isometría, o sea $\boxed{d = d'}$

O sea, $\|T(P) - T(Q)\| \overset{?}{=} \|P - Q\|$, $\forall P, Q$,

$\|(P + \cancel{v_0}) - (Q - \cancel{v_0})\| = \|P - Q\|$. ✓

P.D. T es invertible; o sea existe $S: \mathbb{R}^2 \to \mathbb{R}^2$ t.q.

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "T"] & \mathbb{R}^2 \arrow[l, dashed, bend left, "S"] \end{tikzcd}$$

$S \circ T = id_{\mathbb{R}^2}$, $S(T(P)) = P$

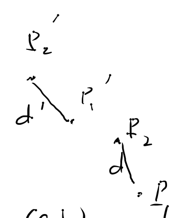
$T \circ S = id_{\mathbb{R}^2}$, $T(S(P)) = P$

[T invertible $\overset{\text{def}}{\iff}$ T es biyectiva $\iff$ iny. + sobre.] ej.

Sea $S: P \longmapsto P - v_0$;

$S(T(P)) = S(P + v_0) = (P + v_0) - v_0 = P$.

$T(S(P)) = T(P - v_0) = (P - v_0) + v_0 = P$.

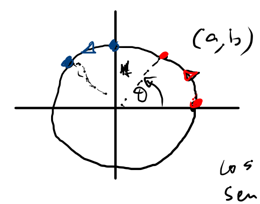


Rotación: $R: (x, y) \longmapsto (ax - by, bx + ay)$, $\boxed{a^2 + b^2 = 1}$

$(1, 0) \longmapsto (a, b)$

$(0, 1) \longmapsto (-b, a)$

P.D. $d = d'$



$\cos\theta = a$
$\operatorname{sen}\theta = b$

$$(d')^2 = \|R(P_1) - R(P_2)\|^2 = \|(ax_1 - by_1,\ bx_1 + ay_1) - (ax_2 - by_2,\ bx_2 + ay_2)\|^2 =$$

$$= \|(a\overbrace{(x_1 - x_2)}^{\Delta x} - b\overbrace{(y_1 - y_2)}^{\Delta y},\ b(x_1 - x_2) + a(y_1 - y_2))\|^2 =$$

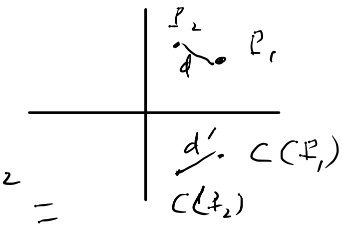
$$= [a\Delta x - b\Delta y]^2 + [b\Delta x + a\Delta y]^2 = a^2(\Delta x)^2 + b^2(\Delta y)^2 - \cancel{2ab\Delta x\Delta y}$$

$$+ b^2(\Delta x)^2 + a^2(\Delta y)^2 + \cancel{2ab\Delta x\Delta y}$$

$$= ((\Delta x)^2 + (\Delta y)^2)(\underbrace{a^2 + b^2}_{=1}).$$

$$d^2 = (\Delta x)^2 + (\Delta y)^2$$ ✓

$$C : (x, y) \longmapsto (x, -y)$$



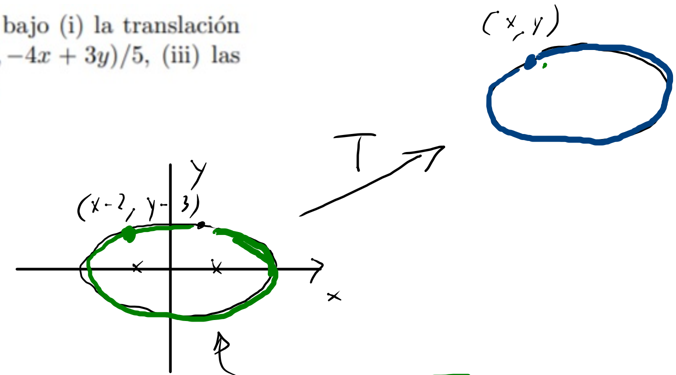
isometria: $P_i = (x_i, y_i)$, $i = 1, 2$.

$$(d')^2 = \|C(P_2) - C(P_1)\|^2 = \|(x_2, -y_2) - (x_1, -y_1)\|^2 =$$

$$= \|(\Delta x, -\Delta y)\|^2 = (\Delta x)^2 + (-\Delta y)^2 = (\Delta x)^2 + (\Delta y)^2 = \|P_2 - P_1\|^2.$$

4. Encuentra una ecuación para la imagen de la elipse $\boxed{2x^2+3y^2=1}$ bajo (i) la translación $T:(x,y)\mapsto(x,y)+(2,3)$, (ii) la rotación $R:(x,y)\mapsto(3x+4y,-4x+3y)/5$, (iii) las composiciones $R\circ T$ y $T\circ R$ de las dos transformaciones anteriores.

(i) $\boxed{2(x-2)^2+3(y-3)^2=1}$ ✓



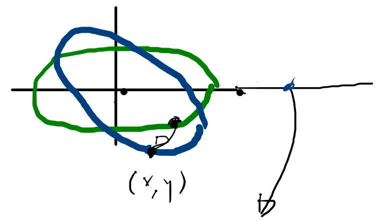
$\boxed{2x^2+3y^2=1}$

(ii) $R^{-1}(x,y)=(3x-4y,\,4x+3y)/5$

$$\boxed{2\left(\frac{3x-4y}{5}\right)^2+3\left(\frac{4x+3y}{5}\right)^2=1}$$

(iii) $(R\circ T)^{-1}=T^{-1}\circ R^{-1}$

ej



$(1,0)\mapsto(3/5,\,-4/5)$

$$(R\circ T)^{-1}(x,y)=T^{-1}\left(R^{-1}(x,y)\right)=$$

$$=T^{-1}\left(\frac{3x-4y}{5},\frac{4x+3y}{5}\right)=\left(\frac{3x-4y}{5}-2,\ \frac{4x+3y}{5}-3\right)$$

$$\Longrightarrow\quad 2\left(\frac{3x-4y}{5}-2\right)^2+3\left(\frac{4x+3y}{5}-3\right)^2=1$$

$$Ax^2+Bxy+\ldots\ldots=0.$$