Maintaining Visibility of a Moving Holonomic Target at a Fixed Distance with a Non-Holonomic Robot

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***Abstract*— In this paper we consider the problem of maintaining surveillance of a moving the target by a nonholonomic mobile observer. The observer's goal is to maintain visibility of the target from a predefined, fixed distance, l. The target escapes if (a) it moves behind an obstacle to occlude the observer's view, (b) it causes the observer to collide with an obstacle, or (c) it exploits the nonholonomic constraints on the observer motion to increase its distance from the observer beyond the surveillance distance l.**

We deal specifically with the situation in which the only constraint on the target's velocity is a bound on speed (i.e., there are no nonholonomic constraints on the target's motion), and the observer is a nonholonomic, differential drive system having bounded speed. We develop the system model, from which we derive a lower bound for the required observer speed. Finally, we consider the effect of obstacles on the observer's ability to successfully track the target.

I. Introduction

In this paper, we consider the surveillance problem of maintaining visibility at a fixed distance of an unconstrained mobile target by a nonholonomic mobile robot equipped with sensors (the observer). This is an extension to our previous work that has considered variations of this problem, including the case for which there is delay but no velocity bounds for the observer [19], and for which there is no delay, but the observer velocity is bounded [20]. In [21] we found, the optimal motion for the target to escape. Symmetrically, an optimal motion strategy for the observer to always maintain visibility of the target is determined.

The distinguishing feature of our current work is the consideration of nonholonomic constraints on the motion of the observer. Such constraints qualitatively change the solutions to the surveillance problems that we have previously considered.

As is well known in mobile robotics research, constraints that are defined in terms of time derivatives of configuration variables and that cannot be integrated to eliminate these derivatives are known as nonholonomic constraints [11], [13]. Motion planning for robots with nonholonomic constraints has been an active research area since the nineties (see, e.g., [2], [12], [16], [17], [26]). From the point of view of path planning, an important consequence of nonholonomic constraints is that the existence of a path in the configuration space does not necessarily imply the existence of a feasible path for the system [13].

In addition to the nonholonomic constraints on observer motion, we assume that both the observer and target have bounded speed, and that each has access to the full state of the other. Under these conditions, we address the problem of maintaining visibility of the target in the presence of obstacles, which produce both motion and visibility constraints.

A. Previous Work

Our problem is related to pursuit-evasion games. A great deal of previous research exists in the area of pursuit and evasion, particularly in the area of dynamics and control in the free space (without obstacles) [1], [7], [10]. The pursuit-evasion problem is often framed as a problem in non cooperative dynamic game theory [1].

A pursuit-evasion game can be defined in several ways. One formulation consists in *finding* an evasive target with one or more mobile observers that sweep the environment so that the target does not eventually sneak into an area that has already been explored. Deterministic [6], [22], [25], [27] and probabilistic algorithms [8], [28] have been proposed to solve this problem. Alternatively, the observers might have as a goal to actually "catch" the targets, that is, move to a contact configuration or closer than a given distance [10].

As mentioned above, our problem is related to the problems of pursuit-evasion. However, the previous problems are not the same as ours. In this paper, the problem consists of determining an observer motion strategy to always maintain the visibility between the target and the observer. We assume that initially the observer can establish visibility with the target. Such a task is sometimes referred to as target tracking.

Previous research has studied the motion planning problem for maintaining visibility of a moving target (target tracking). Game theory is proposed in [14] as a framework to formulate the tracking problem and an online algorithm is presented.

In [4], an algorithm is presented that operates by maximizing the probability of future visibility of the target. This algorithm is also studied with more formalism in [14]. This technique was tested in a Nomad 200 mobile robot with relatively good results. However, the probabilistic model assumed by the planner was often too simplistic, and accurate models are difficult to obtain in practice.

The work in [5] presents an approach that takes into account the positioning uncertainty of the robot observer. Game theory is also proposed as a framework to formulate the tracking problem. One contribution of [5] is a technique that periodically commands the observer to move into a region that has no localization uncertainty (a landmark region) in order to re-localize and better track the target afterward.

The approach presented in [18] computes a motion strategy by maximizing the *shortest distance to escape* —the shortest distance the target needs to move in order to escape the observer's visibility region. In this work the targets are assumed to move unpredictably, and the distribution of obstacles in the workspace is assumed to be known in advance. This planner has been integrated and tested in a robot system that includes perceptual and control capabilities. The approach has also been extended to maintain visibility of two targets using two mobile observers.

In [9], a technique is proposed to track a target without the need of a global map. Instead, a range sensor is used to construct a local map of the environment, and a combinatoric algorithm is then used to compute a differential motion for the observer at each iteration.

The problem of planning an observer's motions to maintain visibility of a moving target has received a good deal of attention in the motion planning community recently. Several techniques have been reported in the literature, and a variety of strategies have been proposed to perform the tracking. However, the decision problem - answering the question: can the target escape - has not been solved for the case of a nonholonomic observer. Answering this question is one of the goals of this paper.

II. Problem definition

The target and the observer are represented as points. The target is visible to the observer whenever the line segment connecting the two does not intersect an obstacle. We refer to this line segment as the *rod* due to an analogy with the motion planning problem studied in [24]. Thus, violation of the visibility constraint corresponds to collision of the rod with an obstacle in the environment. The target controls the position of the rod's origin (x, y) and the observer controls the rod's orientation ϕ and must compensate to maintain a fixed rod length l, where l is the predefined surveillance distance.

Obstacles are modeled as polygonal barriers and we assume that the observer is provided with a map of the environment.

We assume an antagonistic target. The target can defeat the observer by hiding behind an obstacle (breaking the rod with a vertex), by making the observer collide with and obstacle (a segment or a vertex), or by preventing the observer from being at the required fixed distance.

The target moves continuously; its global trajectory is unknown but its maximal speed is known. We assume that full state feedback is available, i.e., the target velocity is measured (or reported) without delay to the observer, and symmetrically, that the target has access to full state information for the observer. Both observer and target are limited to move with bounded speed.

This paper focus on the decision problem which corresponds to answering the question: *can the target escape the observer surveillance?*

III. System Model

Figure 1 shows the geometric description of the system. The variables $x_T(t), y_T(t), x_O(t), y_O(t)$ denote the target and observer positions with respect to the global reference frame. The variable $\theta(t)$ is the angle of the observer's wheels with respect to the global x axis, and $\phi(t)$ represents the angle between the rod and the global x axis.

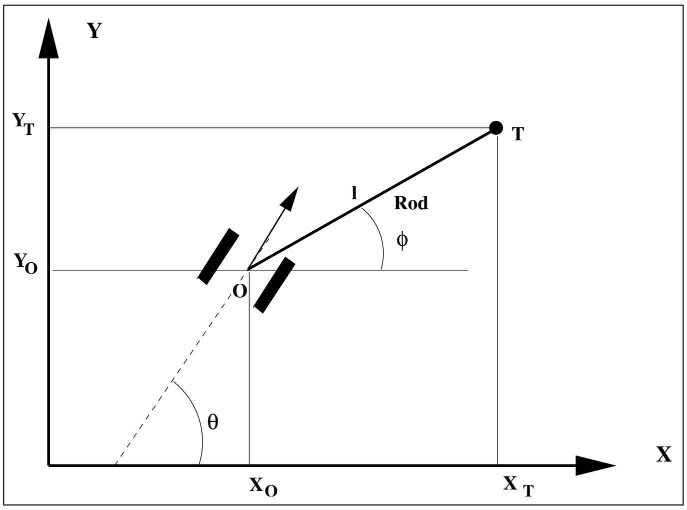

Fig. 1. The geometric model of the observer-target system

Since the observer is a differential drive robot, we use the usual assignment of control inputs

$$u_1 = w_r(t) + w_l(t)$$
$$u_2 = w_r(t) - w_l(t)$$

in which $w_r(t)$ and $w_l(t)$ are the angular speeds of the left and right wheels respectively. When $u_1 = 0$, the robot rotates without translation, and when $u_2 = 0$ the robot translates without rotation.

The bounds on the observer's speed derive from bounds on the rate at which the wheels can spin, and are thus naturally expressed as bounds on u_1 and u_2. We denote these bounds by

$$\begin{aligned} u_1^* &= \max u_1 = \max\{w_r(t) + w_l(t)\} \\ u_2^* &= \max u_2 = \max\{w_r(t) - w_l(t)\} \end{aligned}$$

so that u_1^* is the maximum linear speed of the observer and u_2^* is the maximum rate of rotation of the observer.

Using this system formulation, the system dynamics are given by

$$\begin{pmatrix} \dot{x}_O(t) \\ \dot{y}_O(t) \\ \dot{\theta}(t) \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \cos\theta(t) \\ \sin\theta(t) \\ 0 \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} u_2 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u_3 \tag{1}$$

in which u_3 is a “free” degree of freedom. This system is redundant (or over actuated). This is the typical model of a mobile manipulator (see [3]).

When the surveillance constraints are satisfied, the relationship between observer and target positions is given by

$$\begin{pmatrix} x_T(t) \\ y_T(t) \end{pmatrix} = \begin{pmatrix} x_O(t) + l\cos\phi \\ y_O(t) + l\sin\phi \end{pmatrix} \tag{2}$$

and differentiating this expression we obtain an expression for the target velocities that maintain the fixed required l distance between the target and the observer

$$\begin{pmatrix} \dot{x}_T(t) \\ \dot{y}_T(t) \end{pmatrix} = \begin{pmatrix} \cos\theta & -l\sin\phi \\ \sin\theta & l\cos\phi \end{pmatrix} \begin{pmatrix} u_1 \\ u_3 \end{pmatrix} \tag{3}$$

If we define the matrix A as

$$A = \begin{pmatrix} \cos\theta & -l\sin\phi \\ \sin\theta & l\cos\phi \end{pmatrix} \tag{4}$$

we find

$$\det A = l\cos(\theta - \phi) \tag{5}$$

which implies that the observer can maintain the visibility of the target only when $(\theta - \phi) \neq \pm\frac{\pi}{2}$. In other words, the rod cannot have a relative angle to the observer wheels equal to $\pm\frac{\pi}{2}$ because this would require infinite observer speed (see equation 5 and figure 7).

A. Surveillance in the absence of obstacles

We begin by considering the necessary minimum values for u_1^* and u_2^* required to maintain surveillance in the absence of obstacles. Following the system given by equation 3, we see that the bound on u_2 does not play a direct role in maintaining surveillance. With respect to the problem of pursuit, the only relationship to be considered is between the velocity of the target and the linear velocity of the observer.

$$\begin{aligned} \dot{x_T}^2 + \dot{y_T}^2 &= (u_1 \;\; u_3) A^T A \begin{pmatrix} u_1 \\ u_3 \end{pmatrix} && (6) \\ &= {u_1}^2 + 2u_1u_3 l\sin(\theta - \phi) + l^2{u_3}^2 && (7) \end{aligned}$$

Note that equation 7 defines an ellipse in the u_1-u_3 plane (see Figure 2). Suppose the target's velocity is bounded to have unit norm, $\dot{x}_T^2 + \dot{y}_T^2 \leq 1$. Then the constraint on u_1 and u_3 is that they should be inside the ellipse

$$f(u_1, u_3) = {u_1}^2 + 2u_1u_3 l\sin(\theta - \phi) + l^2{u_3}^2 = 1$$

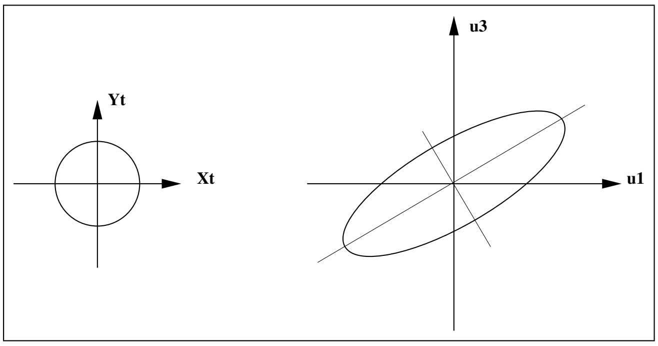

Fig. 2. Velocity bounds in x_T-y_T plane and in the u_1-u_3 plane

We can now determine the minimum value for u_1^* necessary to maintain surveillance. This amounts to projecting the ellipse in equation 7 onto the u_1 axis. Let α denote the maximal projection of the ellipse onto the u_1 axis (see figure 3). Then it is necessary that $u_1^* \geq \alpha$ to maintain surveillance.

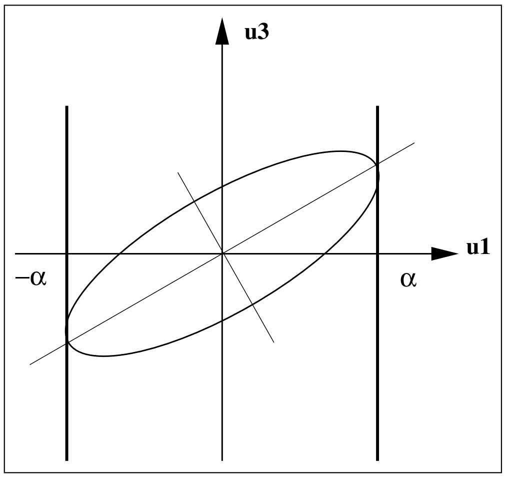

Fig. 3. The lower bound on u_1^* is given by α, the projection of the ellipse onto the u_1 axis.

To determine α we first solve for the value of u_3 that corresponds to the maximal projection of the ellipse in the u_1 direction

$$\frac{\partial f}{\partial u_3} = 0 \rightarrow u_3 = -\frac{u_1\sin(\theta - \phi)}{l}$$

We now substitute this value into $f(u_1, u_3) = 1$, and solve for $u_1 = \alpha$ as follows

$$\begin{aligned} 1 &= {u_1}^2 - 2u_1^2\sin^2(\theta - \phi) + {u_1}^2\sin^2(\theta - \phi) \\ &= u_1^2(1 - \sin^2(\theta - \phi)) \end{aligned}$$

which implies that

$$\alpha = \frac{1}{|\cos(\theta - \phi)|} \leq u_1^* \tag{8}$$

This is a *local analysis*, implying that when the inequality given in (8) holds, there is a control such that the observer can follow the target moving at unit velocity, whatever direction it chooses. Note that, this condition is independent of the surveillance distance l. Also note that this property is local, and does not say anything about the possible evolution of the target position that may tend to make $(\theta - \phi)$ converge to $\pm\frac{\pi}{2}$.

As can be seen from the constraint given in (8), as the difference $\theta - \phi$ approaches zero, the necessary value for u_1^* approaches its minimum. As a consequence, we adopt an observer strategy that attempts to minimize $\mid \theta - \phi \mid$. This can be accomplished by setting $u_2 = u_3$.

Using an analysis analogous to that used to derive α as a lower bound for u_1^*, we derive β as a lower bound on u_2^*. In particular, as shown in figure 4, we project the ellipse $f(u_1, u_3) = 1$ onto the u_3 axis (since we have set $u_2 = u_3$), and after manipulations similar to those above we obtain

$$\beta = \frac{1}{l\cos(\theta - \phi)} \leq u_2^* \tag{9}$$

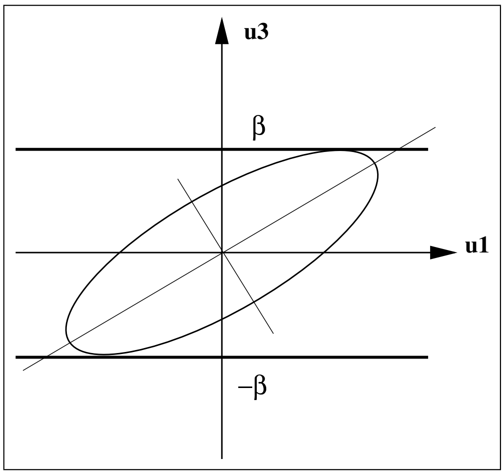

Fig. 4. The lower bound on u_2^* is given by β, the projection of the ellipse onto the u_3 axis.

Proposition I *If the constraints given by (8) and (9) hold at time $t = 0$, then in the absence of obstacles, the strategy $u_2 = u_3$ will guarantee that surveillance is maintained for all t.*

IV. Dealing with obstacles: Geometric Modeling

To maintain surveillance, it is necessary that the line segment connecting the observer and target (the rod) not intersect any obstacle in the environment (this would result in occlusion of the target).

Our approach consists in partitioning the workspace into non-critical regions separated by critical curves [19], [20], [24]. These curves are projections onto the plane of configuration space surfaces that bound forbidden rod configurations [19]. These rod configurations are forbidden either because they generate a violation of the visibility constraint (corresponding to a collision of the rod with an obstacle in the environment [19]) or because they require the observer to move with speed greater than its maximum [20].

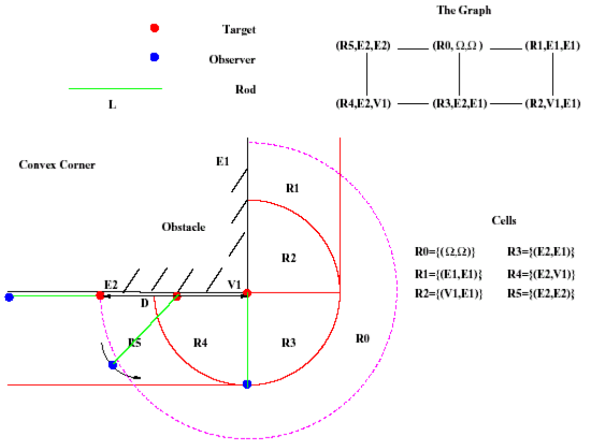

Fig. 5. An environment containing a single convex corner

In order to avoid a forbidden rod configuration, the observer must change the rod configuration to prevent the target to escape. We call this observer motion the rotational motion [20]. This type of motion will be finished either when the observer brings the rod to a configuration that avoids an escapable cell [19], when the observer reaches a inflection ray (aspect graph line) [23] associated to a reflex vertex (those which internal angle is greater than π) or, when the observer is able to move the rod in contact with an obstacle [20].

If the observer has bounded speed then the rotational motion must be started early enough for any forbidden rod configuration. The observer must have enough time to change the rod configuration before the target brings the rod to a forbidden one. There are critical events that tell the observer to start changing the rod configuration before it is too late. These critical events depend on the geometry of the environment, the initial location of the target, the relative configurations of the observer and target, the final rod configuration that prevents the target to escape, and the maximal observer and target speeds. For more details see [20].

To better clarify our description, we present one simple example. This example shows a convex corner (see figure 5). Solid lines indicate the critical curves at l distance from the obstacles and dashed lines indicate the critical events as a function of the distance from the first set of critical curves. The dot labeled (T) indicates the target and the dot labeled (O) the observer. A rod of length l is indicated with a segment finished with T and O labels. The graph in the figure indicates cell adjacency in the configuration space. R_i indicates the region labels in the workspace and, (R_i, a, b) the cell labels in the configuration space (a and b can be a segment or a vertex where the rod contact an obstacle), see [19].

When the target is approaching the corner, the observer must rotate around the target to change the rod configuration, otherwise the target can violate the visibility constraint. This can be by making the bar collide with an obstacle or by forcing the observer to move with speed greater than its maximum (see [20]).

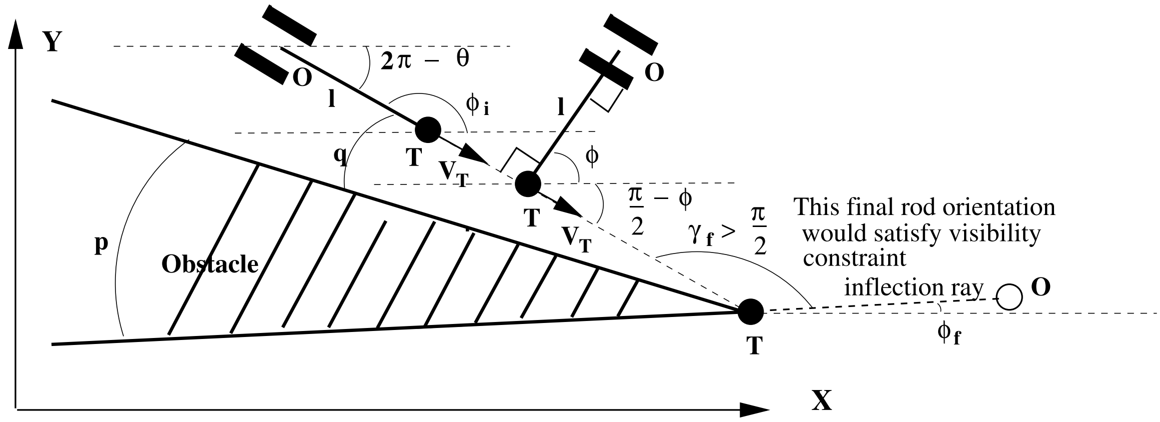

Fig. 6. target wins

The observer can choose to go to anywhere in region $R3$. The shorter rotation in this case is moving just to the border of $R3$.

V. Analysis for the case of obstacles

The following analysis allows us to find the conditions for the target to escape in the presence of obstacles. It corresponds to solving a so called game of kind [1], [10], where the goal is to determine in which conditions one player will win.

As before, the variables $x_T(t), y_T(t), x_O(t), y_O(t)$ denote the target and observer positions with respect to the global reference frame. The variable $\phi(t)$ represents the angle between the rod and the global x axis, while $\theta(t)$ is the angle of the observer's wheels with respect to the global x axis. All of these parameters change as a function of time. Finally, l is the constant length of the rod (see figure 6).

The system dynamics for such a differential drive robot are given by [2], [15].

$$\begin{pmatrix} \dot{x}_O(t) \\ \dot{y}_O(t) \\ \dot{\theta}(t) \end{pmatrix} = \begin{pmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \tag{10}$$

In this model the controls are defined by $u_1(t) = w_r(t)+w_l(t)$ and $u_2(t) = w_r(t) - w_l(t)$, where $w_r(t)$ and $w_l(t)$ are the angular speeds of the left and right wheels respectivey.

The observer position with respect to the target is expressed by:

$$\begin{pmatrix} x_O(t) \\ y_O(t) \end{pmatrix} = \begin{pmatrix} x_T(t) \\ y_T(t) \end{pmatrix} + l \begin{pmatrix} \cos\phi(t) \\ \sin\phi(t) \end{pmatrix} \tag{11}$$

All target velocities that maintain the fixed required l distance between the target and the observer must therefore satisfy:

$$\begin{pmatrix} \dot{x}_O(t) \\ \dot{y}_O(t) \end{pmatrix} = \begin{pmatrix} \dot{x}_T(t) \\ \dot{y}_T(t) \end{pmatrix} + l\dot{\phi}(t) \begin{pmatrix} -\sin\phi(t) \\ \cos\phi(t) \end{pmatrix} \tag{12}$$

From Equations 10 and 12 we obtain two expressoins for $\dot{\phi}(t)$, the first as a function of $x_T(t), u_1(t), \theta(t), \phi(t)$ and l and the second as a function of $y_T(t), u_1(t), \theta(t), \phi(t)$ and l:

$$\dot{\phi}(t) = \frac{\csc(\phi(t))\ [\dot{x}_T(t) - (\cos(\theta(t))\, u_1(t))]}{l} \tag{13}$$

$$\dot{\phi}(t) = \frac{\sec(\phi(t))\ [\sin(\theta(t))\, u_1(t) - \dot{y}_T(t)]}{l} \tag{14}$$

Equating Equations 13 and 14 and solving for $u_1(t)$ we obtain

$$u_1(t) = \frac{[\cos(\phi(t))\, \dot{x}_T(t) + \sin(\phi(t))\, \dot{y}_T(t)]}{\cos(\theta(t) - \phi(t))} \tag{15}$$

Equation 15 determines the appropriate control $u_1(t)$ to maintain the target at the constant distance from the observer. This control is a function of the angles $\theta(t)$ and $\phi(t)$ and the target velocities $\dot{x}_T(t), \dot{y}_T(t)$. If the wheels are perpendicular to the rod then equation 15 is undetermined. In this case, the observer requires unbounded speed, and hence the target can escape observer surveillance. This is, of course, the same result as that given in expression 8 above.

Figure 7 geometrically shows why the observer would require infinite speed if its wheels are perpendicular to the rod. The reason is that the observer's velocity vector does not have a component in $V_O \perp V_T$ direction. To maintain the constant distance from the target, it is required that the observer rotates infinitely fast.

Admissible configurations of the rod with respect to the observer's wheels are disconnected (under bounded observer speed). For the worst case of finite observer speed, the admissible configurations are $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, -\frac{\pi}{2})$. In general, for some given maximal observer speed, this disconnection divides the admissible rod configurations in two separate intervals smaller that π (see figure 7).

If there is an obstacle then the rod must change its orientation from an initial one to an orientation that satisfies the visibility constraint. If during the rod orientation change, the rod is perpendicular to the target velocity vector then the wheels have to be perpendicular to the rod.

In order to clarify our statements let us present an example. Figure 6 shows the target moving in a straight line toward a corner (reflex vertex). In this case if the observer does not change the rod orientation then when the target reaches the corner it can escape by making a sharp turn around the corner. Let us define γ_i as the angle between the straight line target trajectory and the initial rod orientation and, γ_f as the angle between the target trajectory and the final rod orientation. If $\gamma_i < \frac{\pi}{2}$ and $\gamma_f > \frac{\pi}{2}$ then for continuity the rod must be at one moment perpendicular to the target trajectory.

Now, we define ϕ_i as the angle between the x axis and the initial rod orientation. ϕ_f is the angle between the x axis and the final rod orientation.

We show, if the rod is perpendicular to the target trajectory velocity vector $\overrightarrow{V_T}$, then the wheels must be perpendicular to the rod.

From 15 and solving for $\cos(\theta(t) - \phi(t))$, we have:

$$\cos(\theta(t) - \phi(t)) = \frac{\cos\phi(t)\dot{x}_T(t) + \sin\phi(t)\dot{y}_T(t)}{u_1(t)} \tag{16}$$

The target velocities $\dot{x}_T(t)$ and $\dot{y}_T(t)$ can be expressed as:

$$\dot{x}_T(t) = V_T(t)\cos(\frac{\pi}{2} - \phi(t)) = V_T(t)\sin\phi(t) \tag{17}$$

$$\dot{y}_T(t) = -V_T(t)\sin(\frac{\pi}{2} - \phi(t)) = -V_T(t)\cos\phi(t) \tag{18}$$

Substituting 17 and 18 in 16 we have:

$$\begin{aligned} \cos(\theta(t) - \phi(t)) &= \frac{\cos\phi(t)V_T(t)\sin\phi(t) - \sin\phi(t)V_T(t)\cos\phi(t)}{u_1(t)} \\ &= 0 \end{aligned} \tag{19}$$

Therefore, the rod and the wheels are perpendicular (ϕ-$\theta = \pm\frac{\pi}{2}$) when the rod is perpendicular to the target velocity vector $\overrightarrow{V_T}$.

Above, we give necessary conditions for pursuit (see equations 8 and 19). *If the target can exploit obstacles to violate these conditions, then the target can escape.*

There does not exist an observer trajectory that does not require cooperation from the target and allows the observer to change the rod configuration, if the rod at one instant of time is perpendicular to the target velocity vector.

If in the workspace there is an obstacle which requires a rod configuration where the rod has to be perpendicular to the target velocity vector a solution does not exist –target wins.

By definition, both the observer and target know each others states (configuration and velocity). Therefore, if the rod is in a non-admissible configuration then the target can get further from the observer than the fixed surveillance distance.

We assume that the target is antagonistic, and hence it will not cooperate with the target, either helping it to maintain visibility or by inaction. If the target has the opportunity to escape, then it will take the required action to do so. For example, if the target is static, then the observer cannot translate. It will require pointing the wheels perpendicular to the rod and move in a circle around the target. As soon as the observer starts moving, the target can break the rod. If the target is static, the observer can only rotate in place.

In general, for some obstacles and straight line target trajectories, the rod orientation change forces the rod to be perpendicular to the target velocity vector and therefore a solution does not exist. These obstacles and target trajectories can be characterized as follows: Let us define p as the obstacle internal angle and q as the angle between the obstacle and the target trajectory (see figure 6).

If $p + q < \frac{\pi}{2}$ and $\gamma_i < \frac{\pi}{2}$ and $\gamma_f > \frac{\pi}{2}$ then a solution does not exist, target wins.

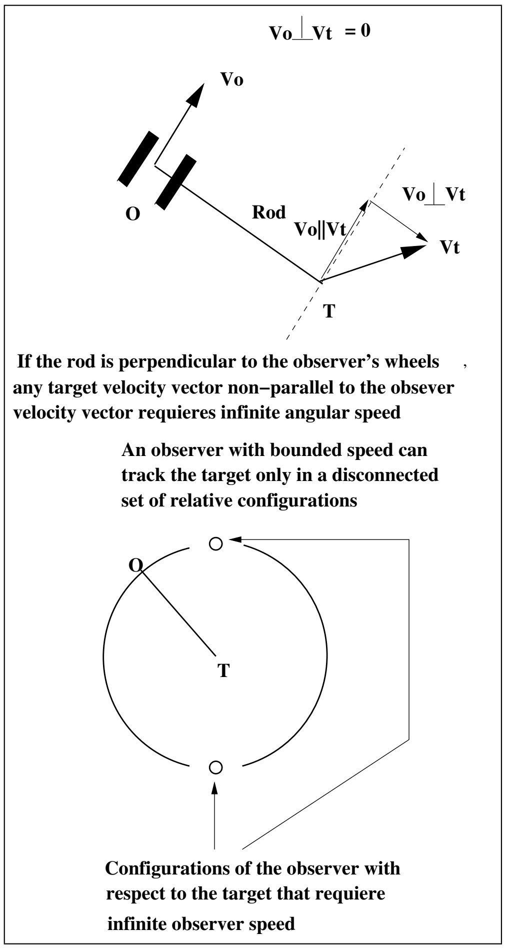

Fig. 7. target escaping if no cooperation

VI. Conclusions and Future Work

This paper solved the game of kind of maintaining visibility at a fixed distance of a moving holonomic target with a differential drive robot (a nonholonomic system) in the presence of obstacles.

Maintaining visibility at a *constant distance* of a holonomic target with a nonholonomic robot in the presence of obstacles has resulted in an enormous constraint. In most of the cases, the observer is not able to accomplish the task. Even when is possible, it requires very accurate control over the observer. A possibility is to relax the constraint and maintain visibility at a variable distance. In some simple cases, that would result in an almost static observer. In general, however, if the observer lets the target to go too far (from the observer) then it may require an tremendous speed to accomplish the task. This may happen because of the obstacles, which can force the observer to change the configuration of a very long rod.

Another option is to maintain the observer very close to the target. However, in most cases, that will correspond to a waste of observer energy. Additionally, the rod is emulating sensor range capabilities and most sensors become blind when a target is closer that a given distance. For this reason, keeping the observer almost touching the target does not seem a practical solution.

We also want to investigate this problem (maintaining visibility at a variable distance). We believe that the key to solving the problem resides in establishing an appropriate cost function and an algorithm based on critical events which can be used to decide when and where to move the observer.

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