Maintaining Visibility of a Moving Target at a Fixed Distance: The Case of Observer Bounded Speed

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***Abstract*— This paper addresses the problem of computing the motions of a robot observer in order to maintain visibility of a moving target at a fixed surveillance distance. In this paper, we deal specifically with the situation in which the observer has bounded velocity. We give necessary conditions for the existence of a surveillance strategy and give an algorithm that generates surveillance strategies.**

I. Introduction

In this paper, we consider the surveillance problem of maintaining visibility at a fixed distance of a mobile evader (the target) using a mobile robot equipped with sensors (the observer), in a workspace containing obstacles.

A great deal of previous research exists in the area of pursuit and evasion, particularly in the area of dynamics and control. This past work typically does not take into account constraints imposed on observer motion due to the existence of obstacles in the workspace, nor visibility constraints that arise due to occlusion. In this paper, we focus on these often neglected geometric aspects of the problem.

In this paper we consider the case for which the observer has bounded velocity, but can react instantaneously to evader motion (i.e., there is no delay in either the sensing or the control system). In our previous research, we have considered variations in which there is neither delay nor velocity bounds for the observer [11], and in which there is delay, but the observer velocity is not bounded [12]. In those cases, as well as in the case we consider here, we are able to express the constraints on the observer dynamics (i.e., delay and velocity bounds) geometrically, as a function of the geometry of the workspace and the surveillance distance.

A. Previous Work

Previous works have studied the motion planning problem for target tracking. Game theory [3] is proposed in [6] as a framework to formulate the tracking problem and an online algorithm is presented.

In [4], a tracking algorithm is presented that operates by maximizing the probability of future visibility of the target. This algorithm is also studied with more formalism in [6]. The approach presented in [10] computes a motion strategy by maximizing the *shortest distance to escape* —the shortest distance the target needs to move in order to escape the observer's visibility region. This planner has been integrated and tested in a robot system that includes perceptual and control capabilities. The approach has also been extended to maintain visibility of two targets using two mobile observers.

The problem of planning observer's motions to maintain visibility of a moving target has received a good deal of attention in the motion planning community over the last years. However, speeds of the observer and evader have never been considered to establish a motion strategy. This is one of the goals of this paper.

II. Problem definition

The target and the observer are represented as points. The environment where they are moving is modeled as a polygon. The visibility between the target and the observer is represented as a line segment and it is called the rod. This rod is emulating the visual sensor capabilities of the observer.

It is assumed that the delay between the target's motion and the observer's is zero. This means that the observer can react immediately to a target motion.

The target moves continuously, its global trajectory is unknown but its maximal speed is known and bounded. We are assuming a feedback control scheme where the target velocity is measured (or reported) without delay. The observer is limited to move with bounded speed. Other than this, no kinematic nor dynamic constraints are imposed on the observer or the target motions.

The target can defeat the observer by hiding behind an obstacle (breaking the rod with a vertex), by making the observer collide with and obstacle (a segment or a vertex) or by preventing the observer from being at the required fixed distance.

This paper focus on computing the motions of a robot observer in order to maintain visibility at a fixed distance of a moving target. This problem is analogous to the path planning problem of a moving rod in the plane [15]. The end points of the rod represent the observer and evader. The rod represents the visibility constraints. Violation of the visibility constraint corresponds to collision of the rod with an obstacle in the environment. The target controls the rod origin (x, y) and the observer controls the rod's orientation θ and must compensate to maintain a fixed rod length Lss.

III. Problem Modeling

We represent the observer and evader as points in the plane. In order to maintain surveillance at a fixed distance, it is necessary that the line segment connecting the pursuer and evader be maintained at a fixed length, and that this line segment not intersect any obstacle in the environment (this would result in occlusion of the evader). In this form, the surveillance problem shares many features with the traditional robot motion planning problem of of moving a rod in the plane. To solve this problem, Schwartz and Sharir represent the robot's configuration space, $\mathcal{C}$, by a cellular decomposition that can be constructed directly from the combinatoric representation of the workspace [15]. Here, we extend this representation to solve our surveillance problem.

For a rod moving in the plane, the configuration space can be represented as $\mathcal{C} = \Re^2 \times S^1$, and the workspace can be represented by $\Re^2$. The representation introduced in [15] and further developed in [2] is defined implicitly, in terms of a set of critical curves. These critical curves comprise the set of points at which the structure of the configuration space obstacle region above the xy-plane undergoes a qualitative change. Indeed, when such a curve is crossed, either the set of configuration space obstacle faces that are intersected by a line perpendicular to the xy-plane at the current position changes, or the number of intersection points changes [7].

The critical curves partition the plane into a set of noncritical regions, and this partition induces a cylindrical decomposition on $\mathcal{C}$. In particular, above any noncritical region in the plane, there will be a set of simply connected cells, each of which lies either entirely in the free configuration space or entirely within the configuration space obstacle region. This cellular decomposition can be represented by a connectivity graph, G, whose vertices correspond to free cells, such that two vertices are connected by an edge when the corresponding cells are adjacent.

IV. CONDITIONS FOR SOLVING THE ESTABLISHED PROBLEM

In this section, we describe three necessary conditions for the existence of a surveillance strategy.

Central to our approach is the notion of an *escapable cell* in the decomposition of the free configuration space described above.

Definition 1: For cell $K \subset \mathcal{C}$ above region $R \subset \Re^2$, if $\exists$ R' adjacent to R such that there is not a K' adjacent to K projecting onto R' then cell K is an escapable cell.

If the configuration of the rod lies in an escapable cell, then the evader can escape by merely moving into the region R' in the definition above. Since there is no free cell that projects onto R', there is no admissible position from which the observer can view the evader at the desired surveillance distance.

To determine the existence of a surveillance strategy, we recursively eliminate escapable cells from the connectivity graph, G, until either no cell K is eliminated (the condition is satisfied) or all the cells K_i corresponding to a single region R are eliminated (the condition is not satisfied). This is the first condition to the existence of a solution.

Proposition 1: If $\exists$ R such that all its corresponding cells K_i are escapable, then there does not exist a surveillance strategy for the given environment.

The proof for this proposition is given in [11].

The second condition for the existence of a solution is related to the bounded observer velocity. We first define an *escape point*:

Definition 2: An escape point is a point on a critical curve associated to an escapable cell in G, or a point in a critical curve bounding an obstacle region (see figure 5).

An interesting, especial case of escape points correspond to reflex vertices (those with interior angle larger than π) of the polygonal workspace.

Merely reaching an escape point does not guarantee that the evader can escape the surveillance. An escape point is a point from which the evader may escape for *some set* of observer positions (i.e., for some set of configurations, (x, y, θ) of the rod). Thus, when the evader nears an escape point, the observer must take action to ensure future visibility of the evader. Since the observer has bounded velocity, it must react before the escape point is reached by the evader.

We denote by $L^*(x, y, \theta)$ the minimal distance from an escape point such that, if the evader is further than $L^*(x, y, \theta)$ from the escape point, the observer will have sufficient time to react and prevent escape. Thus, it is only when the evader is nearer than $L^*(x, y, \theta)$ to an escape point that the observer must take special care.

Proposition 2: If there exists an escape point, p, such that the distance from evader to p is less than $L^*(x, y, \theta)$, the evader can escape the surveillance. The proposition follows immediately from the definition of $L^*(x, y, \theta)$. If the target is exactly at $L^*(x, y, \theta)$ distance from the an escape point, it signals the observer to start the rotation around the target before it is too late.

The distance $L^*(x, y, \theta)$ has to be computed based on: the geometry of the environment, the initial location of the evader, x, y, and on the relative configurations of the observer and evader θ, the final rod configuration that avoid the evader to escape and, the maximal observer and evader speeds. An upper bound of $L^*(x, y, \theta)$, that we call $L(x, y, \theta)$, is explained in detail in section V-B.

Corollary: Because of the bounded velocity the existence of a solution will always depend on the initial rod configuration (position and orientation), even in an environment without escapable cells.

Because of the bounded velocity, there are situations when the observer is not able to determine a motion that guarantees to have the target in sight, this is another condition for the existence of a solution. We call this condition no determinable motion for a single pursuer.

Proposition 3: If there are two or more escape points at $L^*(x, y, \theta)$ distance from the evader a solution does not exist.

Proof of proposition 3: The evader can move to any of the escapable points at $L^*(x, y, \theta)$ distance from it. The observer however can only choose one of them, therefore planning a

motion that ensures target visibility is not possible. ■
An example of this situation is shown in figure 1.

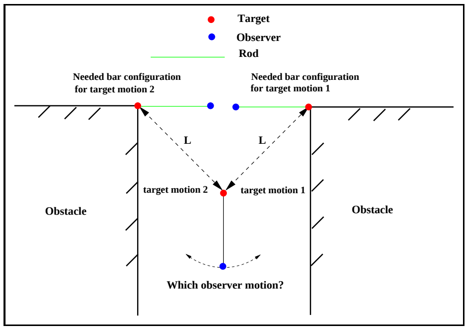

Fig. 1. No determinable motion for a single pursuer

In these situations more than a single observer is required to guarantee target visibility. Note, that this condition can also happen because of escape points that are on critical curves associated to escapable cells.

V. THE MOTION STRATEGY

The target controls the rod origin (x, y) and the observer controls the rod's orientation θ and must compensate to maintain a fixed rod length Lss.

There are 3 types of motion strategies: (1) the reactive motion rm used when the target is farther away from $L(x, y, \theta)$ distance of an escape point, (2) the observer rotational motion used when the evader is at $L(x, y, \theta)$ distance or closer from any escape point, (3) the compliant motion, used when the rod is in contact with an obstacle.

A. Observer reactive motion

At all times the observer must move to a position that respects the fixed sensor range. In the free space, there may be many positions that satisfy such a constraint. For instance, to move the observer with exactly the same velocity vector that the target is using. We call this movement the reactive motion rm.

Other motion strategy consists in moving the observer in the direction of the target. This moves the observer as little as possible. If the evader is moving in straight line the observer motion can be expressed by the Tractrix curve [9], [8] (also called hound curve, see figure 2). Its parametric equations, which determine the observer position, are:

$$x_o(t) = t - \tanh t; \;\; y_o(t) = \frac{1}{\cosh t}$$

B. Observer rotational motion to avoid evader escaping

When the target is at $L^*(x, y, \theta)$ distance or closer to an escape point the observer must do a rotational motion around the target in order to reach a position that satisfies visibility constraints.

The computation of $L^*(x, y, \theta)$ requieres an optimal observer motion. This observer motion strategy needs to have some properties: 1) it must maintain the target at the requiered fixed distance, 2) the observer must travel with bounded speed at all times, 3) it also must move with saturated speed and 4) the observer motion must minimize the time to complete the bar configuration change. Our proposed strategy has properties 1 through 3 but not necessarly 4. This means that it defines an upper bound on the computation of $L^*(x, y, \theta)$ that we call $L(x, y, \theta)$.

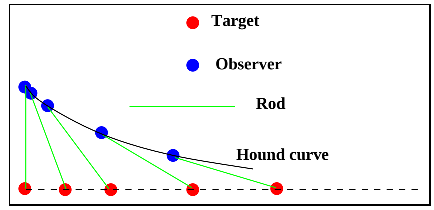

Fig. 2. The Tractrix curve

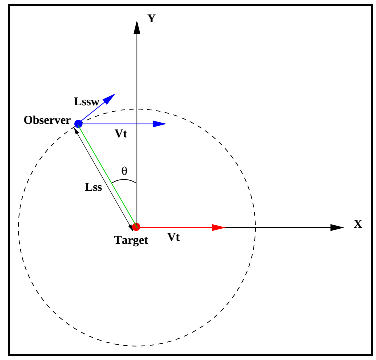

Fig. 3. Observer rotational motion around the target

The motion strategy in this case consists in applying the same velocity vector to the observer as the one that the target applies and an additional vector to get a observer rotation around the target.

This motion can be expressed in a $\mathbf{u_t}, \mathbf{u}_\theta$ basis.

$$\vec{V_o} = V_t\vec{\mathbf{u_t}} + (wLss)\vec{\mathbf{u}_\theta}$$

The vector applied to the observer can be expressed also in the x-y basis by defining the x axes as the one where the target is moving along (see figure 3).

$$\vec{V_o} = (V_t + Lssw\cos\theta)\vec{\mathbf{x}} + wLss\sin\theta\vec{\mathbf{y}}$$

If the target is at $L(x, y, \theta)$ distance from the any escape point and the target is antagonist then an observer motion requires to saturate the observer speed. The observer is constrained to be at a constant distance from the target (Lss must be constant), therefore to saturate the total observer speed its angular speed must vary.

This observer motion is tracing a curve similar to a cycloid. The cycloid is the locus of a point on the rim of a circle of constant radius rolling along a straight line. However, the cycloid is traced by a wheel (or rod) that is turning to uniform angular speed. In our case the angular speed of the turning rod varies.

The magnitude of V_o is obtained by using L_2 norm. $L_2 = \sqrt{X_1^2 + X_2^2 + \ldots + X_n^2}$.

$$\|V_o\| = \sqrt{(V_t + wLss\cos\theta)^2 + (wLss\sin\theta)^2}$$

Squaring in both sizes:

$${V_0}^2 = {V_t}^2 + 2V_t wLss\cos\theta + Lss^2 w^2(\sin^2\theta + \cos^2\theta)$$

Rearranging the equation:

$$Lss^2 w^2 + 2V_t wLss\cos\theta + {V_t}^2 - {V_o}^2 = 0$$

Solution to an algebraic equation of second order $ax^2 + bx + c = 0$ is given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, thus:

$$w = \frac{-2V_t Lss\cos\theta \pm \sqrt{4{V_t}^2 Lss^2\cos^2\theta - 4Lss^2({V_t}^2 - {V_o}^2)}}{2Lss^2}$$

Rearranging the equation:

$$w = \frac{\pm\sqrt{{V_0}^2 - {V_t}^2\sin^2\theta} - V_t\cos\theta}{Lss}$$

The variation of the observer angular speed (angular acceleration) can be expressed using the chain rule.

$$\frac{dw}{dt} = \frac{dwd\theta}{d\theta dt} \quad \text{where} \quad \frac{d\theta}{dt} = \dot{\theta} = w = f(\theta)$$

$$\frac{dw}{dt} = \frac{dw}{d\theta}f(\theta); \qquad dt = \frac{d\theta}{f(\theta)}$$

It is necessary to solve the next integral to obtain the time required to make the observer rotational motion.

This time is function of the given target and observer speeds, the initial and final rod configuration θ_o, θ_f and the constant length of the rod Lss.

$$\int_{t_0}^{t_f} dt = Lss\int_{\theta_0}^{\theta_f} \frac{d\theta}{\sqrt{{V_O}^2 - {V_t}^2\sin^2\theta} - V_t\cos\theta}$$

Multiplying by the conjugate:

$$\int_{t_0}^{t_f} dt = Lss\int_{\theta_0}^{\theta_f} \frac{d\theta}{\sqrt{{V_O}^2 - {V_t}^2\sin^2\theta} - V_t\cos\theta} C$$

Where

$$C = \frac{\sqrt{{V_O}^2 - {V_t}^2\sin^2\theta} + V_t\cos\theta}{\sqrt{{V_O}^2 - {V_t}^2\sin^2\theta} + V_t\cos\theta}$$

Rearranging the equation:

$$\int_{t_O}^{t_f} dt = \frac{Lss}{{V_O}^2 - {V_t}^2}\left[V_O\int_0^{\phi}\sqrt{1 - k^2\sin^2\theta}\,d\theta + V_t\int_{\theta_0}^{\theta_f}\cos\theta d\theta\right]$$

Where $\int_0^{\phi}\sqrt{1 - k^2\sin^2\theta}\,d\theta$ is the incomplete elliptic integral of second kind, with $k = \frac{V_t}{V_0}$, corresponding to a sector of the arc length of an ellipse. Where ϕ is called the amplitude and k is the elliptic module.

A solution to the incomplete elliptic integral of the second kind cannot be expressed by elementary functions[1], actually it is used to defined a form solution denoted by $E[\phi|m]$ [1]. Where $m = k^2$ is called the parameter [1]. This solution $E[\phi|m]$ is implemented in Mathematica [9].

Thus, the time required by the observer to make the rotational motion is defined by:

$$t|_0^{t_f} = \frac{LssV_O}{{V_O}^2 - {V_t}^2}E[\phi|m] + \frac{LssV_t}{{V_O}^2 - {V_t}^2}\sin\theta|_{\theta_0}^{\theta_f}$$

In order to maintain target visibility, the time taken for the target to reach the escape point cannot be smaller than $t = \frac{L(x,y,\theta)}{V_t}$, therefore.

$$L(x,y,\theta) = \frac{LssV_t}{{V_O}^2 - {V_t}^2}\left[V_O E[\phi|m] + V_t\sin\theta|_{\theta_0}^{\theta_f}\right]$$

This type of motion will be finished either when the observer brings the rod to a configuration that avoids an escapable cell (see figure 8), when the observer reaches and aspect graph line [14] (also curve type 3 of the cell decomposition for ladder motion planning [15]) associated to a reflex vertex (see figure 7) or, when the observer is able to move the rod in contact with an obstacle.

Note that the observer can perform a rotational motion around the target and maintain the fixed distance form it, only if its speed is strictly greater that the target speed.

C. Observer compliant motion

If a reactive motion rm would cause the rod to collide, the observer must rotate the minimum angle that makes the rod be in a collision free configuration (while keeping the rod configuration in a non escapable cell).

There are two general cases for the previous condition

- The observer is forced onto an obstacle. In this case it must move along the boundary of the obstacle region (this is the minimal rotation that keeps the rod at Lss). The velocity vector that the observer must applied to stay in contact with the line segment is (see figure 4):

$$V_o = V_{ty}\tan\phi + V_{tx}$$

where

$$V_{tx} = V_t\cos\theta;\ \ V_{ty} = V_t\sin\theta$$

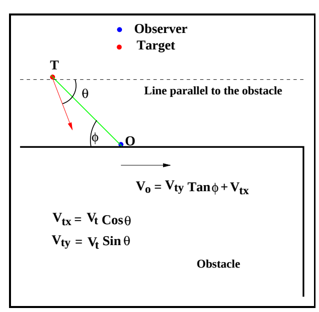

Fig. 4. Motion in contact with a segment

- The rod is in contact with a vertex. Note that if the evader is farther away from $L^*(x,y,\theta)$ distance of the escapable

[1] There is a controversy about whether or not a function expressed as an integral is a closed form solution.

point then this motion is possible, the strategy to achieve this motion is as follows. The observer must rotate away form the vertex to keep sight of the target. To get a minimal observer motion the rod must stay in contact with the vertex. The velocity vectors that the target must

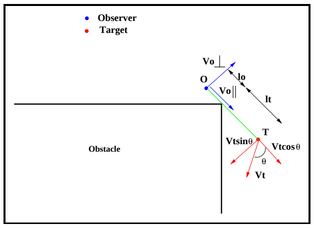

Fig. 5. Motion in contact with a reflex vertex

applied to stay in contact with the vertex are:

$$V_o \parallel = V_t \cos\theta; \;\; V_o \perp = V_t \sin\theta$$

Where

$$\|V_o\| = \sqrt{V_o \parallel^2 + V_o \perp^2}; \; \|V_o\| = V_t \sqrt{1 + \left(\frac{{l_o}^2}{{l_t}^2} - 1\right)\sin^2\theta}$$

In these two cases the rod will show a compliant motion.

VI. A WORST CASE SOLUTION

The solution is based on two sets of critical curves, the first one at Lss distance (the fixed distance) is used to determine the escapable cells.

The second set of curves must be defined at $L^*(x, y, \theta)$ distance from the escape point which corresponds to the worst case and determines the last moment when the observer must start the rotational motion to avoid that the target escapes through the escape point. Remember that, from definition 2, an escape point can be either a point on a critical curve that bounds an escapable cell or a point on a critical curve bounding a obstacle region. Since $L^*(x, y, \theta)$ is a function of the rod configuration, this second set of curves will be dynamic, that is, it will get closer or farther form the defining first set of critical curves or obstacles as the rod configuration changes. Note that target escaping means that it is about to hide behind an obstacle or about to confine the observer onto the obstacle region (brining the rod in an escapable cell) or breaking the rod with an obstacle region.

The existence of a solution depends on the initial rod configuration. Given this configuration, a first $L^*(x, y, \theta)$ can be computed and it can be determined if the target is closer or farther than $L^*(x, y, \theta)$ from the escape point which corresponds to the worst case.

VII. EXAMPLES

In all the examples, the edges are denoted by E_i and the vertices by V_i. The red rectangle indicates the escapable cells. The set of critical curves defined at Lss distance from the obstacles are in red. The second set of critical curves are in purple.

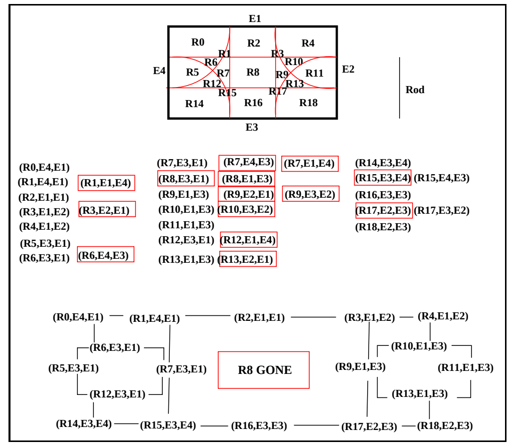

Fig. 6. Rectangle

The first example consists in a polygon (rectangle) having two parallel segments smaller than the rod length, as show in figure 6.

The rectangle has two parallel segments smaller that 2 times the rod length. There are 18 regions in the xy-plane and 32 cells in the configuration space. The rule used to detect non escapable cells is recursively applied to all the cells until all the cells corresponding to a single region are eliminated. Red rectangles indicate the escapable cells. The graph in the figure only contains the cells after elimination of escapable cells. The region 8 is not in the graph. If the target is in region 8, it can leave the region and bring the rod toward an adjacent region (i.e region 9) that does not have a cell adjacent to the rod configuration in region 8. Therefore, a solution does not exist.

The second example shows a convex corner (see figure 7). This example is used to illustrate a pursuit when the target tries to escape around convex corners and how the observer avoids loosing sight in these cases. It is assumed that the target will be antagonist, therefore, it will move along the boundary of the obstacle region. There are 6 regions on the xy-plane and also 6 cells in the configuration space. In this case, the graph representing the polygon contains all the regions on the xy-plane. Therefore, a solution exists for some initial rod configuration.

Let us assume that the target is antagonist and is moving on the boundary of the obstacle. Let us also assume that the target and observer start moving in $R5$.

When the target is at $L^*(x, y, \theta)$ distance from the reflex vertex (escape point) the observer must do a rotation (instead of just a simple reactive motion). The observer could choose to go to anywhere in $R3$. The shorter rotation in this case is moving to just to the border of $R3$.

Figure 8 shows the example of a non-convex corner. This example illustrates a compliant motion and how the observer keeps the rod configuration outside an escapable cell.

After elimination of the escapable cell, there are 5 regions on the xy-plane and 6 cells in the configuration space. The

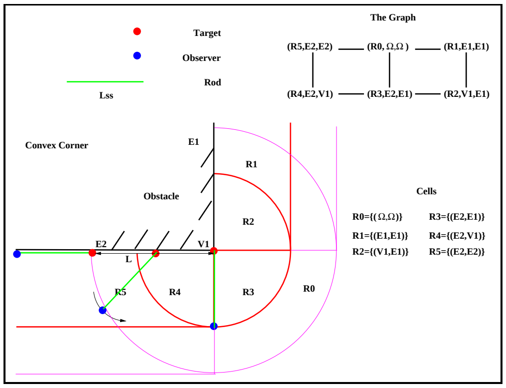

Fig. 7. Convex Corner

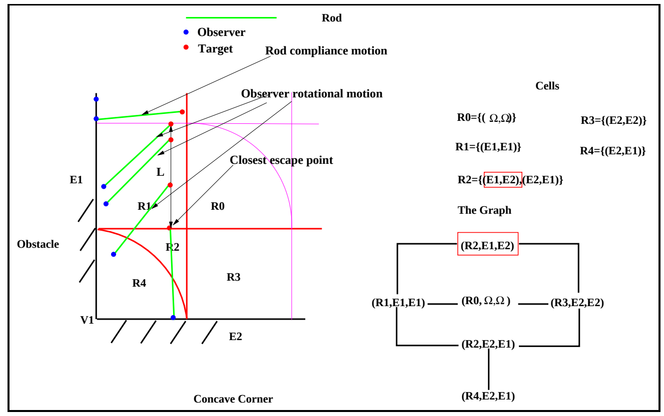

Fig. 8. Non-convex Corner

graph representing the polygon contains all the regions on the xy-plane, then a solution exists for certain initial configurations.

Let us assume that the target moves along the following path: The target starts in $R0$ while the observer is in $R1$; then the target moves towards $E1$ and finally, when it is close to $E1$ it changes direction and moves towards $E2$. Obviously, our algorithm does not know this information in advance (this is for illustration purposes only).

When the target starts moving, it forces the observer onto the obstacle $E1$. Note that the rod must be in a configuration that allows the observer to perform the compliant motion (see section V-C). As a consequence, a rotational observer motion may be required before the observer (endpoint of the rod) gets in contact with the obstacle. When the observer is in contact with the obstacle, it starts sliding along $E1$. Let's assume that it moves towards the corner. When the target is at $L^*(x, y, \theta)$ distance from the escape point (a point on the critical curve associate with an escapable cell), the rod is at configuration $(R1, E1, E1)$ and it would change to $(R2, E1, E2)$ if the current compliant motion was continued. However, this cell is an escapable cell. Therefore, the observer must rotate the rod to keep its configuration outside it and allows the observer to perform the compliant motion. After this, as the target moves closer to $E2$ the observer starts moving along the boundary of this edge.

VIII. Conclusions and Future Work

This work proposes an approach to maintain visibility of a moving evader with a mobile robot in a polygonal environment. The target moves continuously, its global trajectory is unknown but the distribution of obstacles in the workspace is known in advance. It is assumed that the delay between the target's motion and the observer's is zero. The approached consists in partitioning the configuration space and the workspace in non-critical regions separated by critical curves. We give necessary conditions for the existence of a surveillance strategy. A motion strategy that maintains target visibility is proposed. This motion strategy consists of three types of motions: reactive, compliant and rotational.

As future work, we plan on finding the optimal motion to define $L^*(x, y, \theta)$. In this work, it is assumed that the delay between the target and the observer motions is zero. This assumption was done to simplify the analysis, and better understand the problem. However, in order to get a more realistic model, considerable delay must be taken into account. we would also like to find a solution for the case of both delay and bounded observer speed.

References

[1] U.S. National Bureau of Standards, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. *Edited by M Abramowitz and I.A. Stegun*, 1964.

[2] J. Bañon, Implementation and Extension of the Ladder Algorithm *In Proc IEEE Int. Conf. on Robotics and Automation*, 1990.

[3] T. Başar and G. Olsder, Dynamic Noncooperative Game Theory. *Academic Press. 1982*.

[4] C. Becker, H. Gonzlez-Baños, J.-C. Latombe and C. Tomasi, An intelligent observer. In *Int. Symposium on Experimental Robotics*, 1995.

[5] B. Espiau, F. Chaumette, and P. Rives, A new approach to visual servoring in robotics. *IEEE Trans. Robot and Autom.*, 8(3):313-326, June 1992.

[6] S.M. LaValle, H.H. González-Baños, C. Becker and J.-C. Latombe, Motion Strategies for Maintaining Visibility of a Moving Target *In Proc IEEE Int. Conf. on Robotics and Automation*, 1997.

[7] J.-C. Latombe, Robot Motion Planning. Kluwer Academic Publishers, 1991.

[8] J.D. Lawerence, A Catalog of Special Plane Curves. *Dover Pub.*, 1971.

[9] http://mathworld.woldfram.com/ EllipticIntegraloftheSecondKind.html

[10] R. Murrieta-Cid, H.H. González-Baños and B. Tovar, A Reactive Motion Planner to Maintain Visibility of Unpredictable Targets *In Proc IEEE Int. Conf. on Robotics and Automation*, 2002.

[11] R. Murrieta-Cid, A. Sarmiento and S. Hutchinson, A Motion Planning Strategy to Maintain Visibility of a Moving Target at a Fixed Distance in a Polygon *In Proc IEEE Int. Conf. on Advanced Robotics*, 2003.

[12] R. Murrieta-Cid, A. Sarmiento and S. Hutchinson, On the Existence of a Strategy to Maintain a Moving Target within the Sensing Range of an Observer Reacting with Delay *In Proc IEEE Int. Conf. on Intelligent Robots and Optimal Systems*, 2003.

[13] J. O'Rourke, Visibility. *In Handbook of Discrete and Computational Geometry, 467-479*. J.E. Goodman and J. O'Rourke Ed. 1997.

[14] S. Petitjean, D. Kriegman and J. Ponce, Computing exact aspect graphs of curved objects: algebraic surfaces. *Int J. Comput Vis., 9:231-255*, Dec 1992.

[15] J.T. Schwartz and M. Sharir, On the Piano Movers' Problem: I. The Case if a Two-Dimensional Rigid Polygon Body Moving Amidst Polygonal Barriers, *Communications on Applied Mathematics*, 36, 345-398, 1987.