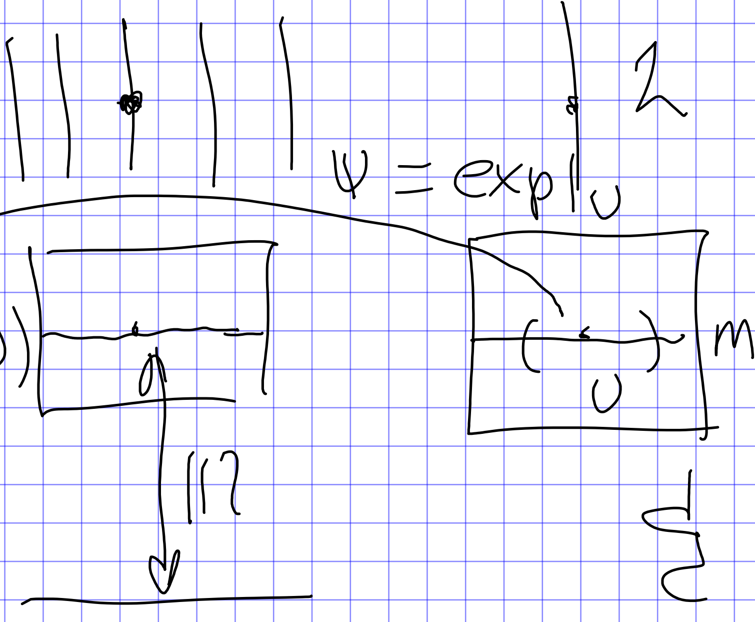
$$\begin{tikzcd}
\mathfrak{h} \arrow[r, "\exp"] \arrow[d, hook] & H \arrow[d, hook] \\
\mathfrak{m} \oplus \mathfrak{h} = \mathfrak{g} \arrow[r, "\exp"] \arrow[d] & G \supseteq \psi(U) \arrow[d, "\pi"] \\
\mathfrak{m} \cong \mathfrak{g}/\mathfrak{h} & G/H \supseteq N_0 \\
U \arrow[u, phantom, "\cup\!\!\mid"] \arrow[ur, bend right] &
\end{tikzcd}$$



Esquema del Lema 4.1 en la página 123

Como $\pi : G \longrightarrow G/H$ es mapeo cociente:

$$\pi(\exp(U)\underbrace{\exp(U_{\mathfrak{h}})}_{\text{en } H}) = \pi(\exp(U)) = \pi(\psi(U)) = N_0$$

$\therefore$ tenemos los homeomorfismos:

$$\begin{array}{ccccc}
0 \in U & \xrightarrow{\ \psi\ } & \psi(U) & \xrightarrow{\ \pi\ } & N_0 \ni p_0 \\
\cap\!\!\mid & \uparrow & \cap\!\!\mid & & \cap\!\!\mid \quad \parallel \\
\mathfrak{m} & \exp & G & & G/H \quad eH
\end{array}$$

$\psi(U)$ es local cross section pues:

$$\begin{tikzcd} G \supseteq \psi(U) \arrow[d, "\pi"'] \\ G/H \supseteq N_0 \arrow[u, bend right, "\pi|_{\psi(U)}^{-1}"'] \end{tikzcd}$$

$\pi\big|_{\psi(U)}^{-1} = \pi^{-1}$

con $\pi\big|_{\psi(U)}$ es homeomorfismo

$\therefore$ $\forall\ x \in G/H$ $\pi^{-1}(x)$ es un elemento en la clase de x:

$$x = \pi^{-1}(x)H$$

En la página 124:

$$\begin{tikzcd} G \times B \arrow[r, "\Phi"] \arrow[d, "I\times\pi"'] & G \arrow[d, "\pi"] \\ G \times N_0 \arrow[r, "\Lambda"] & G/H \end{tikzcd}$$

$\Lambda =$

$= \pi \circ \Phi \circ (I \times \pi)^{-1}$

$\pi = \pi|_B$

$$G \times N_0 \ni (g, xH) \longmapsto gxH$$

$G \times N_0 \subseteq G \times G/H$

Ejemplos de la definición en la página 125:

$$\mathbb{P}^n(\mathbb{C}) \cong GL(n+1,\mathbb{C}) \Big/ \left\{ \begin{pmatrix} * & * \\ 0 & \\ \vdots & * \\ 0 & \end{pmatrix} \right\}$$

$$S^n \cong O(n+1) \big/ O(n) \cong SO(n+1) \big/ SO(n)$$

$$\begin{aligned} O(n) &\hookrightarrow O(n+1) \\ A &\longmapsto \begin{pmatrix} A & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

$$Gr(\mathbb{C}^n, \mathbb{C}^k) = \{ V \subseteq \mathbb{C}^n \mid \dim_{\mathbb{C}} V = k \}$$

$$\cong GL(n,\mathbb{C}) \Big/ \underbrace{\left\{ \begin{array}{c} \begin{array}{cc} k & n-k \end{array} \\ \begin{pmatrix} * & * \\ 0 & * \end{pmatrix} \end{array} \begin{array}{l} \\ k \\ n-k \end{array} \right\}}_{H}$$

$$\cong U(n) \big/ U(k) \times U(n-k)$$

$$(A, B) \longmapsto \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$$

Sobre la isotropía lineal:

$$h \in H \;\therefore\; h \cdot \pi(e) = \pi(e)$$

$$\therefore \quad d\tau(h)_{\pi(e)} : T_{\pi(e)} G/H \longrightarrow T_{\pi(e)} G/H$$

$$d\tau(h)_{\pi(e)} \in GL(T_{\pi(e)} G/H)$$

$$\begin{aligned} H &\longrightarrow GL(T_{\pi(e)} G/H) \\ h &\longmapsto d\tau(h)_{\pi(e)} \end{aligned}$$

cuya imagen es H^*.

Un detalle sobre $Gr(\mathbb{C}^n, \mathbb{C}^k)$.

Construimos:

$$\begin{tikzcd} U(n) \arrow[r, hook] \arrow[d] & GL(n,\mathbb{C}) \arrow[r] & GL(n,\mathbb{C})/H \\ U(n)/U(k)\times U(n-k) \arrow[urr, dashed, "\exists F"] & & \end{tikzcd}$$

biyectivo cont.
homeomorfismo

pues $U(n) \cap H = U(k) \times U(n-k)$

Más aún, se ve que F es difeomorfismo con los argumentos del libro.

Observación:

$$\dim G/H = \dim \mathfrak{m} = \dim \mathfrak{g} - \dim \mathfrak{h} = \dim G - \dim H.$$

Para la proposición 4.4 generalizamos a:

$$N \hookrightarrow G \longrightarrow G/H$$

N <u>no</u> necesariamente es transitivo en G/H. Pero!

$$\begin{tikzcd} N \arrow[r, hook] \arrow[d] & G \arrow[d] \\ N/N\cap H \arrow[r, hook, "\varphi"] & G/H \end{tikzcd}$$

$N/N\cap H$ y G/H son variedades analíticas

$$\varphi\left(N/_{N \cap H}\right) = NeH$$