

$$\mathbb{C}^n \cong \mathbb{R}^{2n} \quad z = (z_1, \dots, z_n) = (x_1 + iy_1, \dots, x_n + iy_n)$$

$$\omega_0 = \sum_{j=1}^n dx_j \wedge dy_j$$

$$\sum_{j=1}^n dz_j \wedge d\bar{z}_j = \sum_{j=1}^n (dx_j + idy_j) \wedge (dx_j - idy_j)$$

$$= \sum_{j=1}^n (-idx_j \wedge dy_j + idy_j \wedge dx_j)$$

$$= -2i \sum_{j=1}^n dx_j \wedge dy_j$$

$$\therefore \frac{i}{2} \sum_{j=1}^n dz_j \wedge d\bar{z}_j = \sum_{j=1}^n dx_j \wedge dy_j = \omega_0$$

---

$$z \in \mathbb{C} : \underbrace{z + \bar{z}} \in \mathbb{R} \ni i(z - \bar{z}) = -2\operatorname{Im}(z)$$

expresión compleja.

---

$$\omega = i \sum_{j,k=1}^n g_{jk} dz_j \wedge d\bar{z}_k$$

$$\omega = -i \sum_{j,k=1}^n g_{j\bar{k}} d\bar{z}_j \wedge dz_k$$

$$= -i \sum_{j,k=1}^n g_{k\bar{j}} (-dz_k \wedge d\bar{z}_j)$$

$$= i \sum_{j,k=1}^n g_{k\bar{j}} dz_k \wedge d\bar{z}_j = \omega$$