Con el grupo $H(n)$ en D_n:

$$z \longmapsto (r^{\frac{1}{2}} t' z', r z_n) \qquad t' \in \mathbb{T}^{n-1}, r > 0.$$

la función $z \longmapsto \operatorname{Im}(z_n) - |z'|^2$
NO es invariante:

$$\operatorname{Im}(z_n) - |t' z'|^2 = \operatorname{Im}(z_n) - |z'|^2 \checkmark$$

$$\operatorname{Im}(r z_n) - |r^{\frac{1}{2}} z'|^2 = r(\operatorname{Im}(z_n) - |z'|^2) \times$$

Otras funciones:

$$|r^{\frac{1}{2}} z_j|^2 = r|z_j|^2 \qquad j = 1, \dots, n$$

$$\operatorname{Re}(r z_n) = r \operatorname{Re}(z_n)$$

$$\operatorname{Im}(r z_n) = r \operatorname{Im}(z_n)$$

que tampoco son invariantes.

Sea $D = G/K$ dominio simétrico acotado donde:

$$G = Aut_0(D)$$
$$K = \{\varphi \in G \mid \varphi(z_0) = z_0\}$$

$z_0 \in D$ fijo. D puede substituirse por otros dominios simétricos (e.g. D_n).

Métrica de Bergman:

$$g_{j\bar{h}}\Big|_z = \frac{\partial^2 \log K(z,\bar{z})}{\partial z_j \partial \bar{z}_h}$$

$K(z,\bar{z})$ es el Kernel de Bergman.

Teorema (Helgason)
Si D es irreducible, entonces la métrica de Bergman g es la única métrica Hermitiana (salvo por $c>0$ multiplicativa) G-invariante:

$$g_{\varphi(z)}(d\varphi_z(\ ),\, d\varphi_z(\ )) = g_z(\ ,\ ).$$

$\forall\, \varphi \in G$.

De aquí se calcula g como sigue:

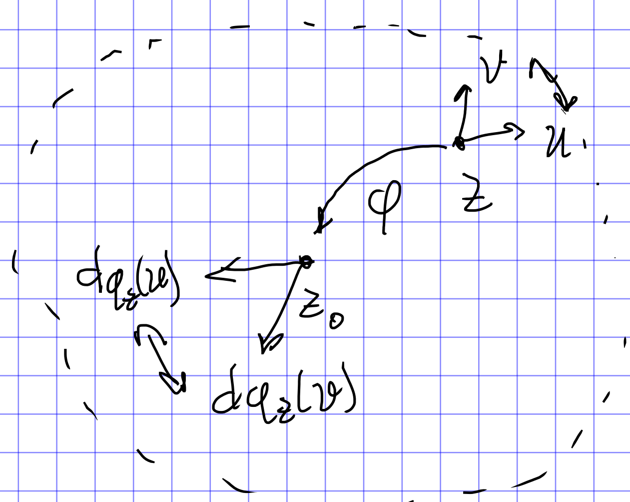
1) Hallamos g_{z_0} tal que:

$$g_{z_0}(d\varphi_{z_0}(\ ),\ d\varphi_{z_0}(\ )) = g_{z_0}(\ ,\ )$$

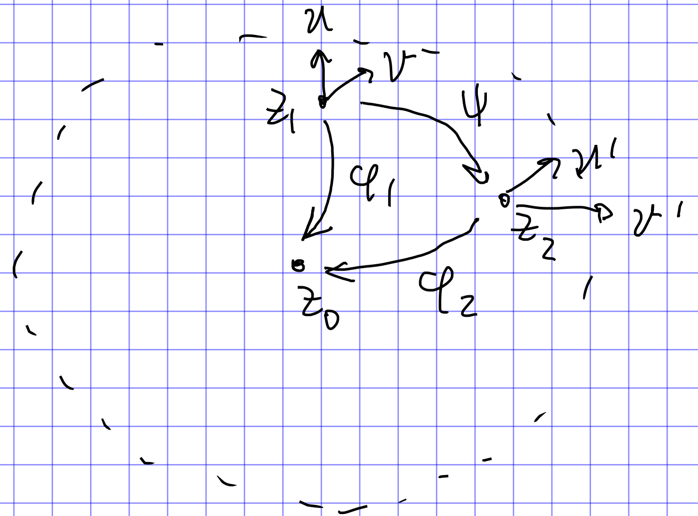
$\forall\ \varphi \in K$. $(\varphi(z_0) = z_0)$.

2) Dado $z \in D$, hallamos $\varphi \in G \ni \varphi(z) = z_0$ y definimos:

$$g_z(\ ,\ ) = g_{\underset{\underset{z_0}{\shortparallel}}{\varphi(z)}}(d\varphi_z(\ ),\ d\varphi_z(\ ))$$



¿g es G-invariante en todo D?

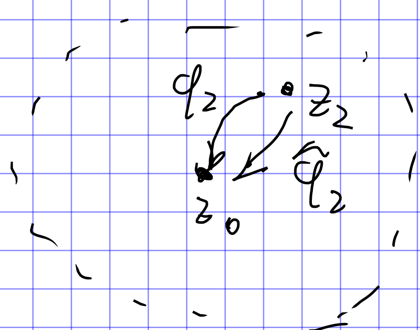


$\psi \in G$

$\psi(z_1) = z_2$

Fig. 1.

Reemplazamos a φ_2 de modo que $\varphi_1 = \varphi_2 \circ \psi$. Se puede hacer porque:



$$\hat{\varphi}_2 = \underbrace{\hat{\varphi}_2 \circ \varphi_2^{-1}}_{\substack{\shortparallel \\ \alpha \in K}} \circ \varphi_2$$

$$\therefore g_{z_2}(\ , \ ) = g_{z_0}(d(\hat{\varphi}_2)_z(\ ), d(\hat{\varphi}_2)_z(\ ))$$

$$= g_{z_0}(d\alpha_{z_0}(d(\varphi_2)_z(\ )), d\alpha_{z_0}(d(\varphi_2)_z(\ )))$$

$$= g_{z_0}(d(\varphi_2)_z(\ ), d(\varphi_2)_z(\ ))$$

Entonces en la figura 1.

$$\underbrace{d(\varphi_1)_{z_1}}_{\text{isom.}} = \underbrace{d(\varphi_2)_{z_2}}_{\text{isom.}} \circ d\psi_{z_1}$$

$\Rightarrow d\psi_{z_1}$ isometría.

Para $\mathbb{D} = \{|z| < 1\}$:

$$z_0 = 0, \quad \mathbb{D} = SU(1,1) \Big/ \underset{\smile}{\mathbb{T}}$$

$$g_0(u, v) = u\bar{v} \qquad u, v \in \mathbb{C}$$

$$\longleftrightarrow dz \otimes d\bar{z} \text{ en } 0.$$

$\forall z \in \mathbb{D}$:

$$\varphi(w) = \frac{w - z}{1 - \bar{z} w}$$

$$\therefore \quad g_z(u, v) = g_0(\varphi'(z) u, \varphi'(z) v)$$

$$\varphi'(w) = \frac{(1 - \bar{z} w) 1 - (w - z)(-\bar{z})}{(1 - \bar{z} w)^2}$$

$$\varphi'(z) = (1 - |z|^2)^{-1}$$

$$\therefore \quad g_z(u, v) = \frac{u\bar{v}}{(1 - |z|^2)^2}$$

$$\therefore \quad g = \frac{dz \otimes d\bar{z}}{(1 - |z|^2)^2}$$