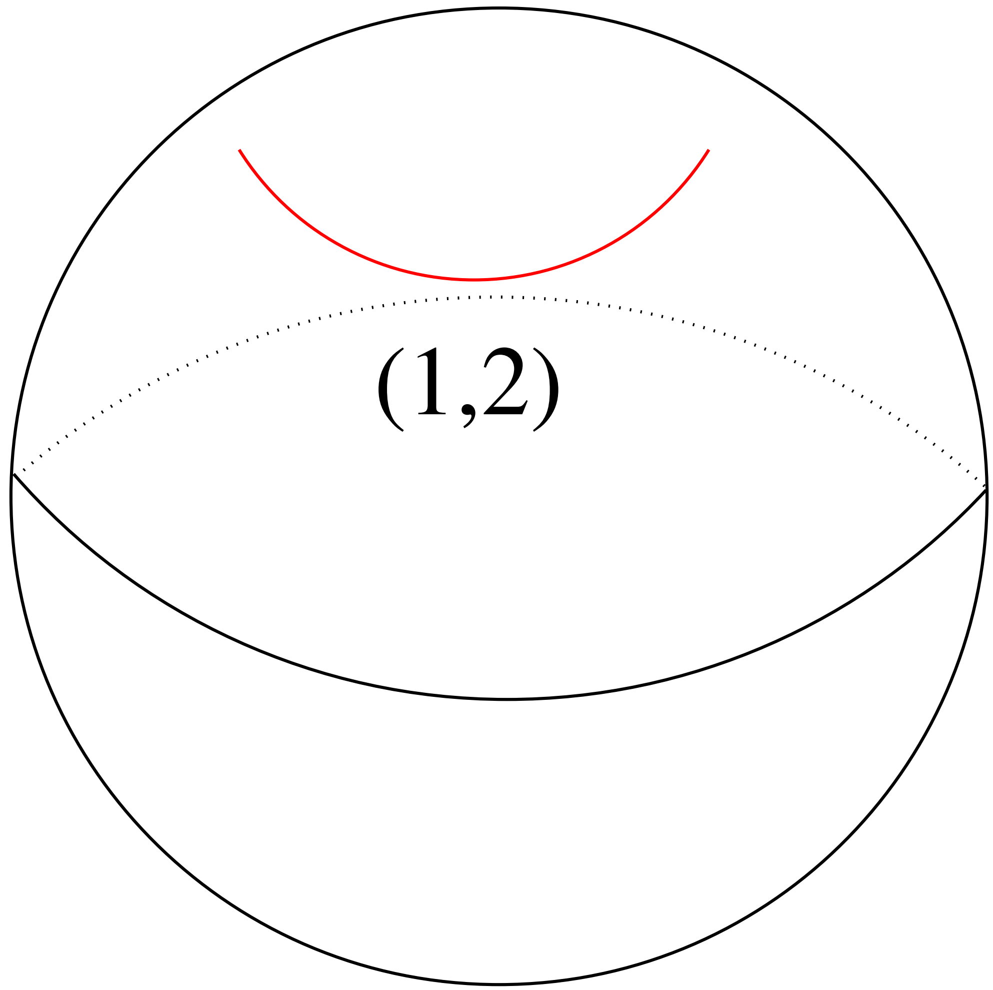
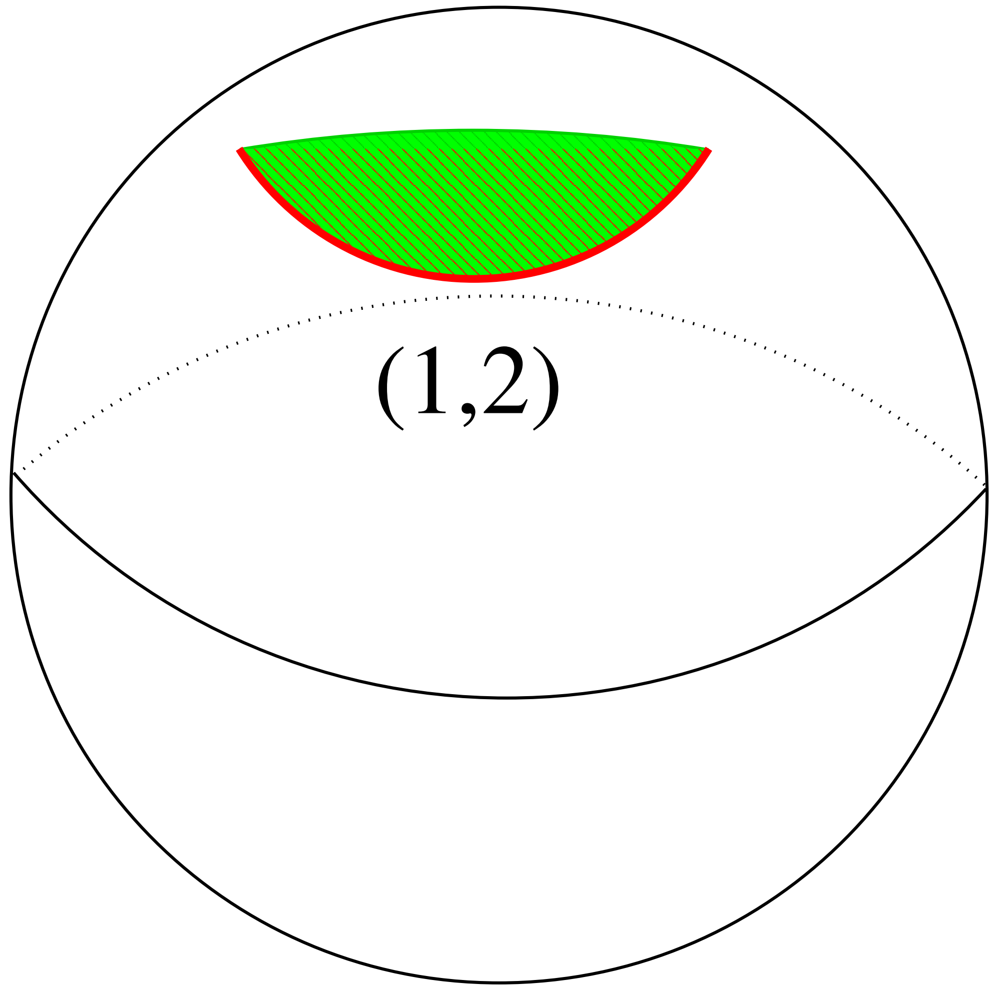
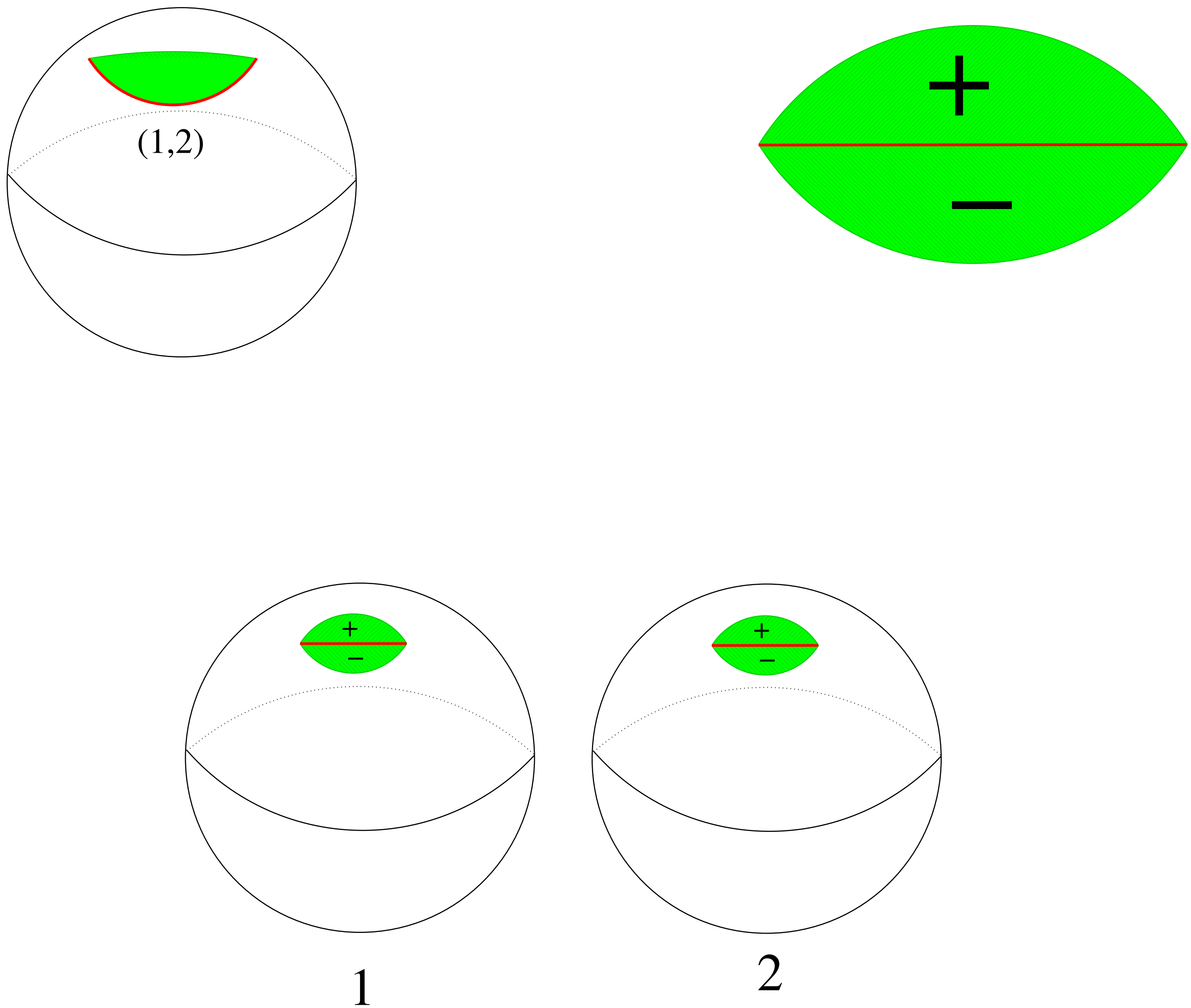
On universal Montesinos knots

Víctor Núñez
Cimat



A 3-ball B with
a properly embedded arc $\alpha \subset B$, and
a permutation $(1, 2) \in S_n$.


(1,2)


(1,2)
+
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1
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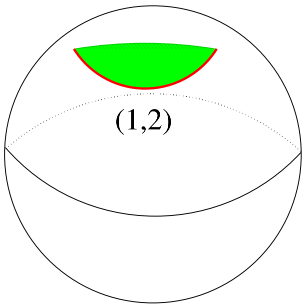
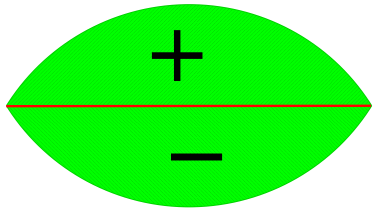

(1,2)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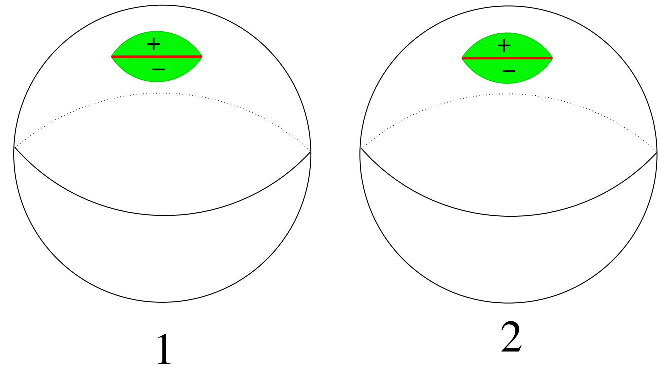
+
−
+
−
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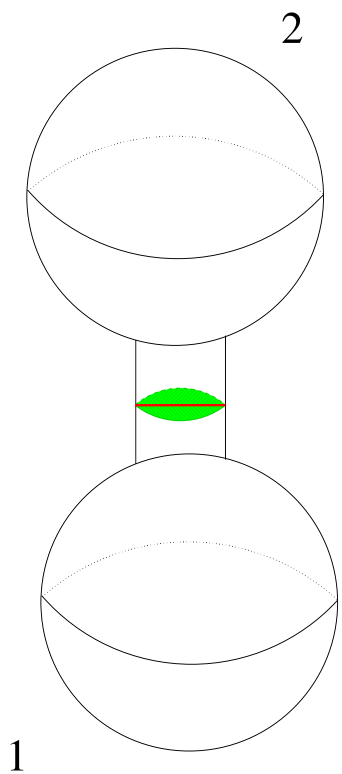
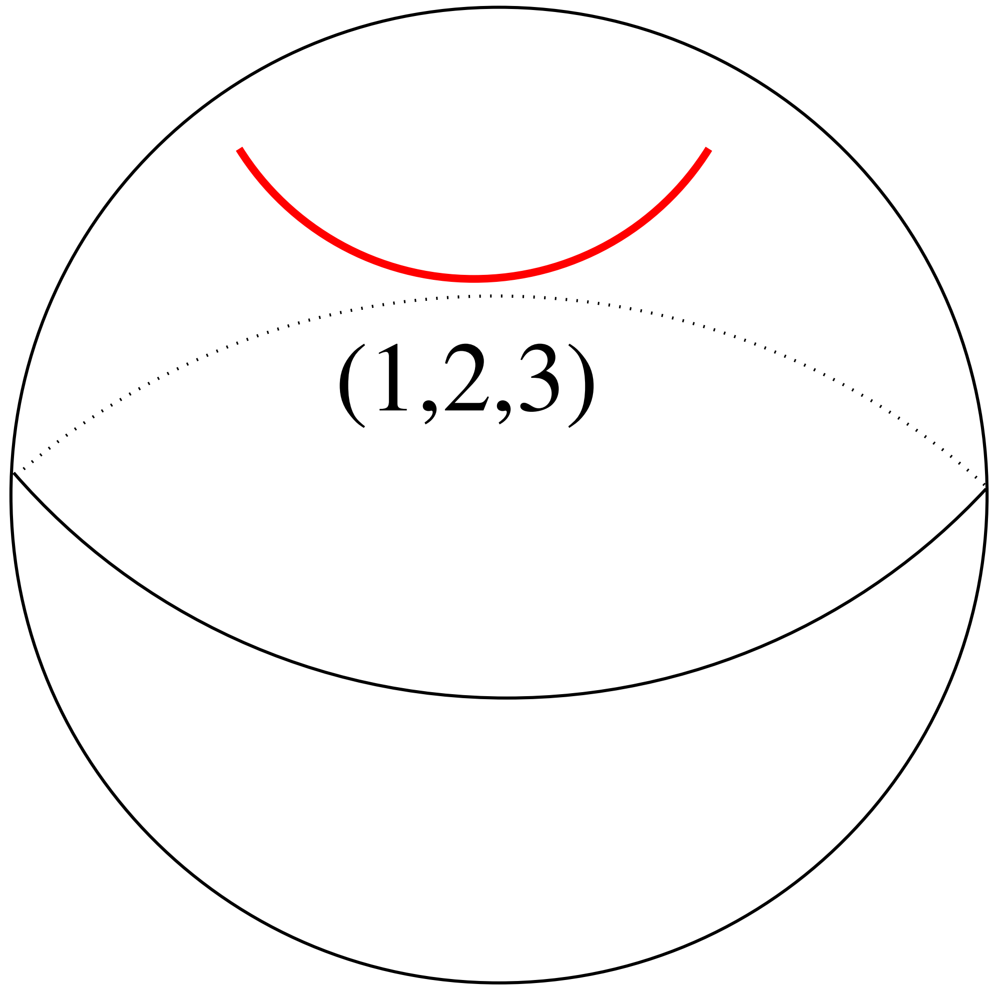
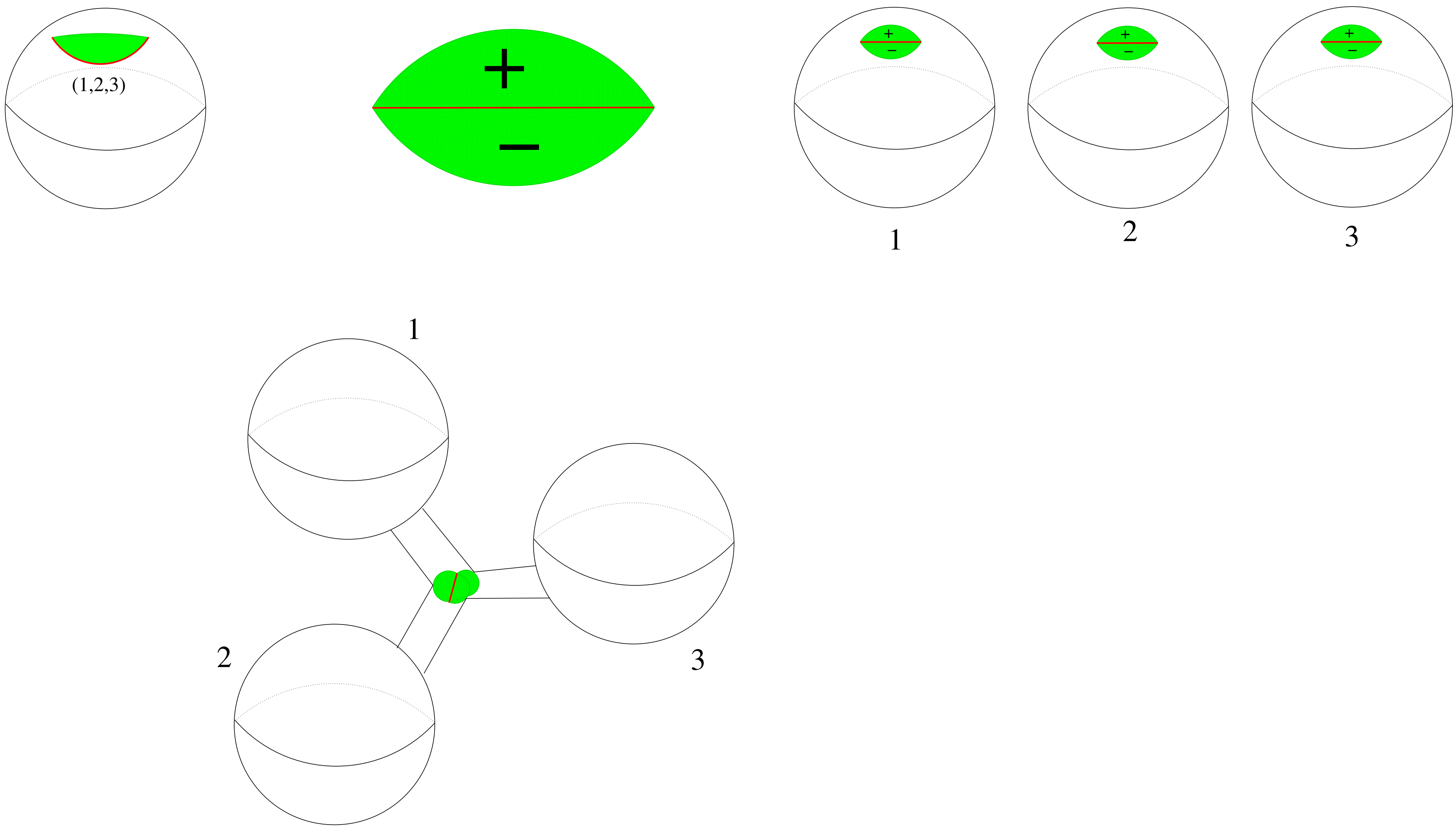
2


2
1


(1,2,3)


(1,2,3)
+
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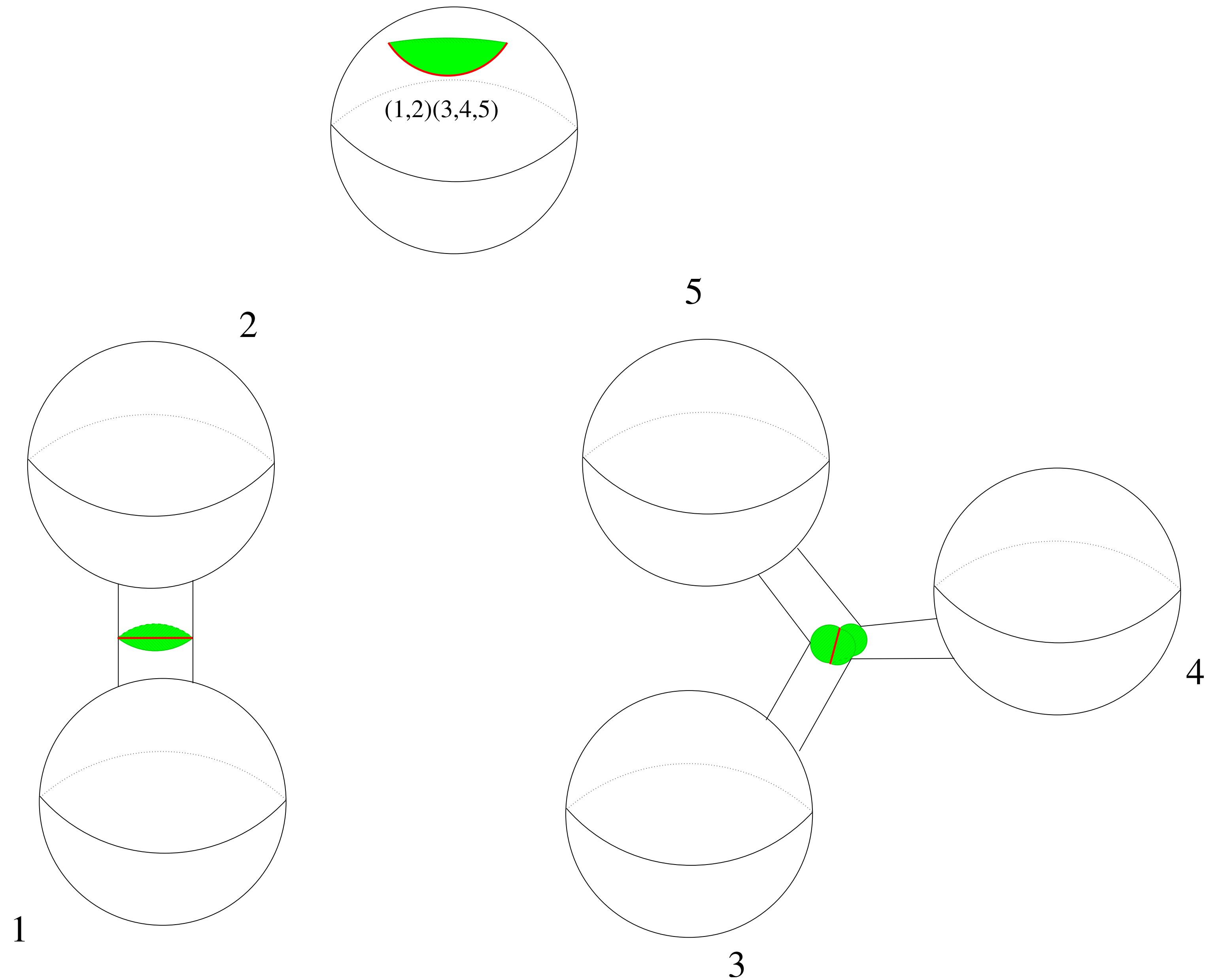
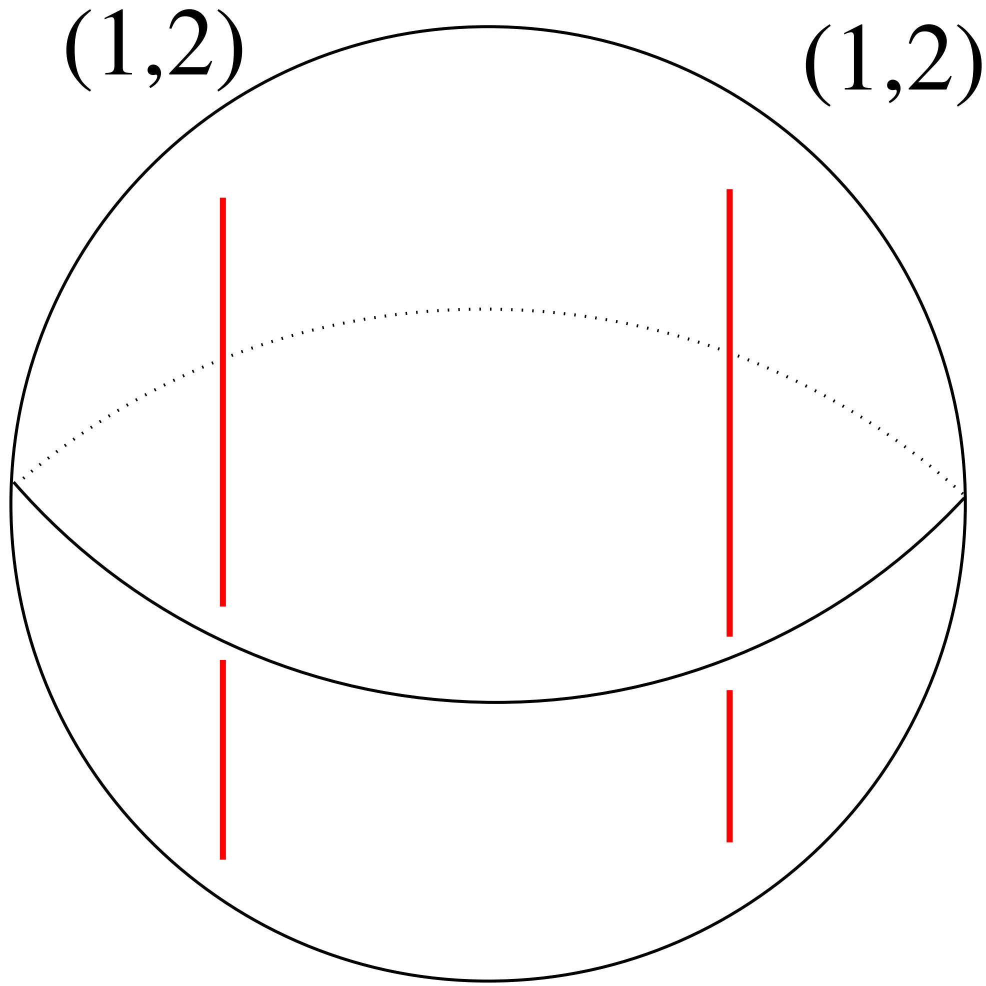
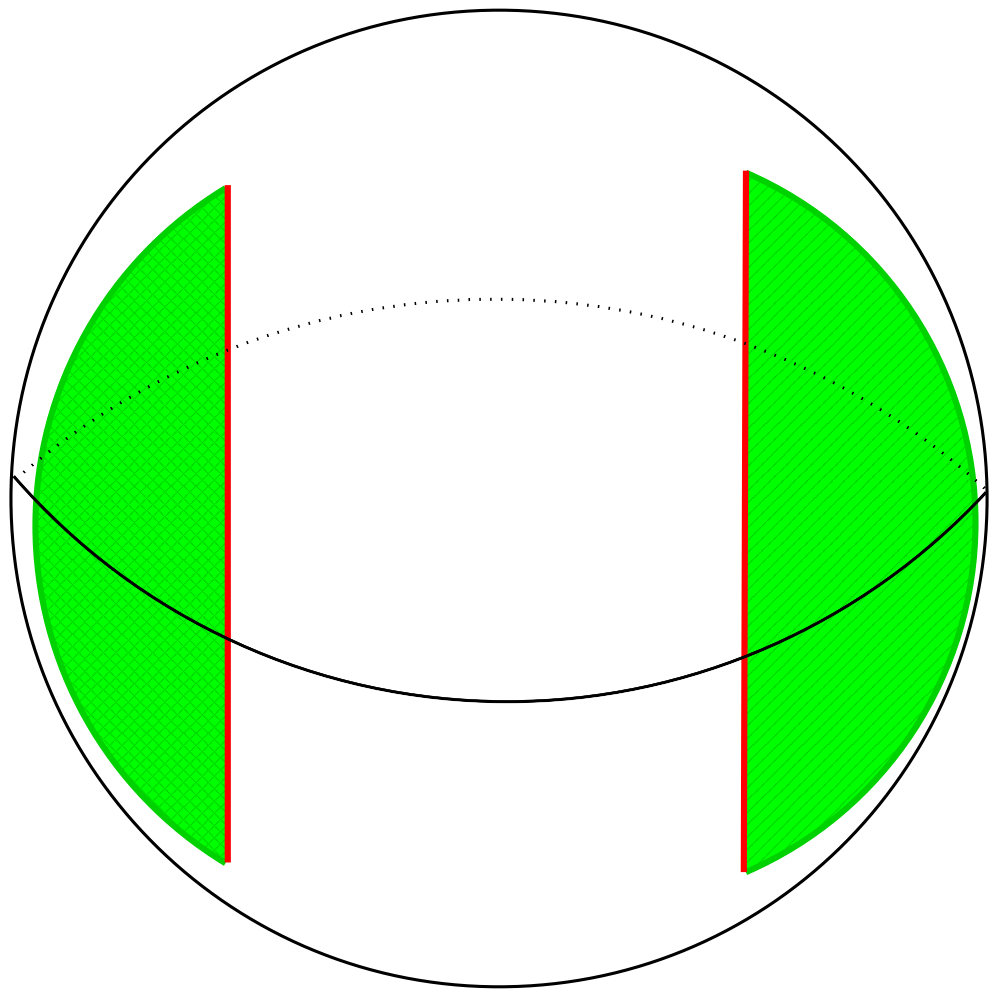
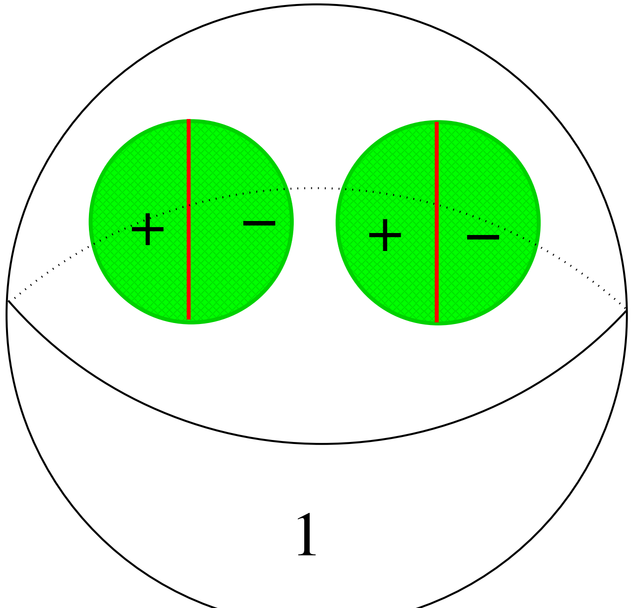
1
2
3
1
2
3

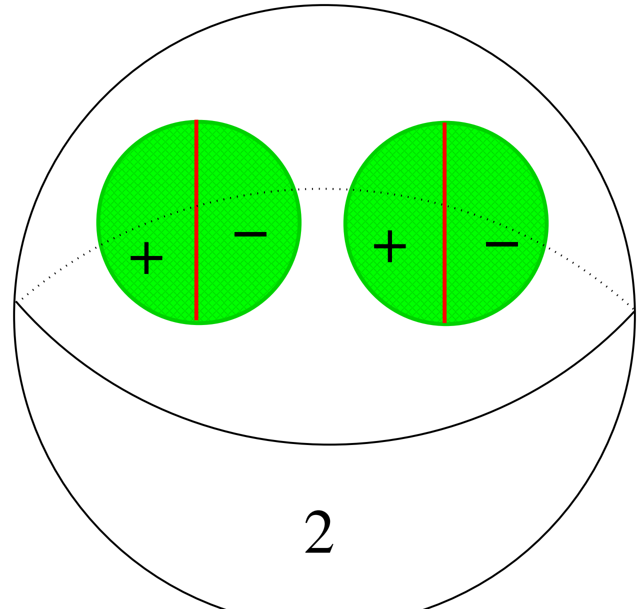

(1,2)(3,4,5)
2
1
5
4
3


(1,2)
(1,2)

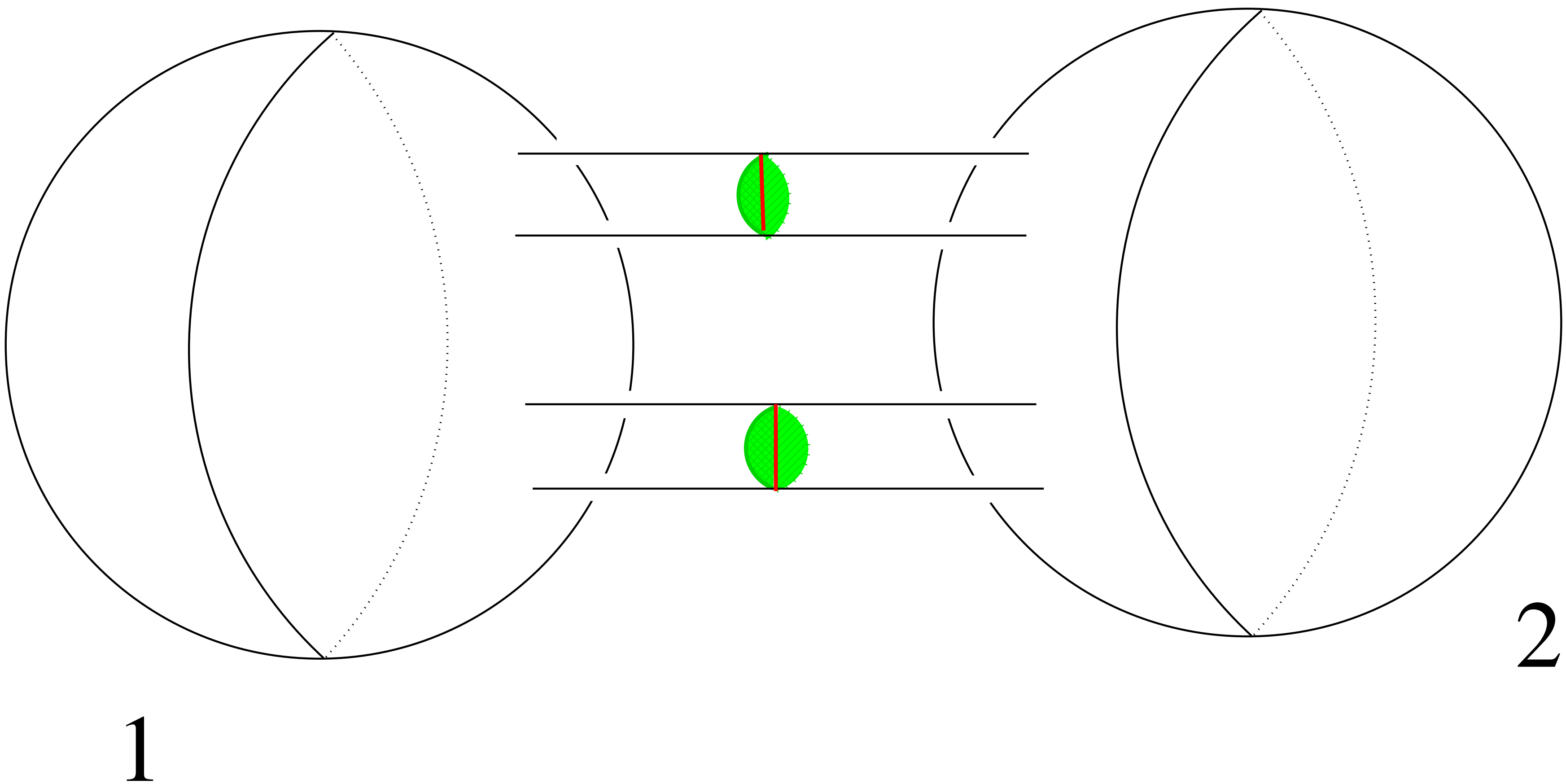




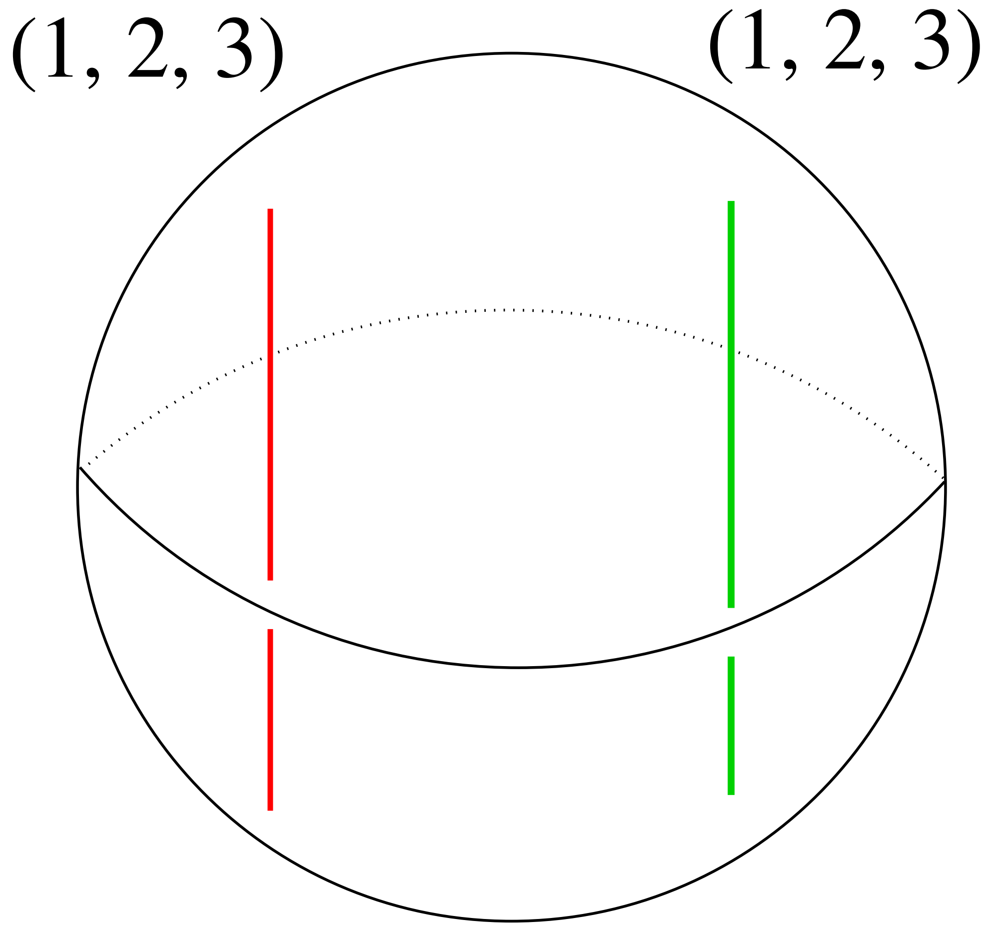
1

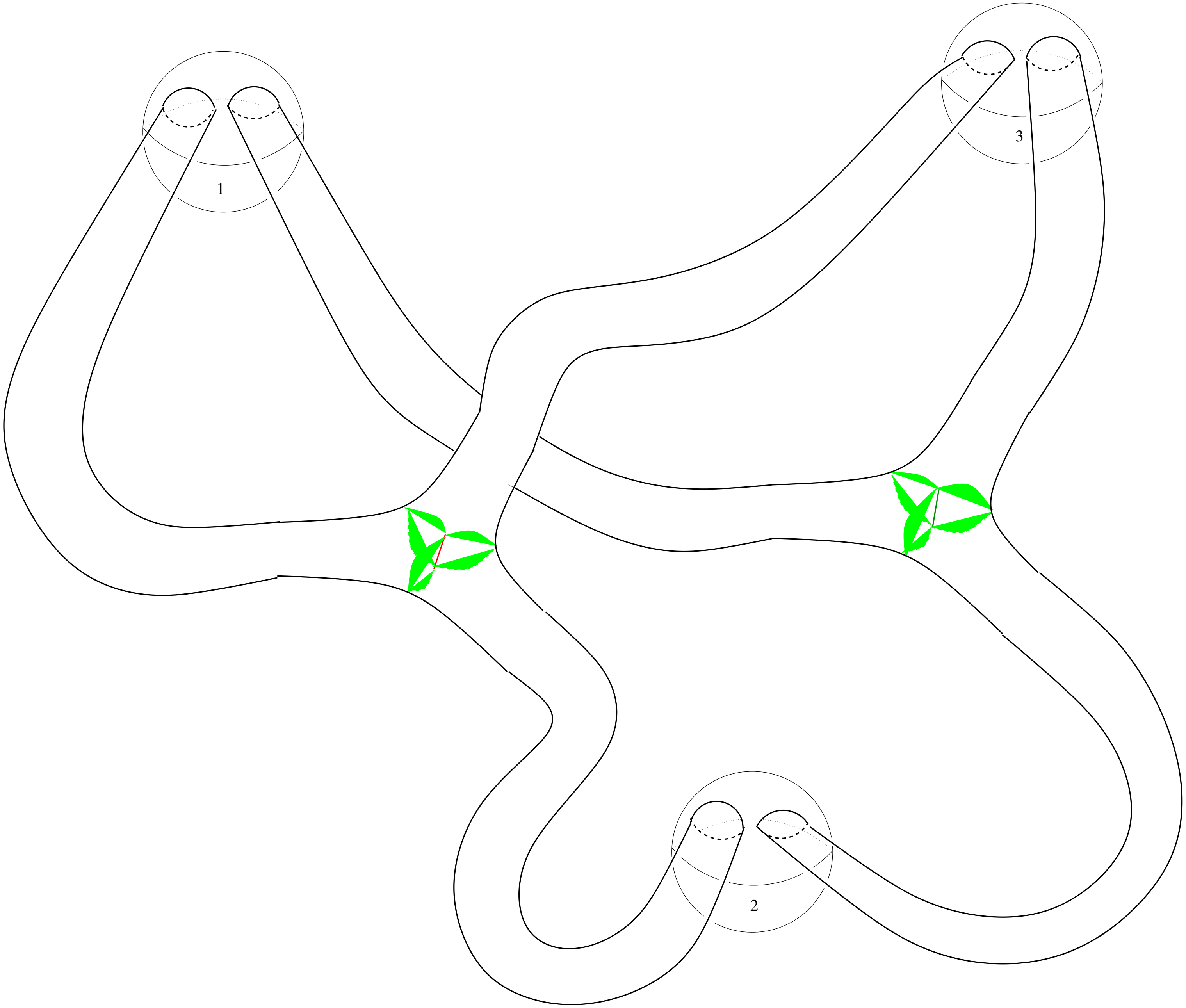


2



1
2


(1, 2, 3)
(1, 2, 3)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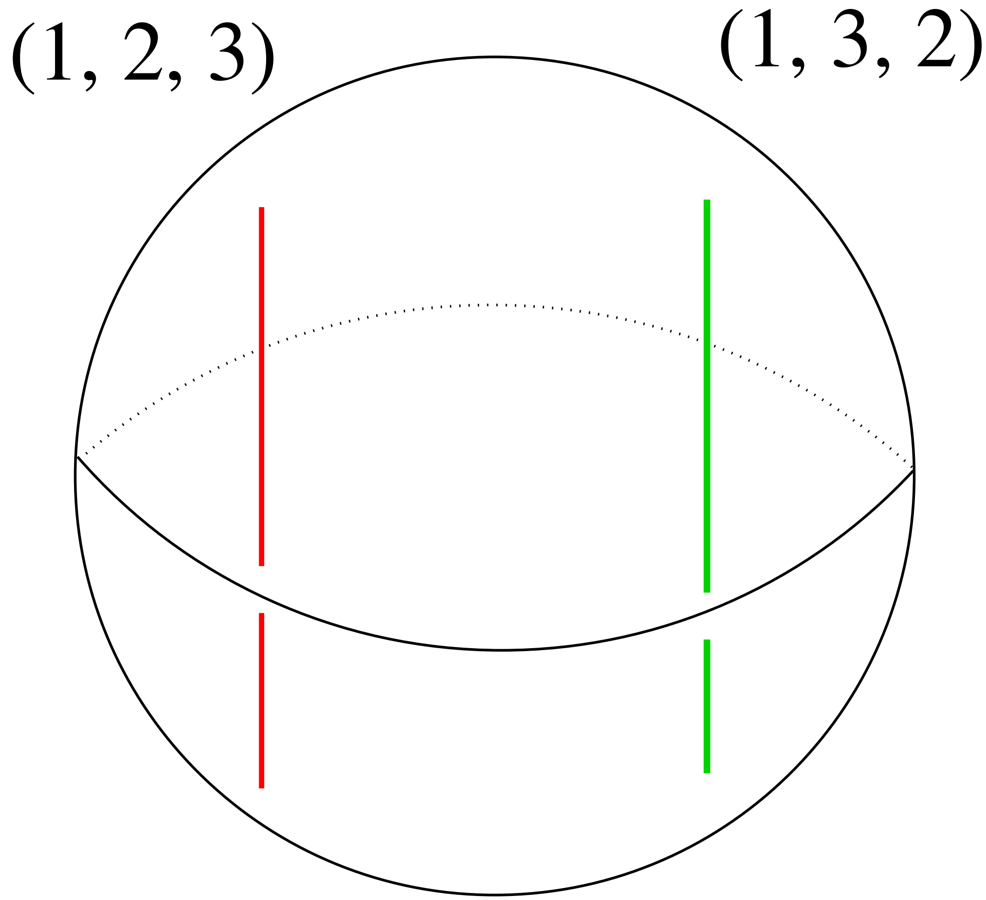
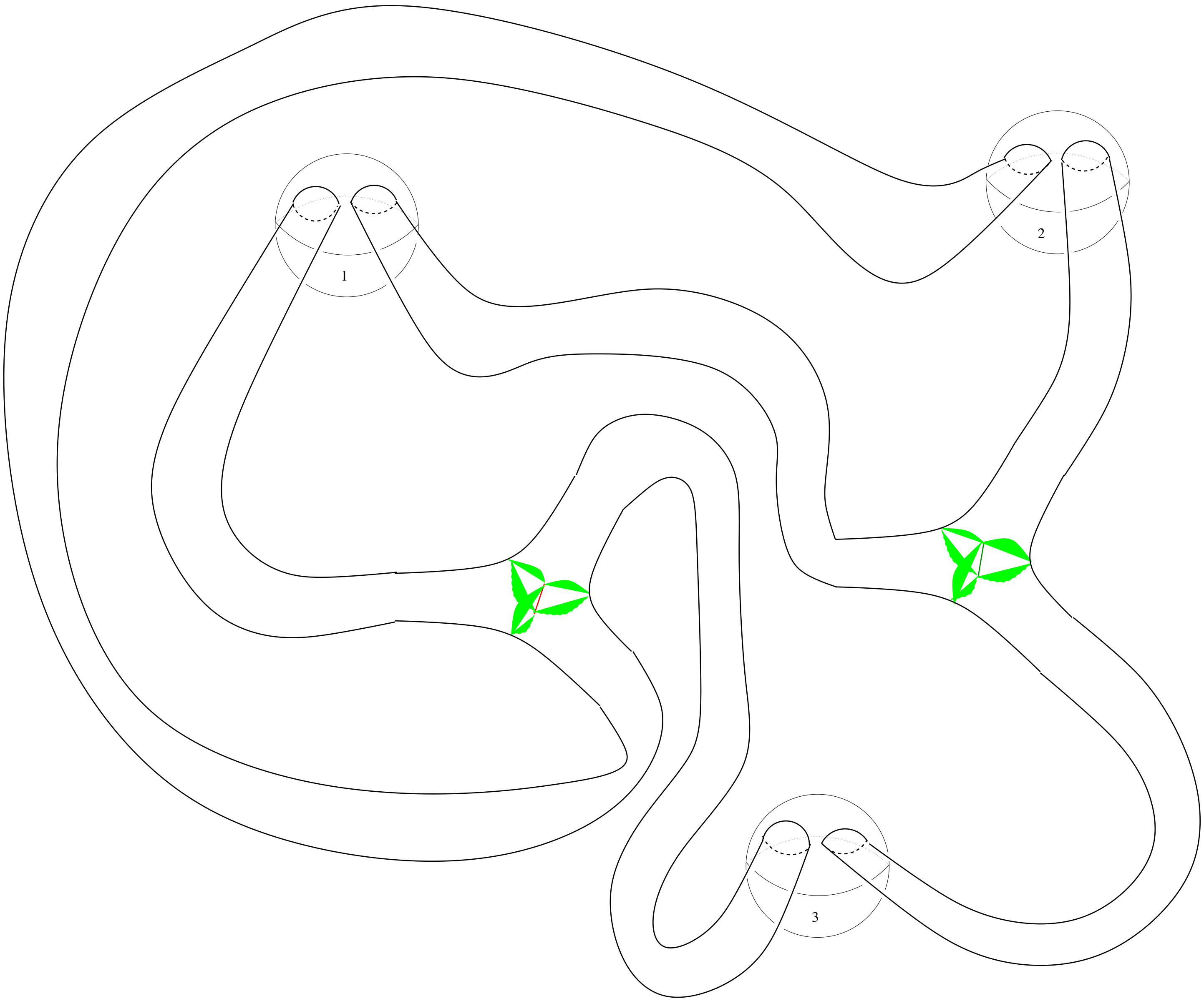
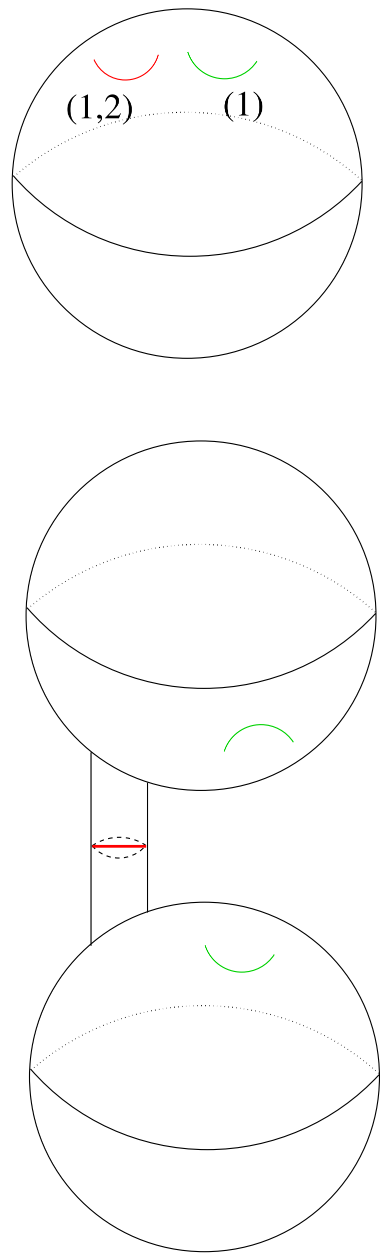
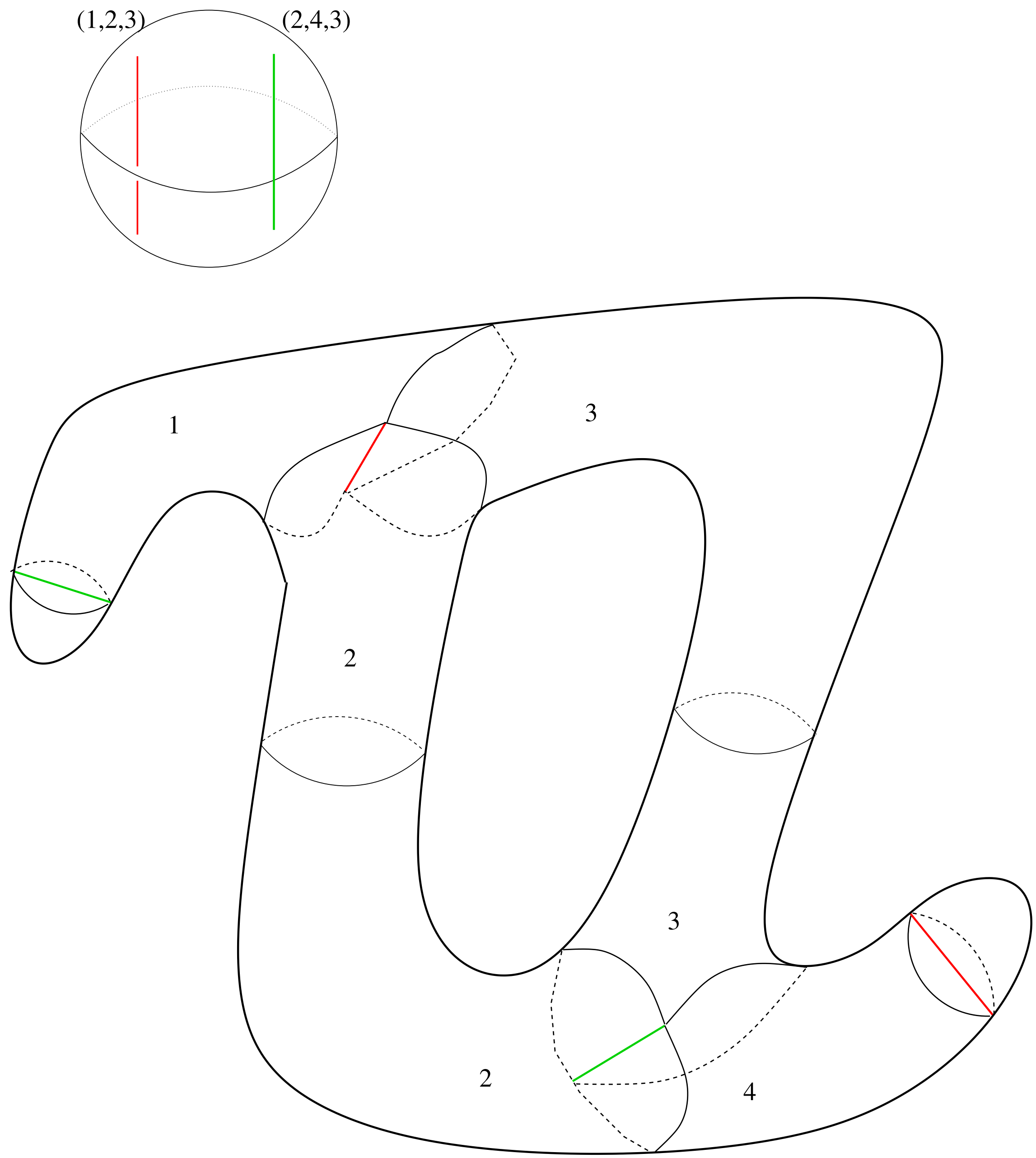
2


(1, 2, 3)
(1, 3, 2)

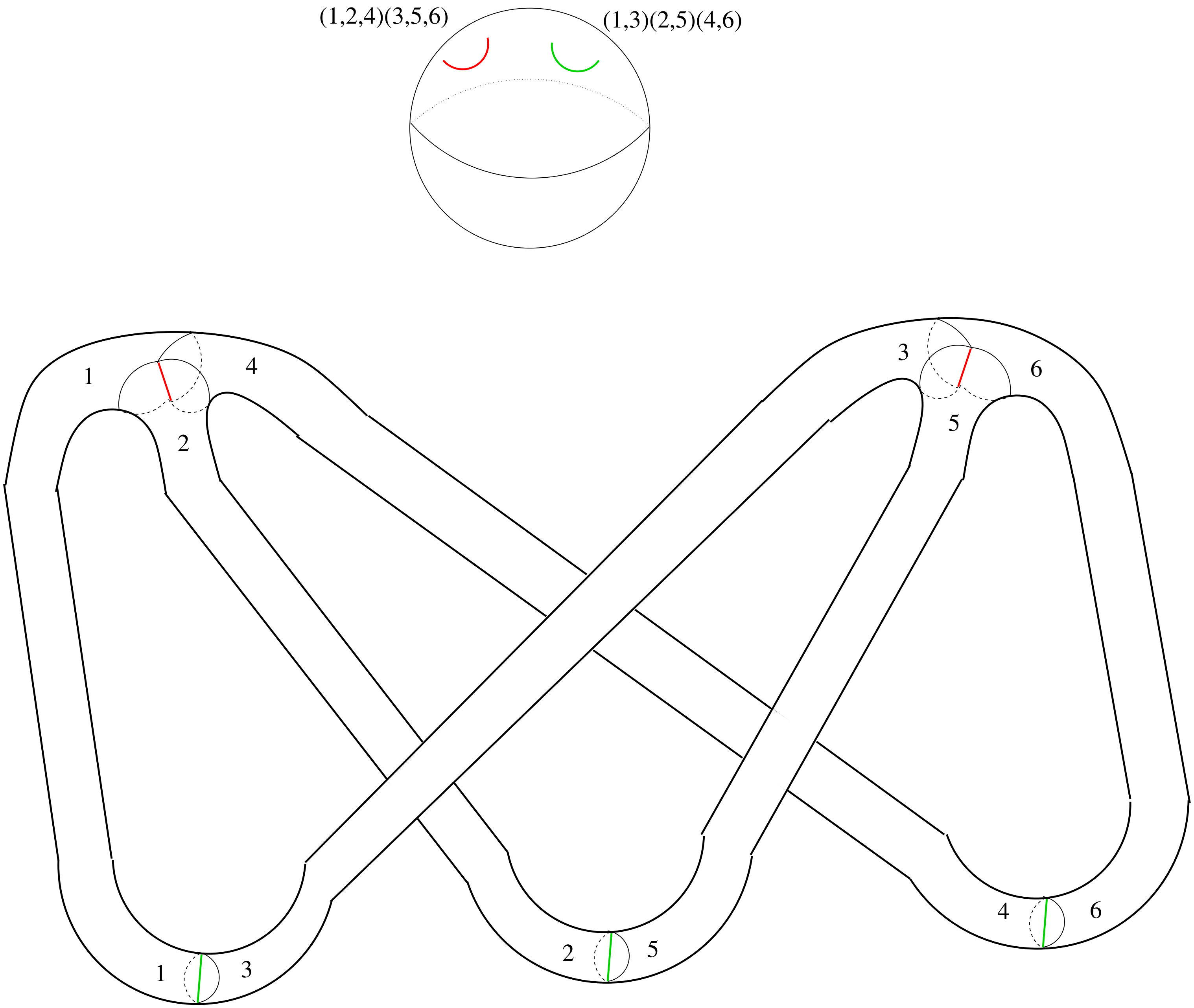

1
2
3



(1)=identity



(1,2,3)
(2,4,3)
1
3
2
3
2
4



(1,2,4)(3,5,6)
(1,3)(2,5)(4,6)
1
4
2
3
6
5
1
3
2
5
4
6

We got a function $\varphi : M \to N$ which is

- continuous,
- open, and
- proper.

For each $x \in N$ the number $\#\varphi^{-1}(x) = n$ is fixed, except for the points of a codimension 2 subset $K \subset N$.

Definition. A function $\varphi : M^m \longrightarrow N^m$ is called an n-fold branched covering if φ is continuous, open and proper, and there exists a codimension 2 submanifold $k \subset N$ such that

$$\varphi : M - \varphi^{-1}(k) \to N - k$$

is an n-fold covering space.
(k is properly embedded in N).

One says that φ is branched along k.

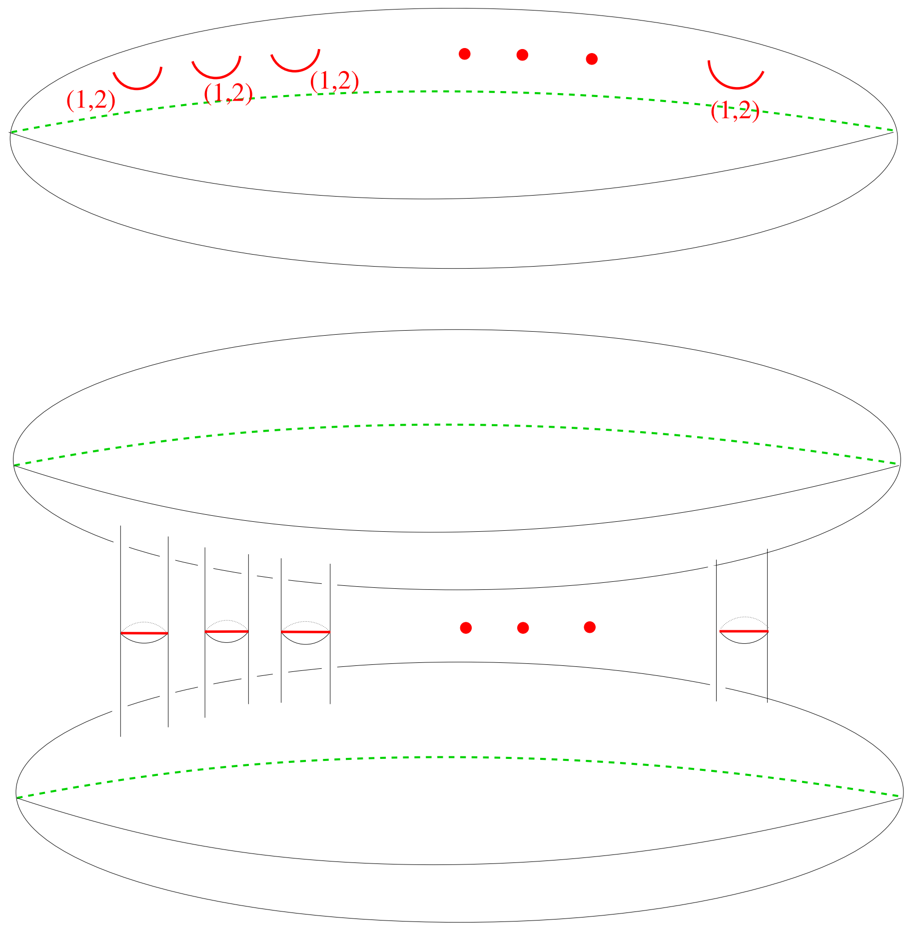
For a given n-fold branched covering $\varphi : M \longrightarrow (N, k)$, one has an associated representation (a homomorphism):

$$\omega_\varphi : \pi_1(N - k) \longrightarrow S_n.$$

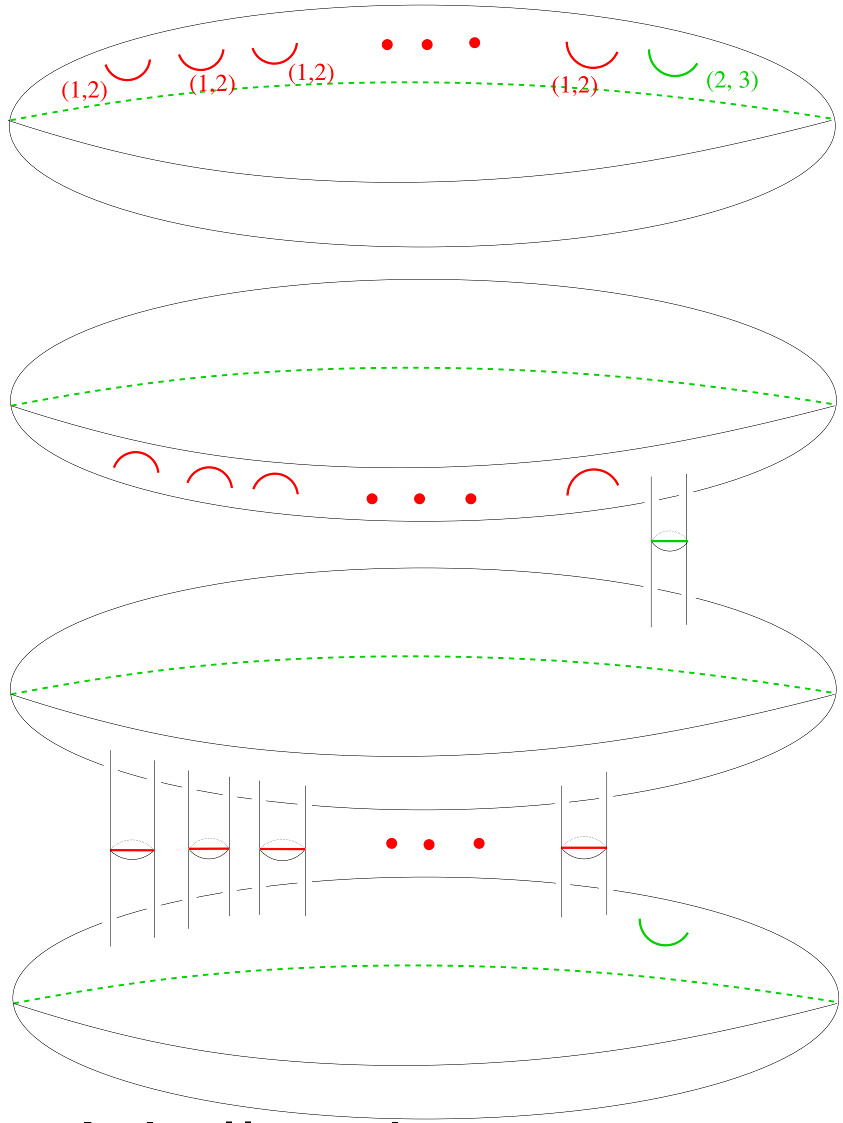
For a given representation $\omega : \pi_1(N - k) \longrightarrow S_n$, one has an associated n-fold branched covering

$$\varphi_\omega : M \longrightarrow (N, k).$$

A branched covering $\varphi : M \longrightarrow (N, k)$ is called simple if its associated representation sends each meridian of k into a 2-cycle.



A ball with $g+1$ arcs
gives a handlebody with g handles.



A ball with $g+2$ arcs
gives a handlebody with g handles.

Theorem. (Heegaard)

M is a closed connected orientable 3-manifold

$\Leftrightarrow$

M is the union of two orientable handlebodies glued along their boundaries.

$$\begin{array}{c} V \\ {\scriptstyle\varphi}\big\downarrow \\ B^3 \end{array} = \begin{array}{c} V_1 \\ \big\downarrow{\scriptstyle\varphi_1} \\ B_1 \end{array} = \begin{array}{c} V_2 \\ \big\downarrow{\scriptstyle\varphi_2} \\ B_2 \end{array}$$

$$f : \partial V_1 \to \partial V_2$$
$$g : \partial B_1 \to \partial B_2$$

$$\begin{array}{ccc} V_1 \sqcup V_2 & \longrightarrow & V_1 \cup_f V_2 \\ {\scriptstyle\varphi_1 \sqcup \varphi_2}\big\downarrow & & \big\downarrow{\scriptstyle\varphi_1 \cup \varphi_2} \\ B_1 \sqcup B_2 & \longrightarrow & B_1 \cup_g B_2 (\cong S^3) \end{array}$$

$$\varphi_1 \cup \varphi_2 \text{ is a map } \Leftrightarrow \begin{array}{ccc} \partial V_1 & \xrightarrow{f} & \partial V_2 \\ {\scriptstyle\varphi_1}\big\downarrow & & \big\downarrow{\scriptstyle\varphi_2} \\ \partial B_1 & \xrightarrow[g]{} & \partial B_2 \end{array} \text{ commutes}$$

Theorem. (Berstein y Edmonds)

Let $\varphi : \partial V \to \partial B^3$ be a d-fold simple branched covering with $d \geq 3$, and let $f' : \partial V \to \partial V$ be a homeomorphism

$\Rightarrow$

There exist $f : \partial V \to \partial V$ and $g : \partial B^3 \to B^3$ homeomorphisms such that f is isotopic to f' and

$$\begin{array}{ccc} \partial V_1 & \xrightarrow{f} & \partial V_2 \\ {\scriptstyle \varphi_1}\downarrow & & \downarrow{\scriptstyle \varphi_2} \\ \partial B_1 & \xrightarrow[g]{} & \partial B_2 \end{array} \quad \text{commutes.}$$

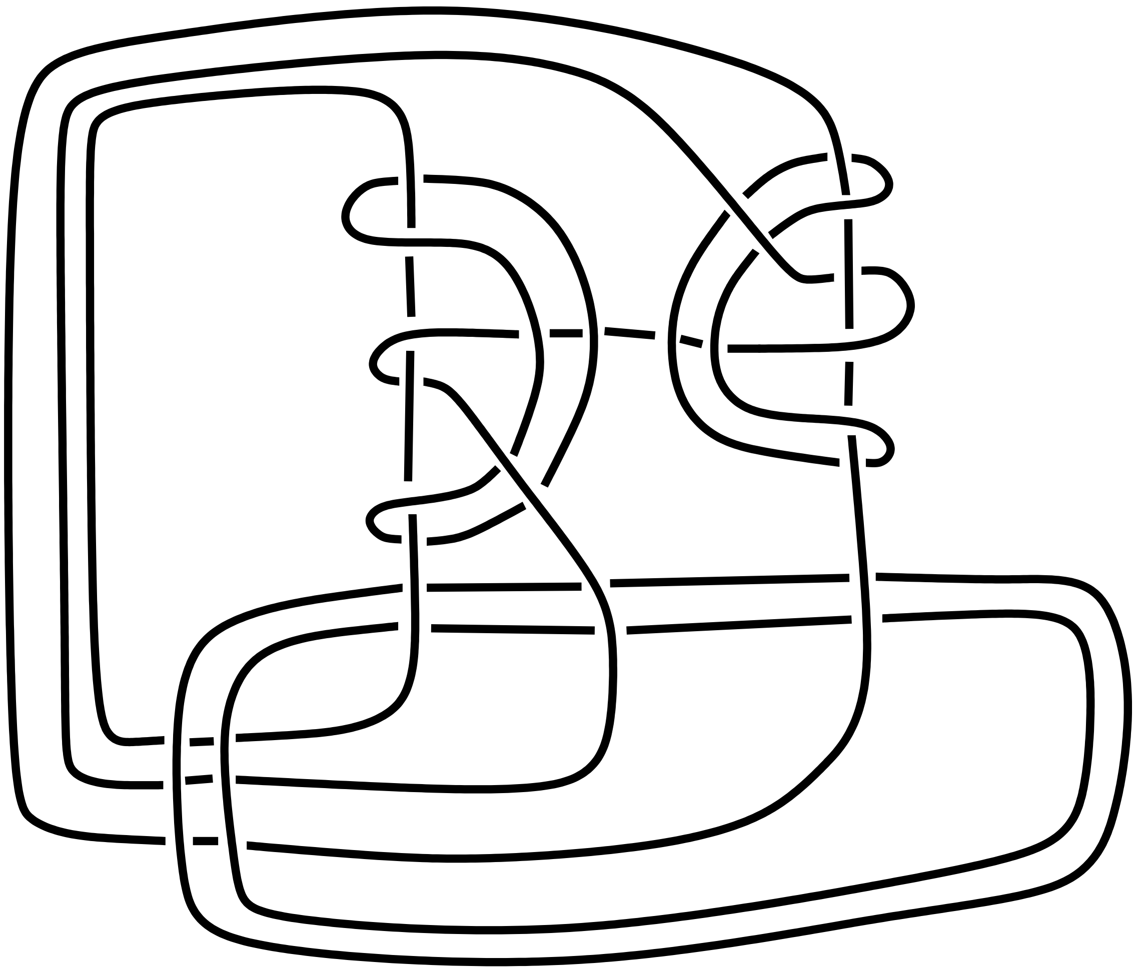
Theorem. (Hilden and Montesinos)

Each closed connected orientable 3-manifold is a branched covering of the 3-sphere S^3 through a 3-fold simple branched covering, and the branching is along a link in S^3.

Question:

Is there a link $L \subset S^3$ such that each closed connected orientable 3-manifold is a branched covering of (S^3, L)?

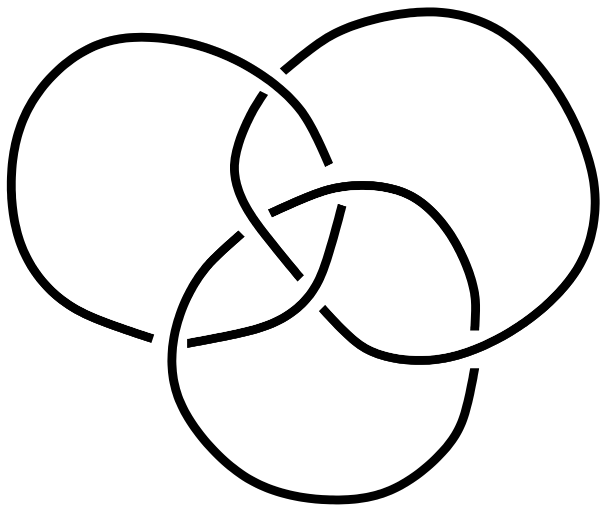
Theorem. (Thurston) The link

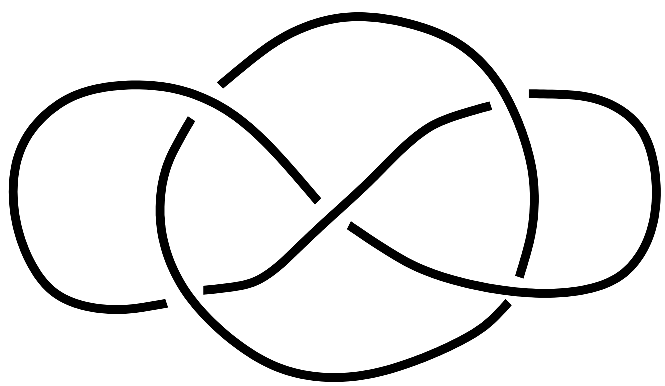


is universal.

Theorem. (Hilden–Lozano–Montesinos)

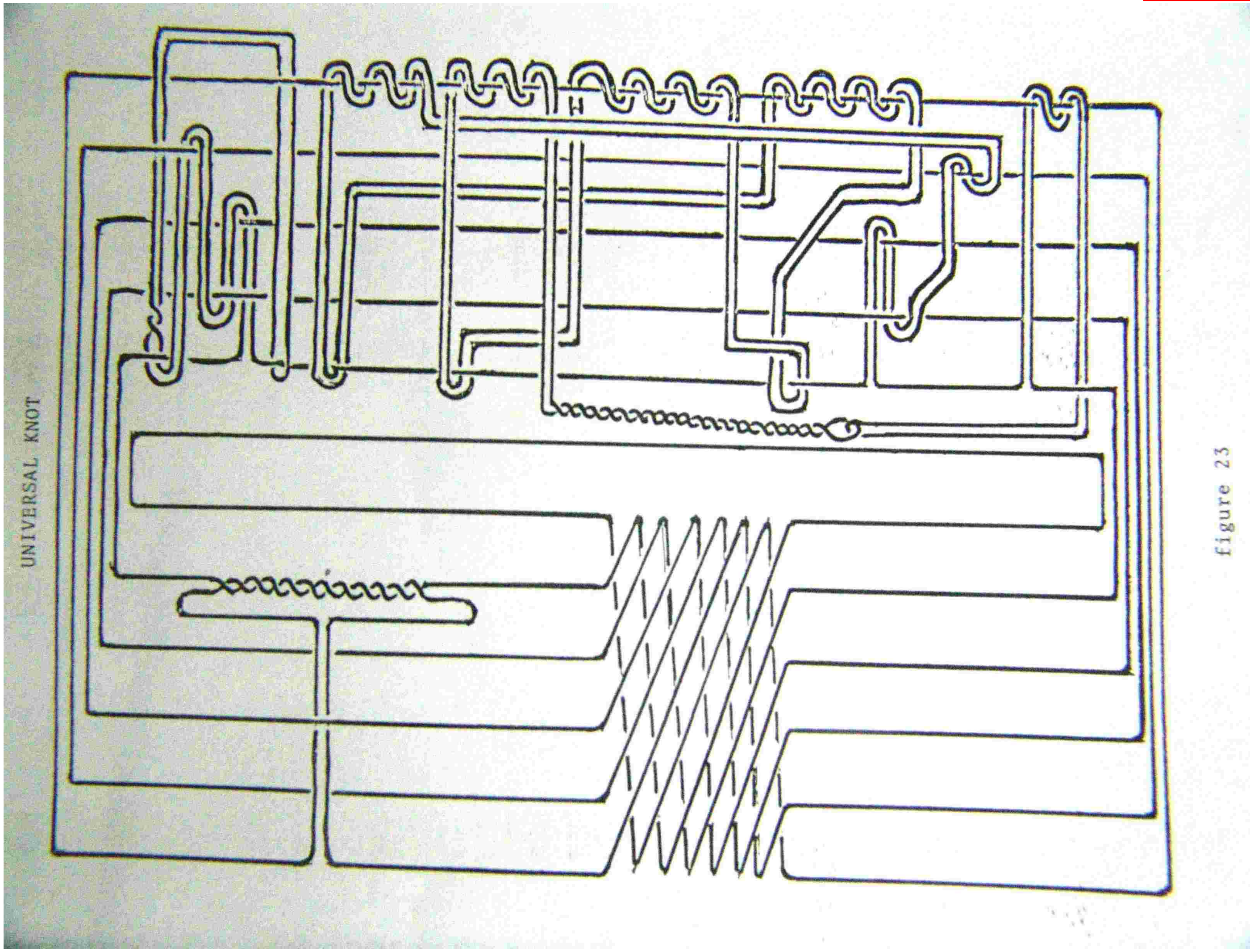
The links





are universal.

Theorem. (Hilden–Lozano–Montesinos) The knot



is universal.

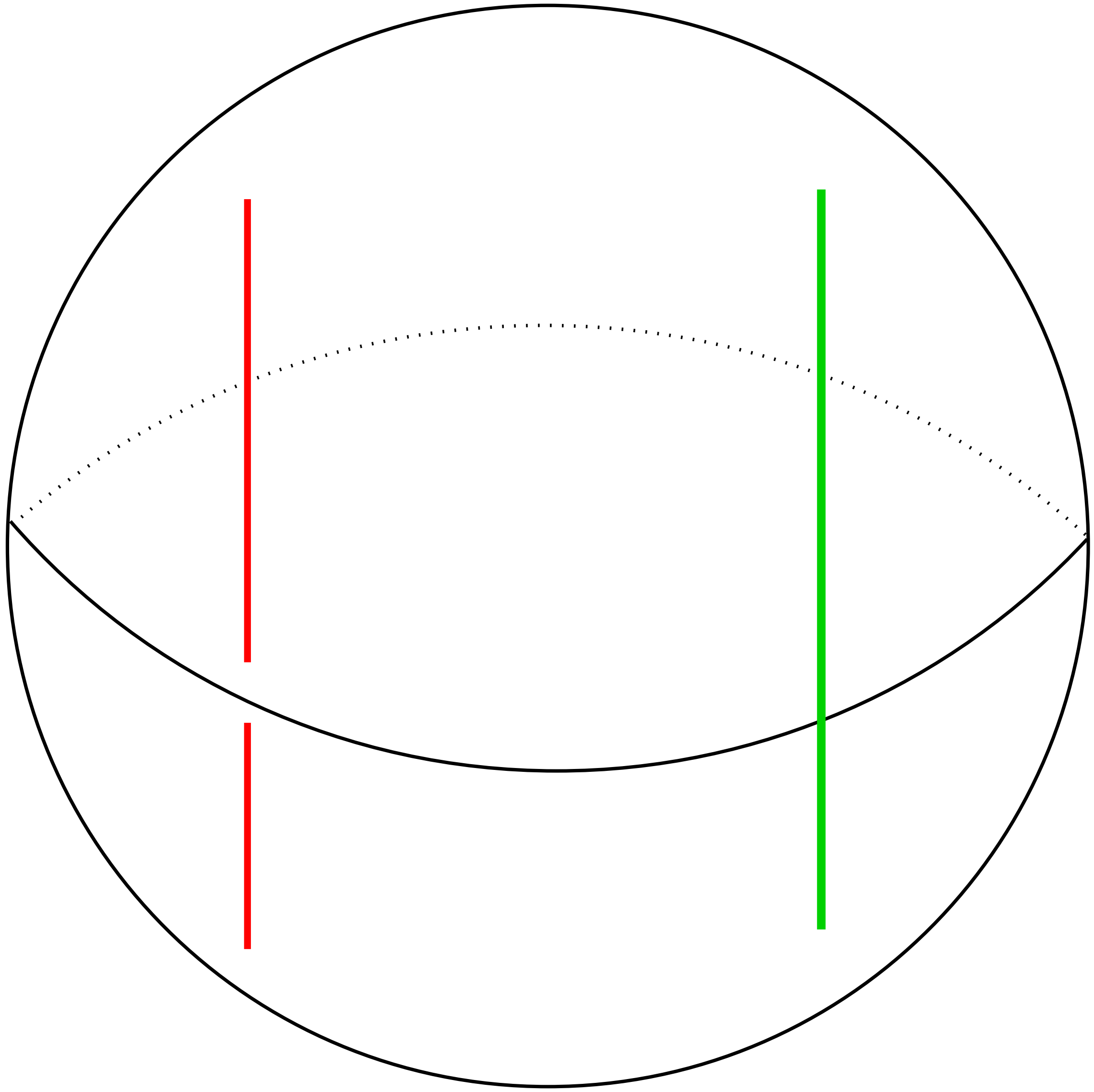
n-bridge knots.

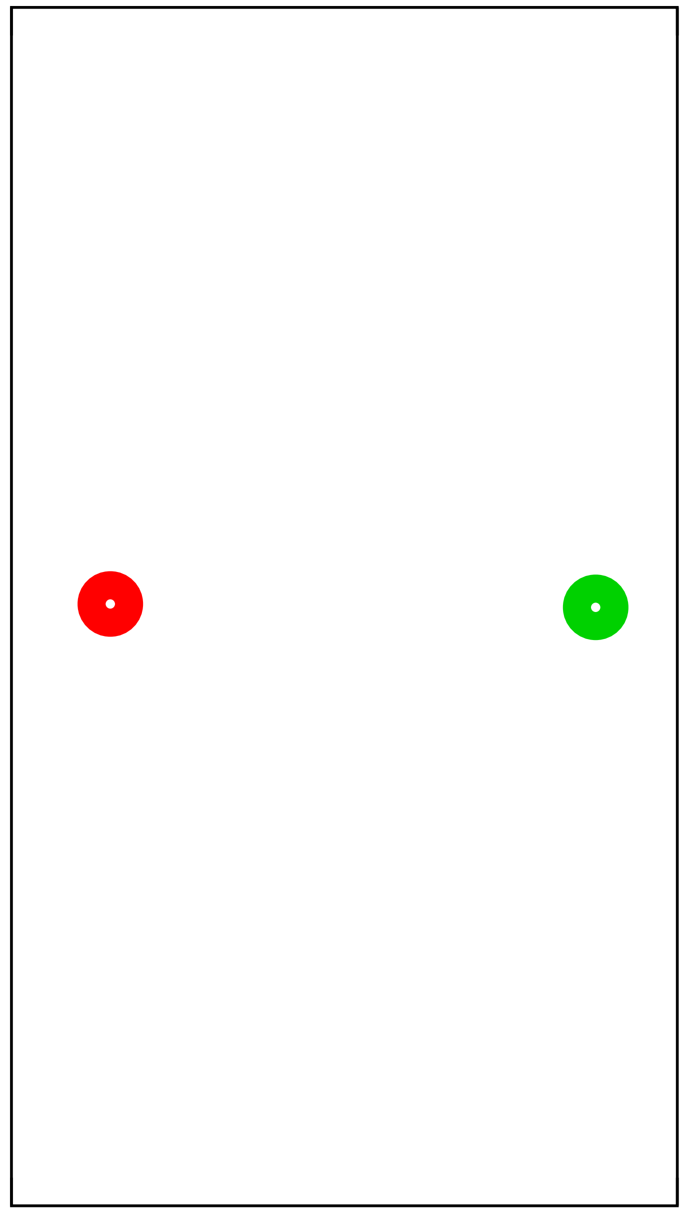
A trivial n-tangle is a pair $(B, \{\alpha_i\}_{i=1}^n)$ where

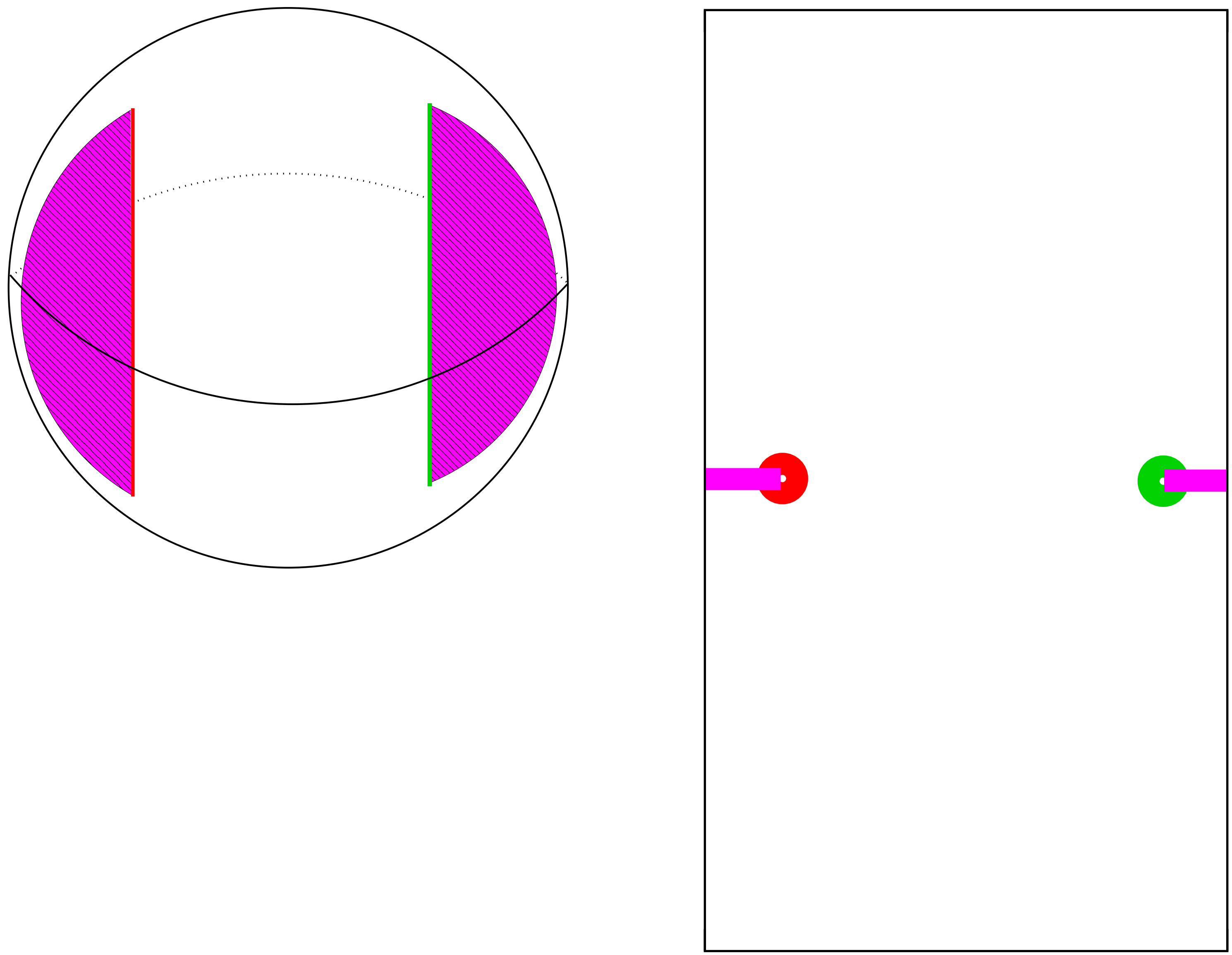
B is a 3-ball,

and $\alpha_1, \dots, \alpha_n \subset B$ are n trivial properly embedded arcs

(That is, there are n disjoint 2-disks $D_1, \dots, D_n \subset B$ such that $\partial D_i = \alpha_i \cup \beta_i$ where $\beta_i \subset \partial B$ and $\partial \alpha_i = \partial \beta_i$.)





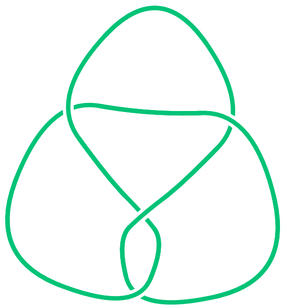


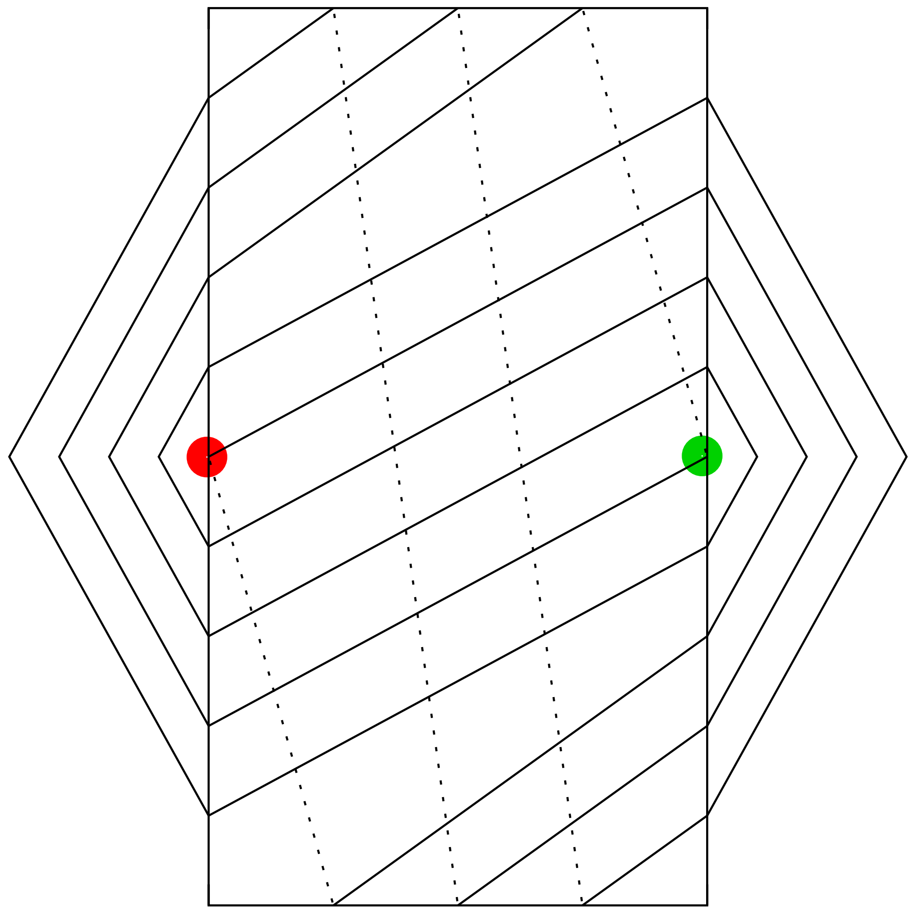
A link $k \subset S^3$ is in $\underline{n\text{-bridge position}}$ if there are two trivial n-tangles, $(B, \{\alpha_i\})$ and $(B', \{\alpha_i'\})$, such that

$$S^3 = B \cup_\partial B'$$

y

$$k = (\sqcup \alpha_1) \cup (\sqcup \alpha_i')$$

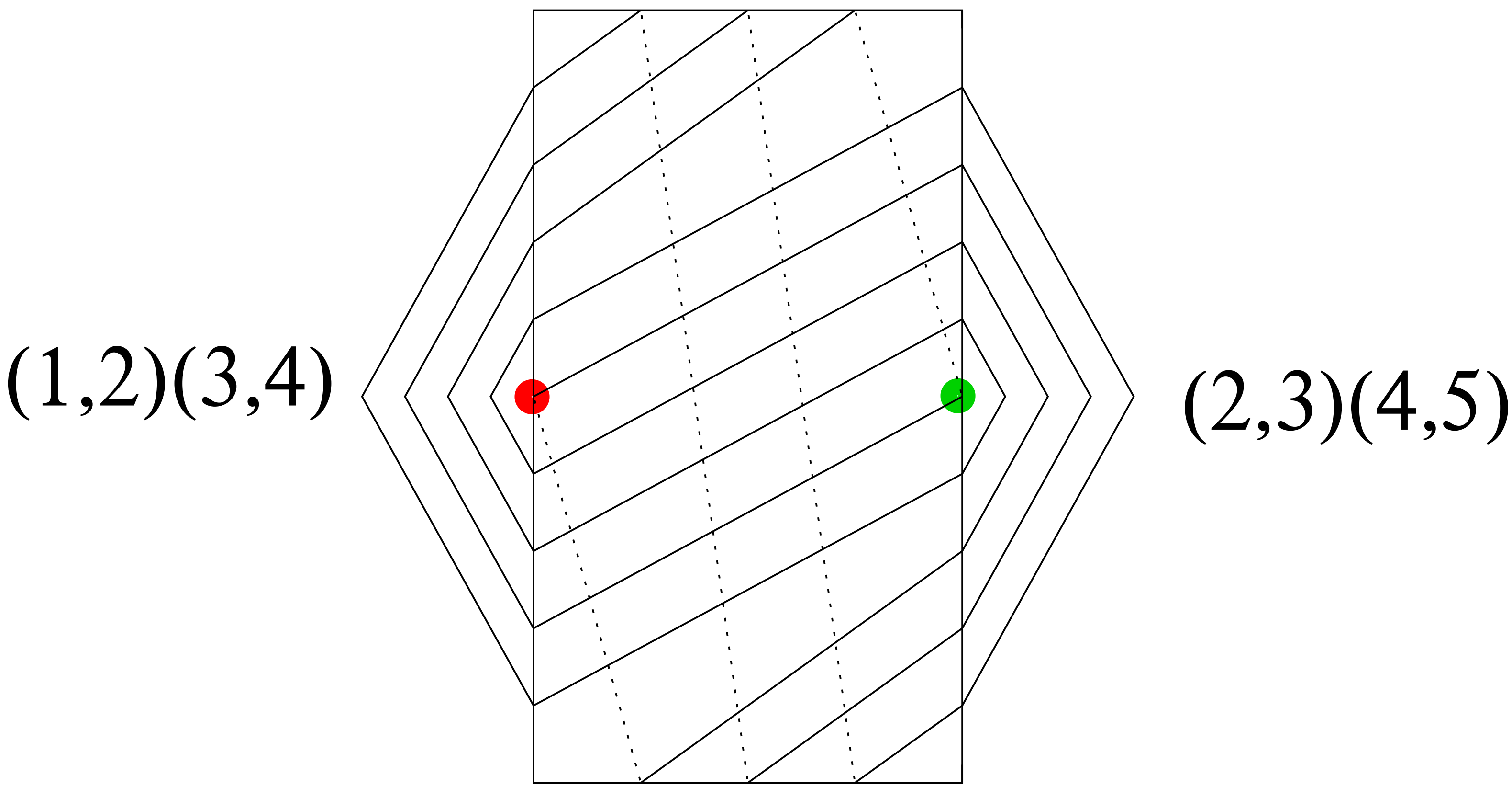




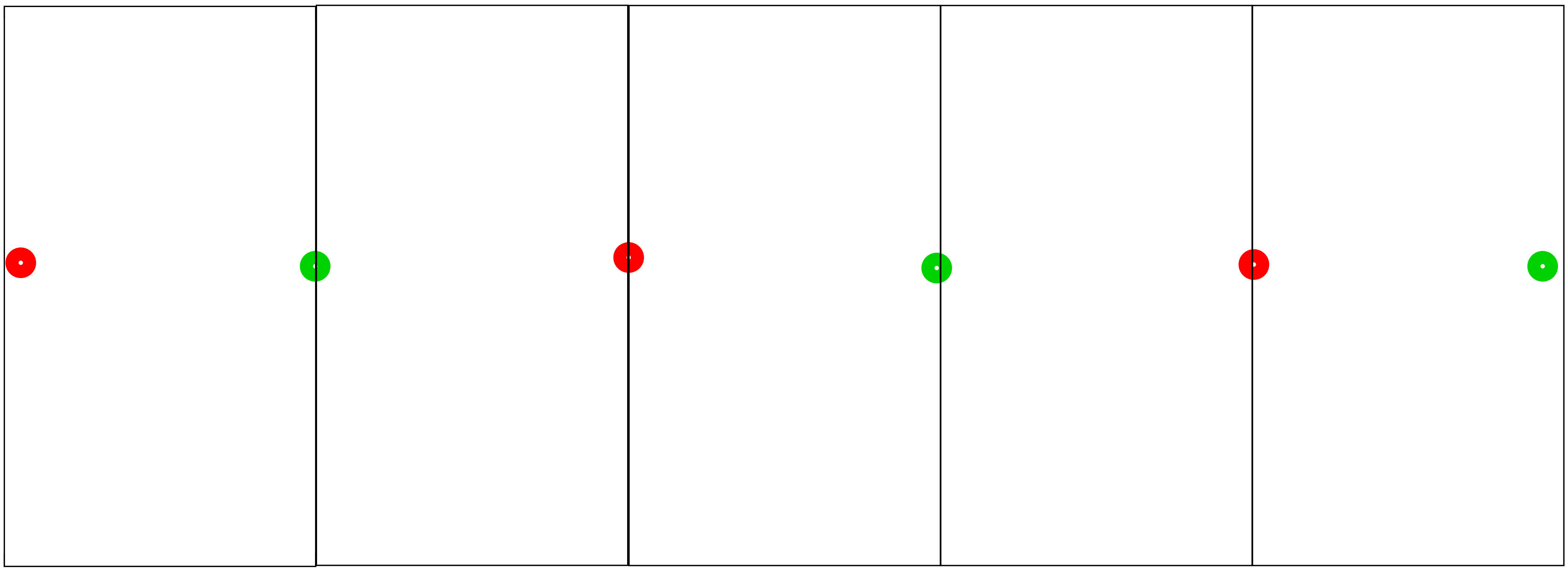
$\ell(5/3)$.

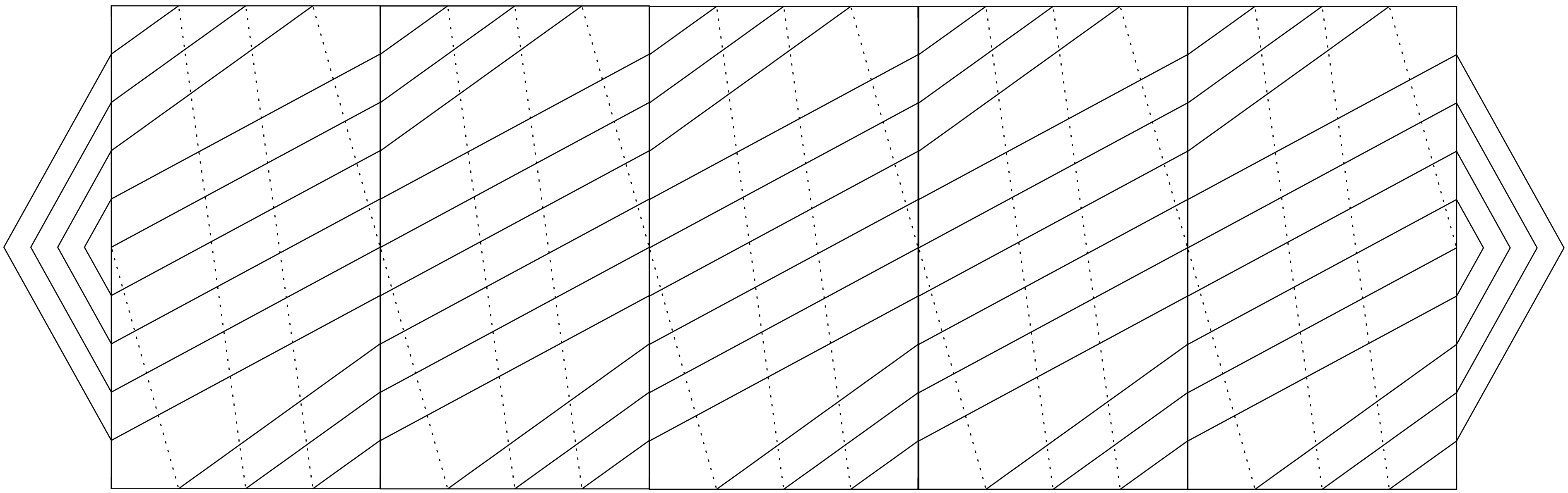
Coverings of 2-bridge knots.

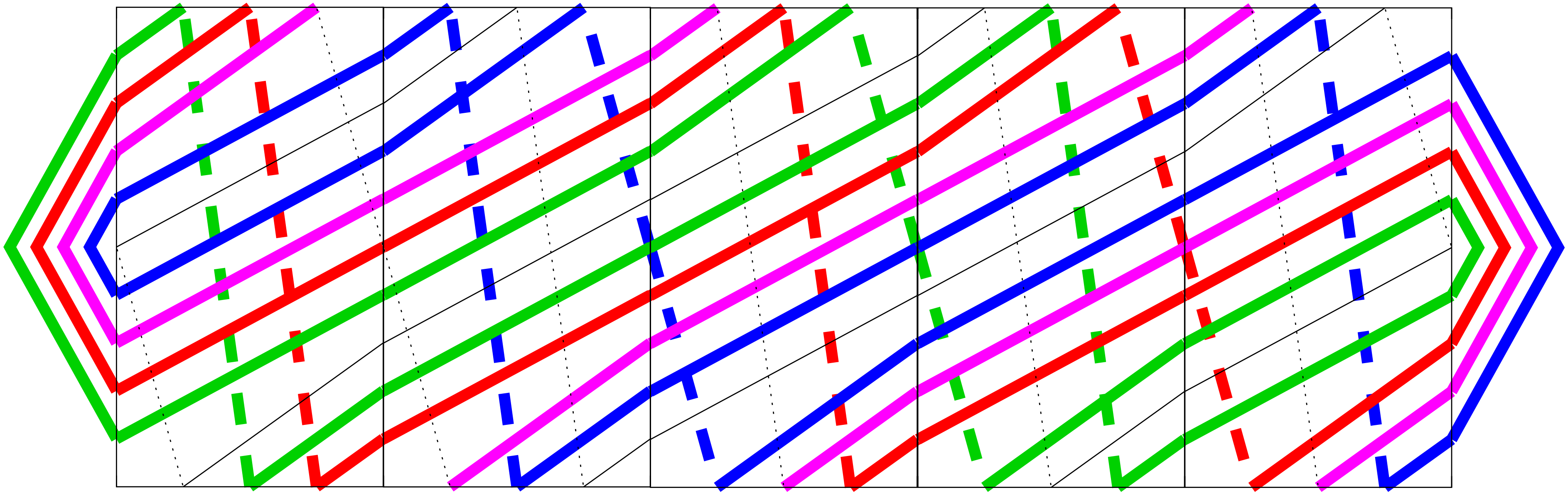
$k = \ell(5/3)$



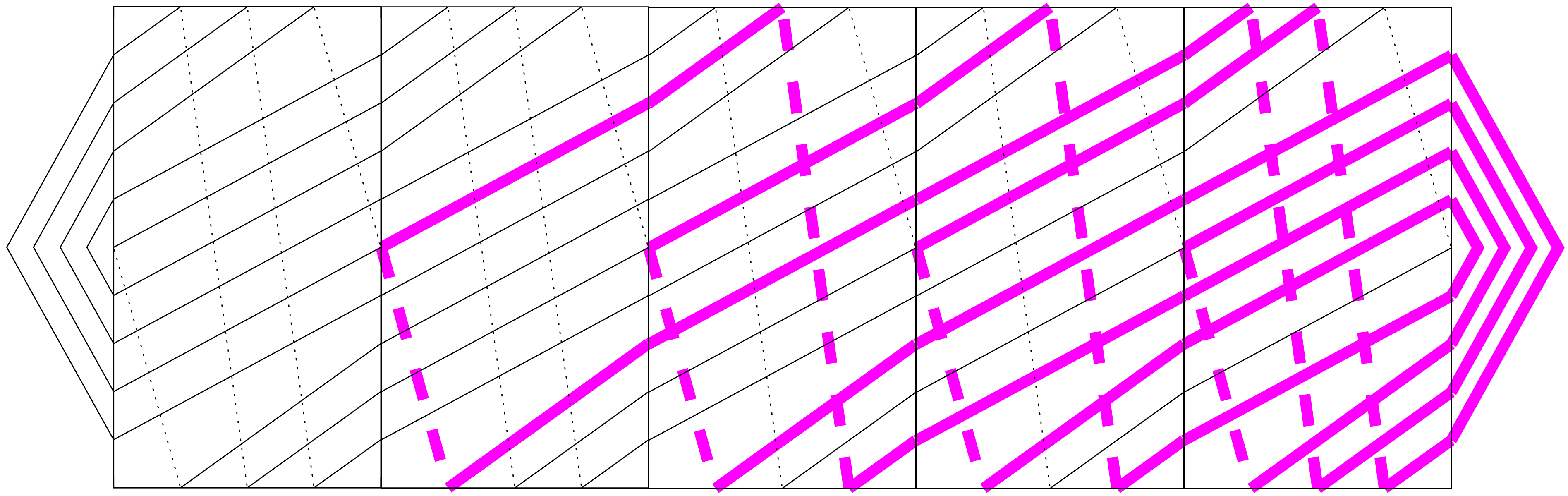
We know the covering of the 3-ball:

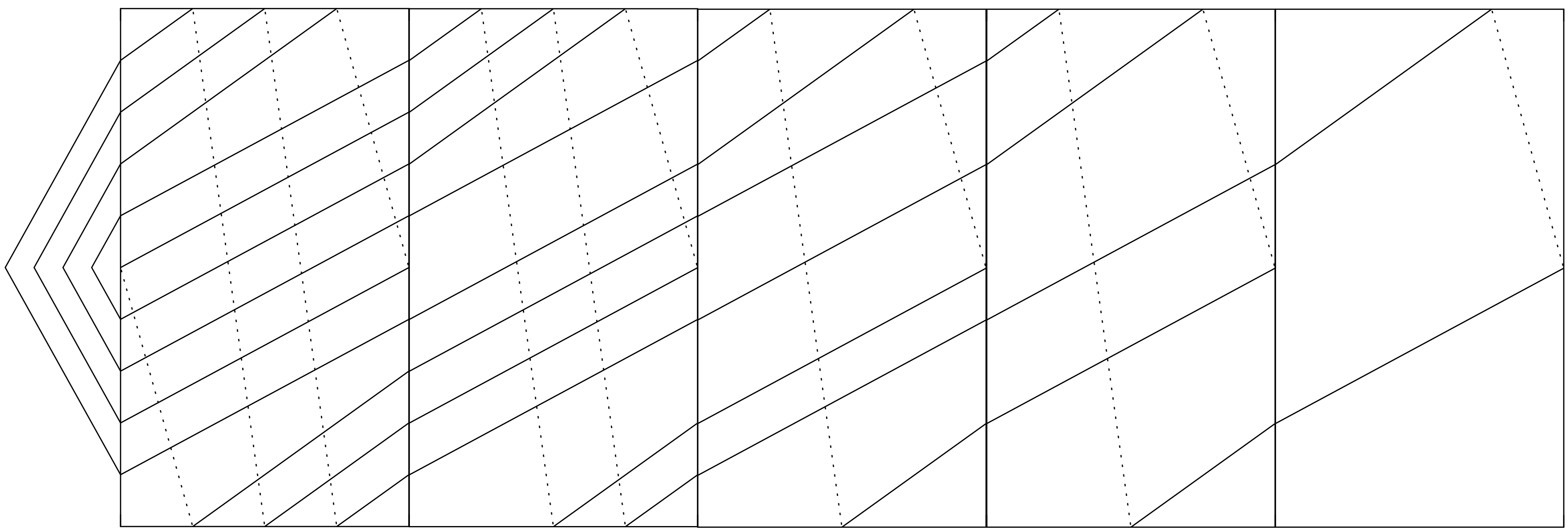


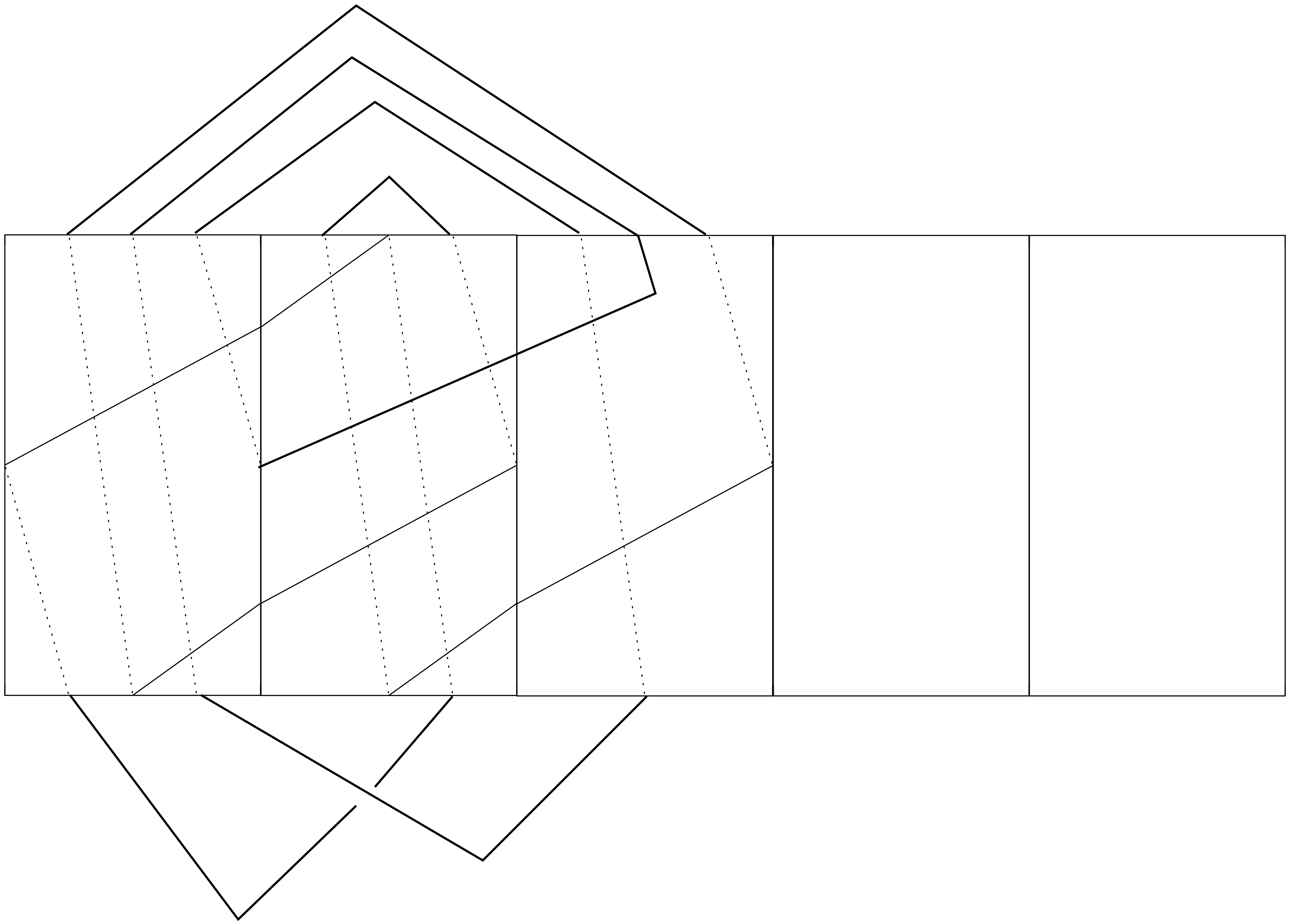


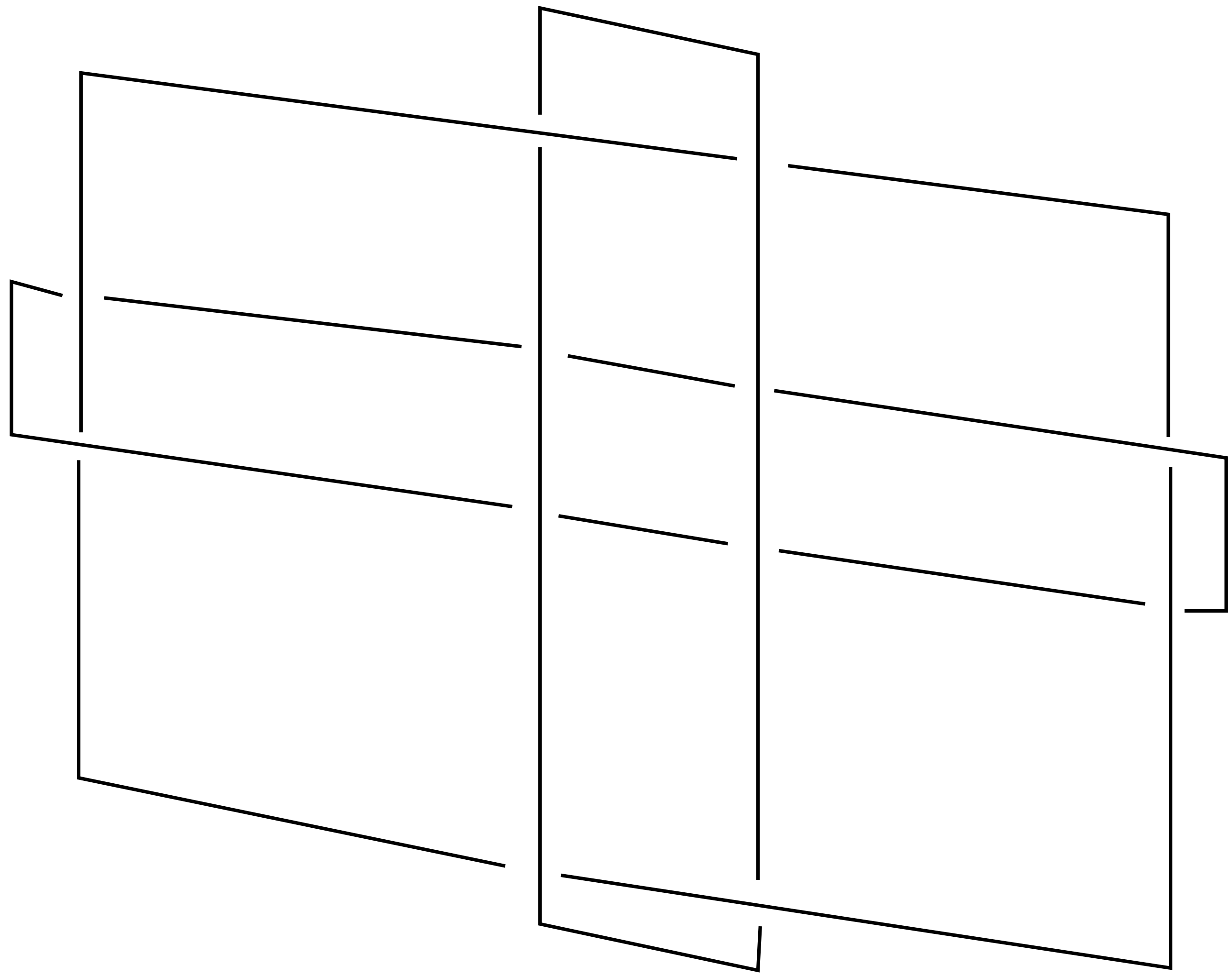


These arcs are ‘unnecessary’:









The Roman link.

5–fold $\varphi_1 : S^3 \to (S^3, \ell(\frac{5}{3}))$, $\varphi_1^{-1}(\ell(\frac{5}{3}))$ =Roman link.

4–fold $\varphi_2 : S^3 \to (S^3, \text{Roman link})$, $\varphi_2^{-1}(\text{Roman link}) \supset \ell(\frac{12}{5})$.

6–fold $\varphi_3 : S^3 \to (S^3, \ell(\frac{12}{5}))$, $\varphi_3^{-1}(\ell(\frac{12}{5})) \supset L_2$.

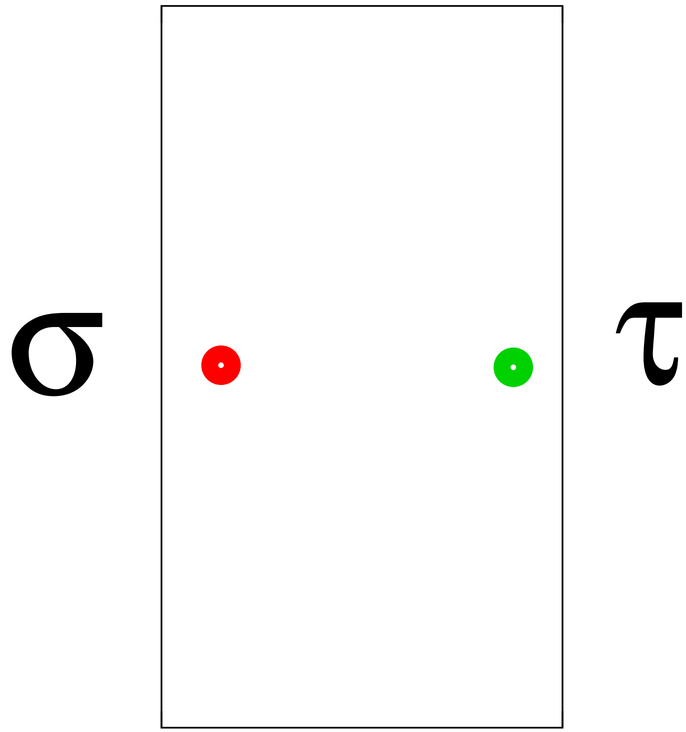
3–fold $\varphi_4 : S^3 \to (S^3, L_2)$, $\varphi_4^{-1}(L_2) \supset L_3$.

3–fold $\varphi_5 : S^3 \to (S^3, L_3)$, $\varphi_5^{-1}(L_3) \supset \text{Borromean rings}$.

Therefore $\varphi = \varphi_5 \circ \varphi_4 \circ \varphi_3 \circ \varphi_2 \circ \varphi_1 : S^3 \to (S^3, \ell(\frac{5}{3}))$
1080–fold, $\varphi^{-1}(\ell(\frac{5}{3})) \supset \text{Borromean rings}$.

Thus $k = \ell(\frac{5}{3})$ is universal.

For $k = \ell(a/b)$ with a odd



$$\sigma = (1,2)(3,4)\cdots(a-2,a-1)$$
$$\tau = (2,3)(4,5)\cdots(a-1,a)$$

Remark: A 2-bridge knot $\ell(b/a)$ is hyperbolic if and only if $b \not\equiv \pm 1 \pmod{a}$.

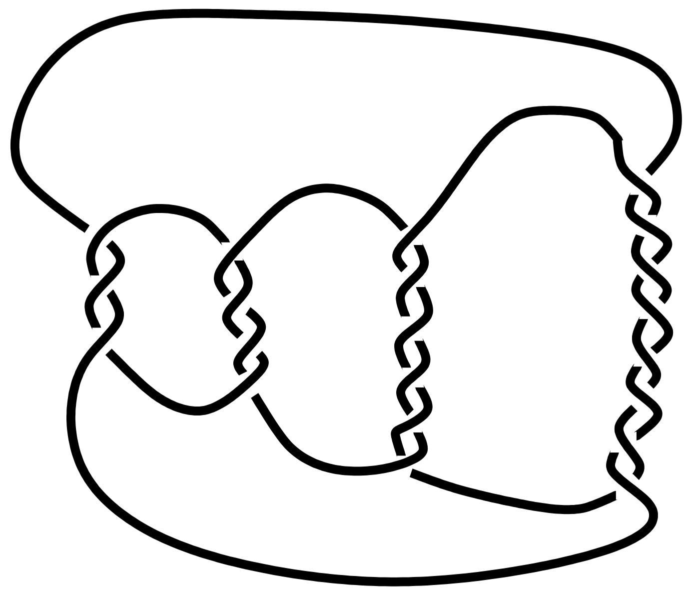
Theorem. (Hilden–Lozano–Montesinos) A 2-bridge knot, k, is universal if and only if k is hyperbolic.

More universal knots.

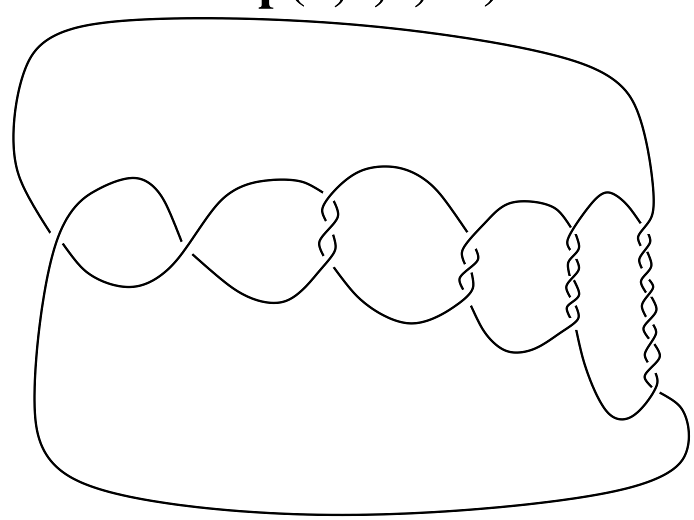
Definition. An Uchida link is a pretzel knot, $p(a_1, a_2, \ldots, a_t)$, with at least two even a's.

Theorem. (Uchida) All Uchida links are universal, except for:

- $p(2s, 2t)$, $s, t \in Z - \{0\}$.
- $p(-2, 2, s)$, $s \in Z - \{0\}$
- $p(\pm 2, \pm 3, \mp 4)$, $p(\pm 2, \mp 3, \mp 6)$, $p(\pm 2, \mp 4, \mp 4)$ y
- $p(-2, -2, 2, 2)$.



p(2,4,6,−8)

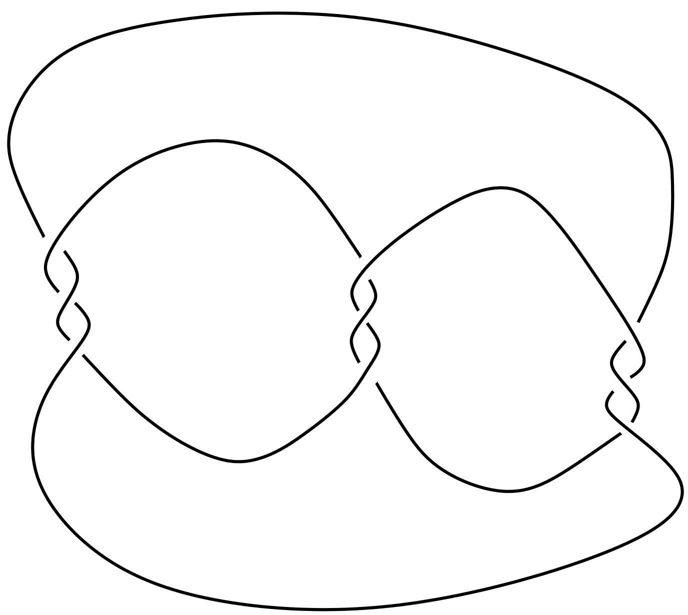


p(1, 1, 2, 3, 5, 8)

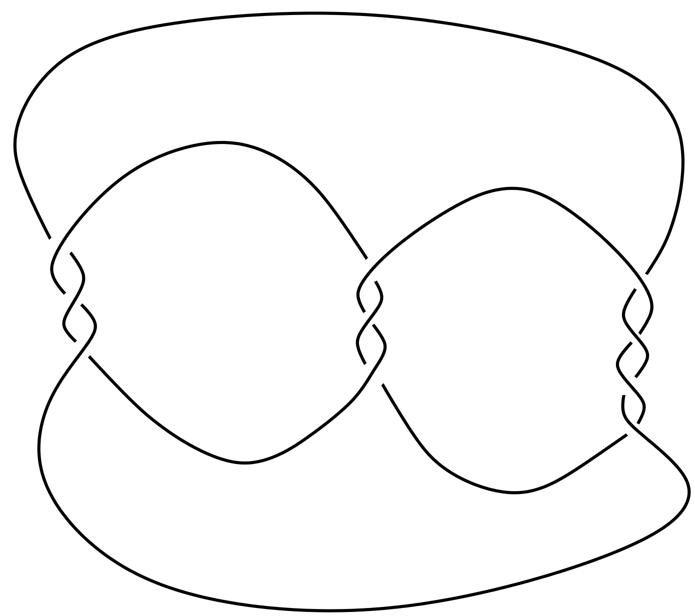
Theorem. (J. Rodríguez)

If $|n| > 1$ and n is odd, then $p(n, n, -n)$ is universal.

If $n \neq 2$ and n is even, then $p(3, 3, n)$ is universal.



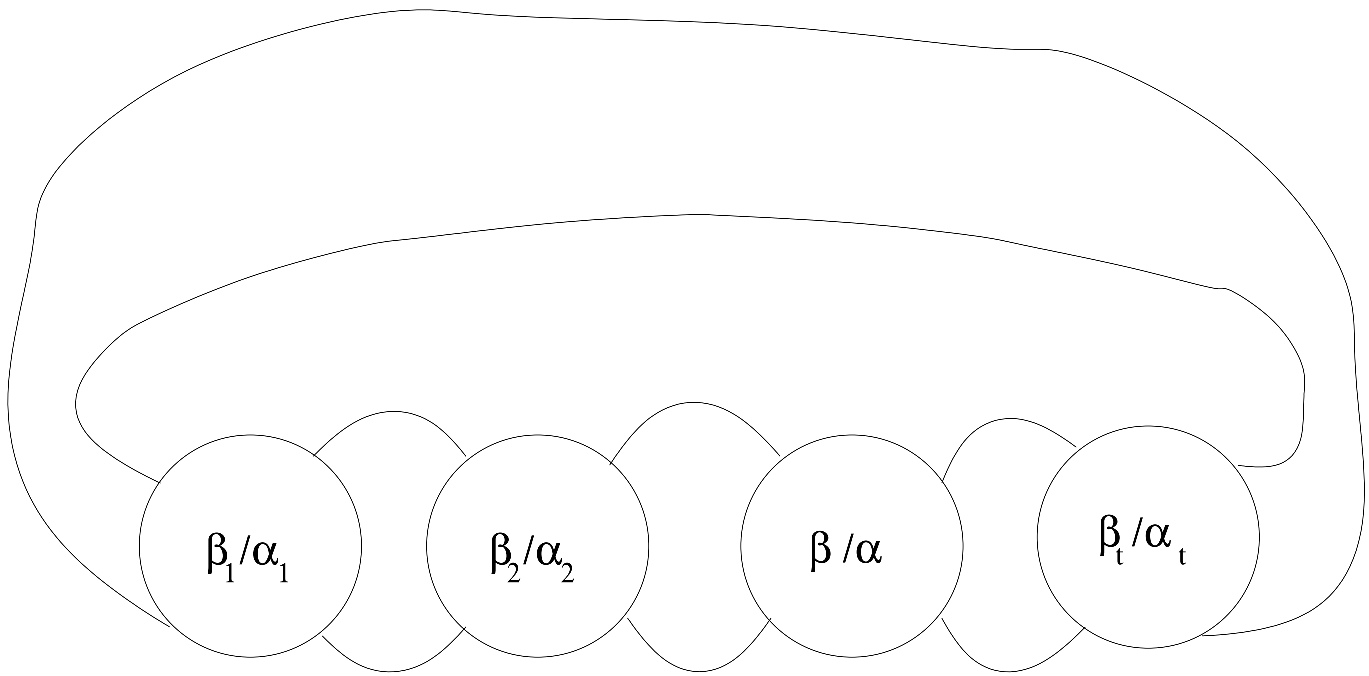
p(3, 3, –3)



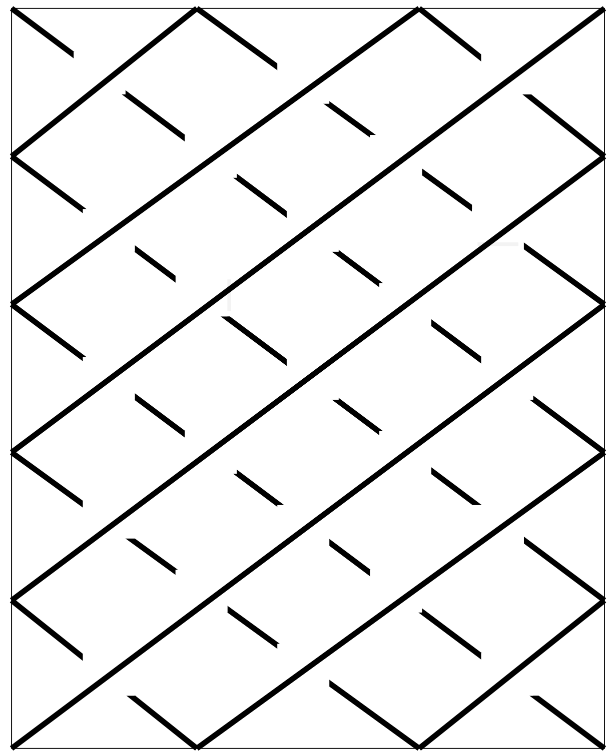
p(3, 3, –4)

Montesinos knots.

A Montesinos knot, $m(\beta_1/\alpha_1, \ldots, \beta_t/\alpha_t)$, is a link of the form:



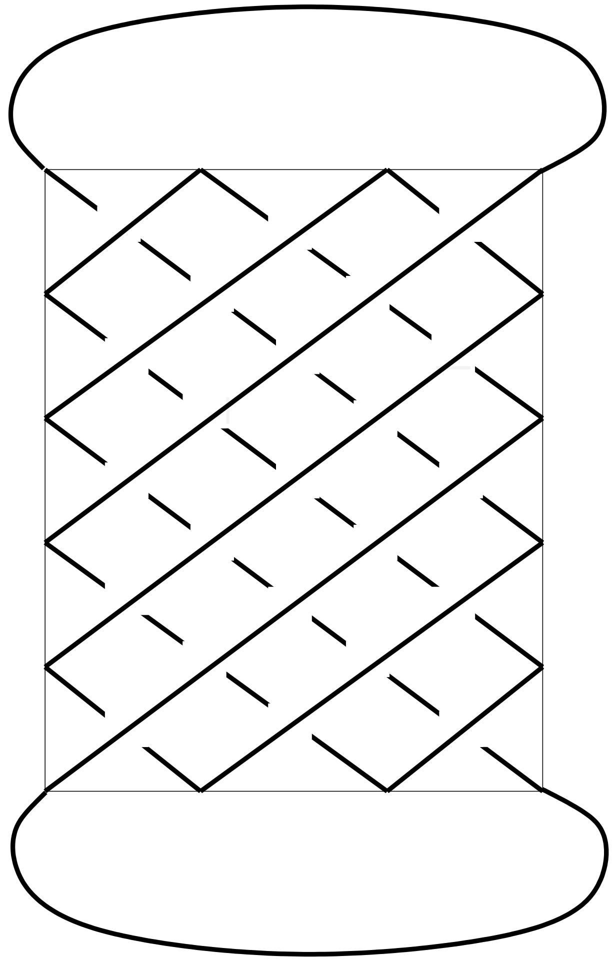
where each little box (each 'square pillowcase') contains a rational 2-tangle.



The rational tangle 3/5

The vertical side is divided into **5** intervals.

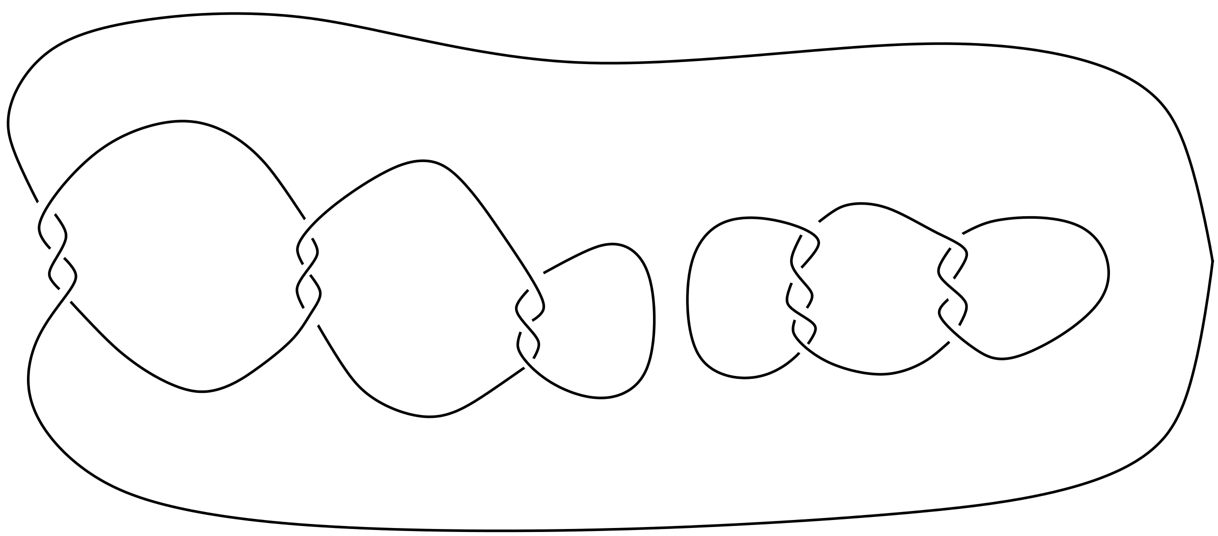
The horizontal side is divided into **3** intervals.



The rational knot $\ell(3/5) = m(3/5)$

We will always assume that for each i, $(\alpha_i, \beta_i) = 1$.

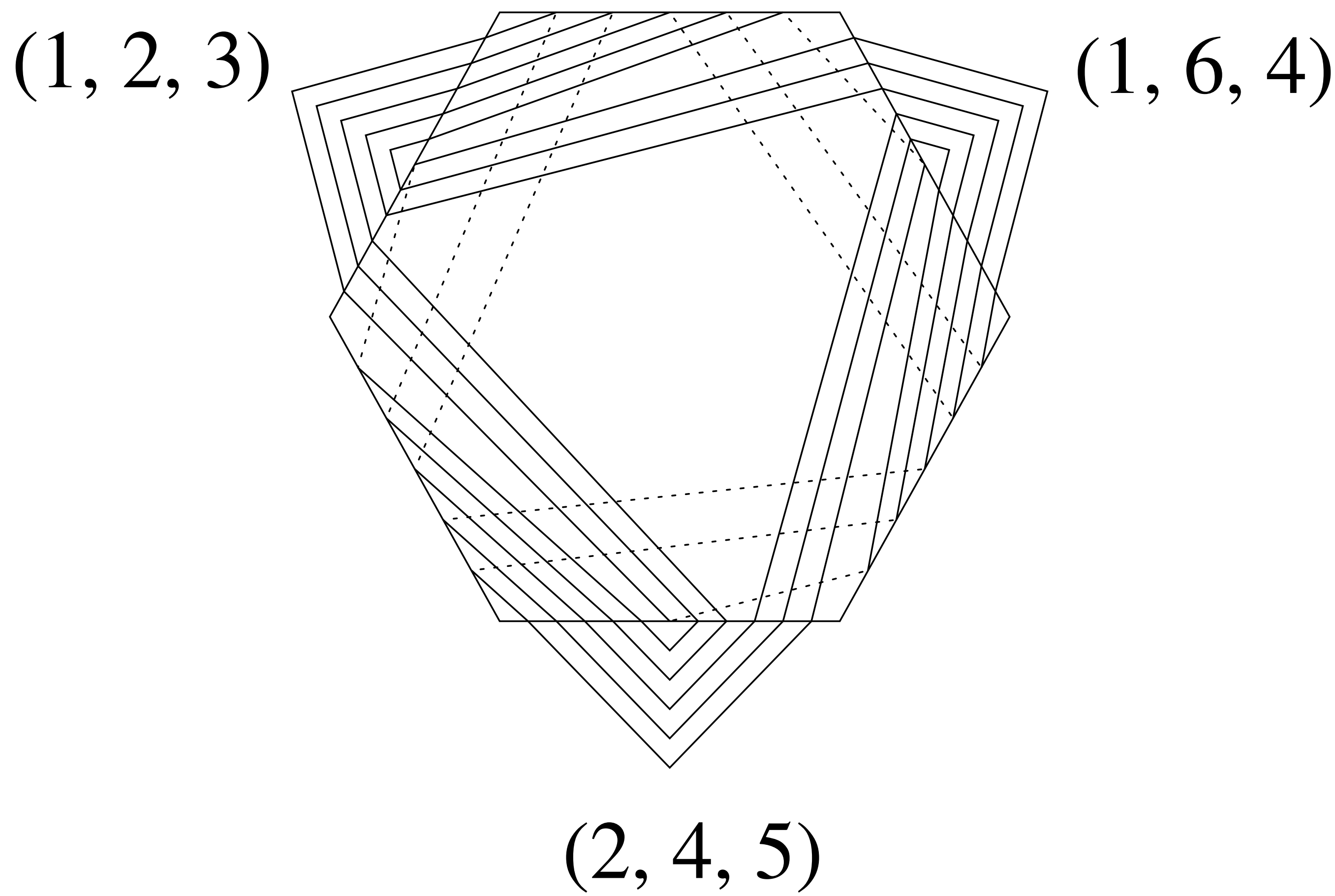
It is convenient to allow that some of the α's are zero (though in this case the Montesinos knot is a union of connected sums of rational links



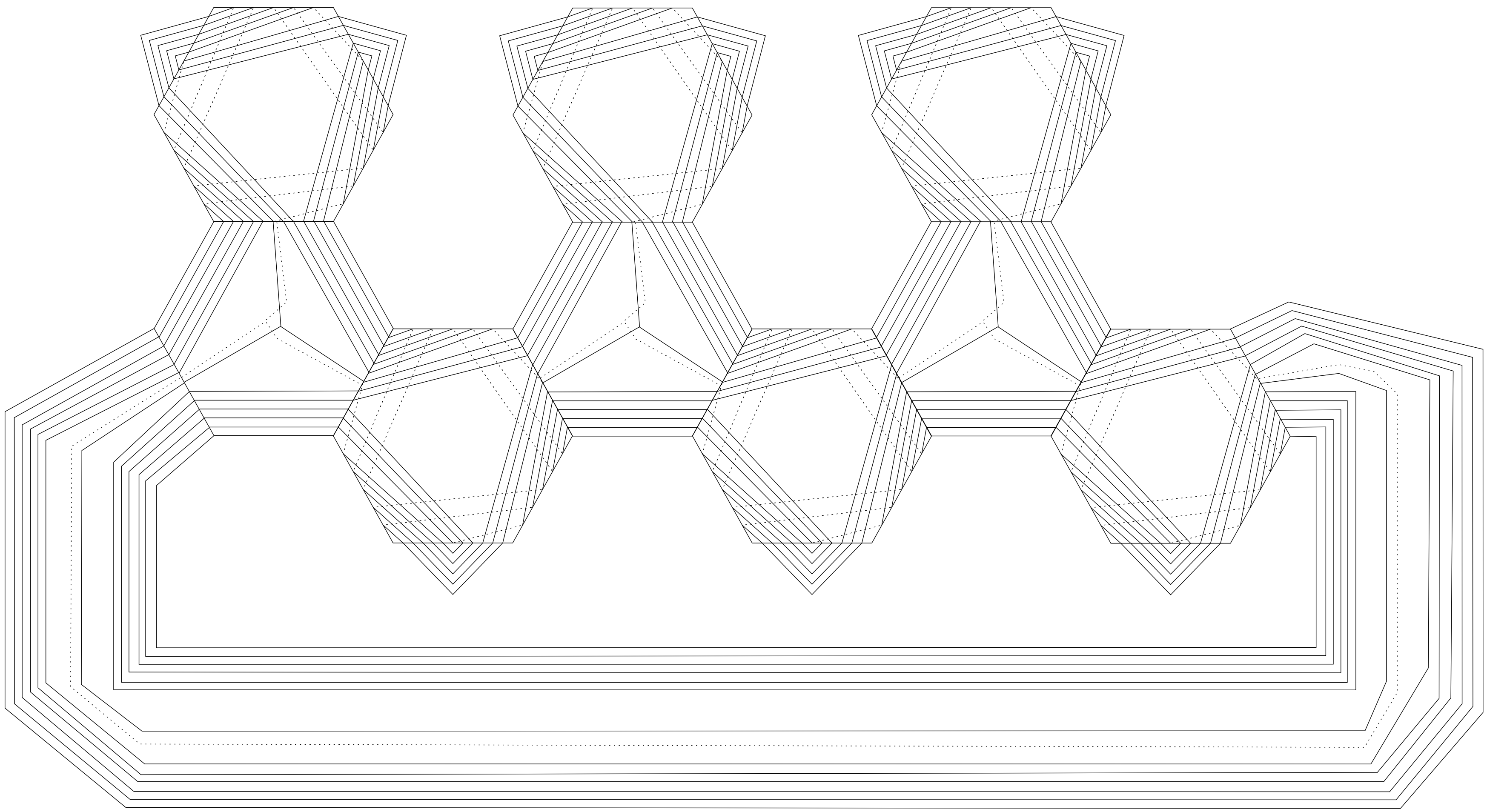
m(1/3, 1/3, –1/3, 1/0, –1/4, 1/3, 1/0)

).

Coverings of Montesinos knots.



$m(1/3, 1/3, 1/3)$



We’ll follow a different approach...

Dihedral quotients.

Let $k \subset S^3$ be a link. We write $B_2(k)$ for the double cyclic branched covering of (S^3, k)

(that is, $B_2(k)$ is the covering obtained by labeling each meridian of k with the permutation $(1, 2)$.)

In this case there is an involution

$$u : B_2(k) \to B_2(k)$$

with quotient the 2–fold cyclic branched covering

$$p : B_2(k) \to (S^3, k)$$

and such that $p(\mathrm{fix}(u)) = k$.

Let $\varphi : M \longrightarrow (S^3, k)$ be a d–fold branched covering. Then φ is called a dihedral quotient if there exists a commutative diagram of branched coverings

$$\begin{array}{ccc} & \tilde{M} & \\ {\scriptstyle q} \swarrow & & \searrow {\scriptstyle \psi} \\ M & & B_2(k) \\ {\scriptstyle \varphi} \searrow & & \swarrow {\scriptstyle p} \\ & (S^3, k) & \end{array}$$

such that ψ is a d–fold cyclic covering space (unbranched).

In this case q is a 2–fold cyclic branched covering branched along the pseudo-branch of φ (this is a very special sublink of $\varphi^{-1}(k)$).

If k is the Montesinos knot $k = m(\beta_1/\alpha_1, \ldots, \beta_t/\alpha_t)$, then $B_2(k)$ is the Seifert manifold

$$B_2(k) = (O, 0; \beta_1/\alpha_1, \ldots, \beta_t/\alpha_t).$$

Theorem. (E. Ramírez and V.) It is possible to compute, in terms of the Seifert invariants, the coverings of the Seifert manifold with symbol $(O, 0; \beta_1/\alpha_1, \ldots, \beta_t/\alpha_t)$.

The pseudo-branch.

For the Montesinos knot $k = m(\frac{\beta_1}{\alpha_1}, \ldots, \frac{\beta_t}{\alpha_t})$ we write $\Delta(k) = \beta_1\alpha_2 \cdots \alpha_t + \alpha_1\beta_2 \cdots \alpha_t + \cdots + \alpha_1\alpha_2 \cdots \beta_t$.

Theorem. (J. Rodríguez and V.) If n is a positive divisor of $\Delta(k)$ and for each $i = 1, \ldots, t$, $(n, \alpha_i) = 1$, then

$$k \sim m(\frac{n \cdot b_1}{\alpha_1}, \ldots, \frac{n \cdot b_t}{\alpha_t}),$$

and there exists an n–fold dihedral quotient $\varphi : S^3 \to (S^3, k)$ such that

$$m(\frac{b_1}{\alpha_1}, \ldots, \frac{b_t}{\alpha_t}) \subset \varphi^{-1}(k).$$

($m(\frac{b_1}{\alpha_1}, \ldots, \frac{b_t}{\alpha_t})$ is the pseudo-branch de φ.)

Corollary.
Assume $(n, \alpha_i) = 1$ for each $i = 1, 2, \ldots, t$.

If $m(\beta_1/\alpha_1, \ldots, \beta_t/\alpha_t)$ is universal,
then $m(n\beta_1/\alpha_1, \ldots, n\beta_t/\alpha_t)$ is universal.

Conway's table: alternating 2-links of 9 crossings

$m(1/4, 2/3, 1/2)$ $[\Delta = 17 \cdot 2]$ $= m(17/4, 17/3, -17/2) \leftarrow m(1/4, 1/3, -1/2)$

$m(3/4, 1/3, 1/2)$ $[\Delta = 19 \cdot 2]$ $= m(19/4, 19/3, -19/2) \leftarrow m(1/4, 1/3, -1/2)$

$m(3/4, 2/3, 1/2)$ $[\Delta = 23 \cdot 2]$ $= m(23/4, 23/3, -23/2) \leftarrow m(1/4, 1/3, -1/2)$

$m(1/3, 1/3, 2/3)$ $[\Delta = 4 \cdot 3^2]$ $= m(4/3, 4/3, -4/3) \leftarrow m(1/3, 1/3, -1/3)$

$m(2/3, 2/3, 2/3) \leftarrow m(1/3, 1/3, 1/3)$

$m(3/5, 1/2, 3/2)$ $[\Delta = 13 \cdot 2^2]$ $= m(13/5, 13/2, -13/2) \leftarrow m(1/5, 1/2, -1/2)$

$m(1/3, 1/2, 5/2)$ $[\Delta = 5 \cdot 2^2]$ $= m(-5/3, 5/2, 5/2) \leftarrow m(-1/3, 1/2, 1/2)$

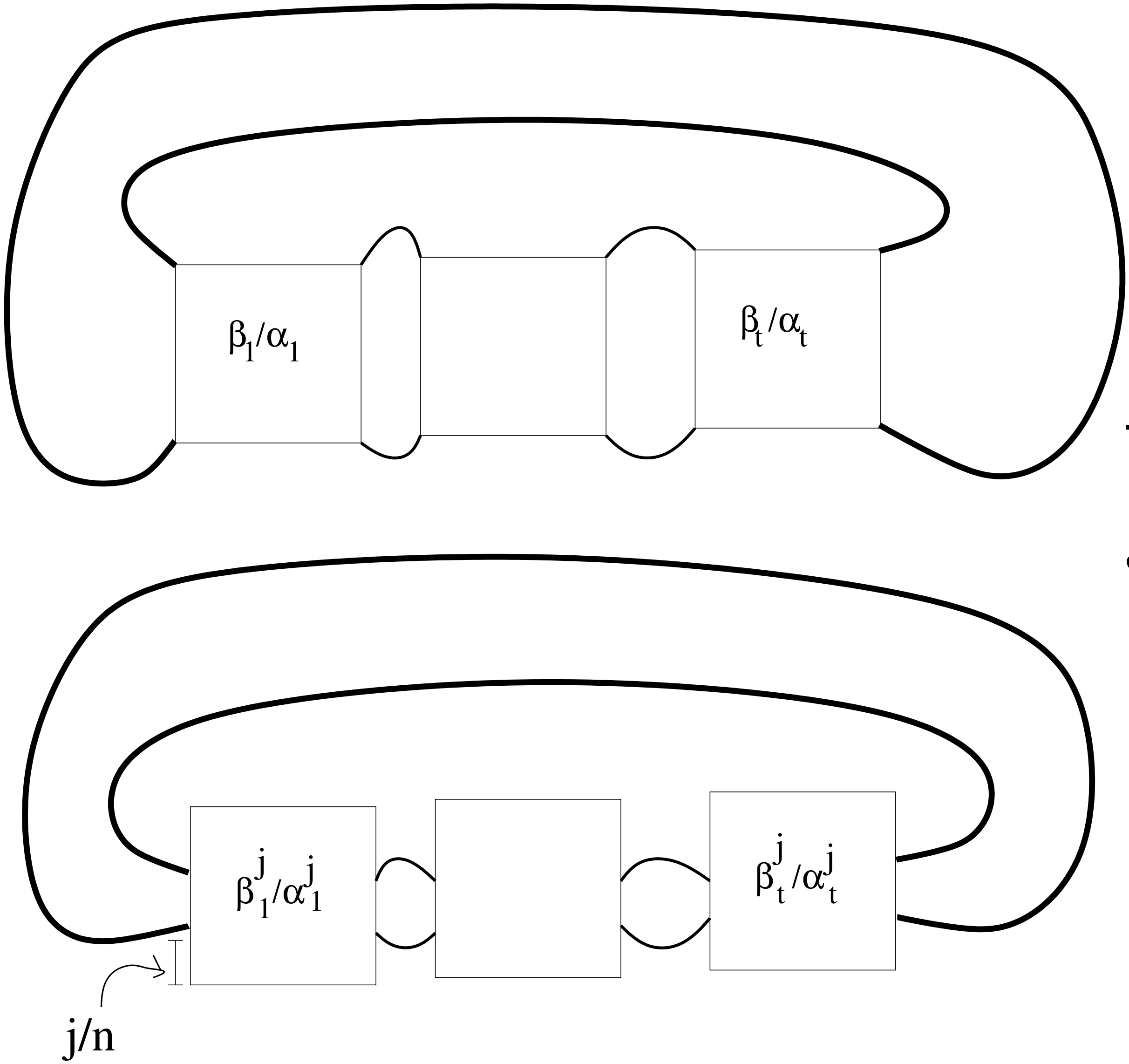
The branch.

If $(n, \alpha_i) = 1$ for each i and $k = m(n\beta_1/\alpha_1, \ldots, n\beta_t/\alpha_t)$ and $\varphi : S^3 \to (S^3, k)$ is an n–fold dihedral quotient, then

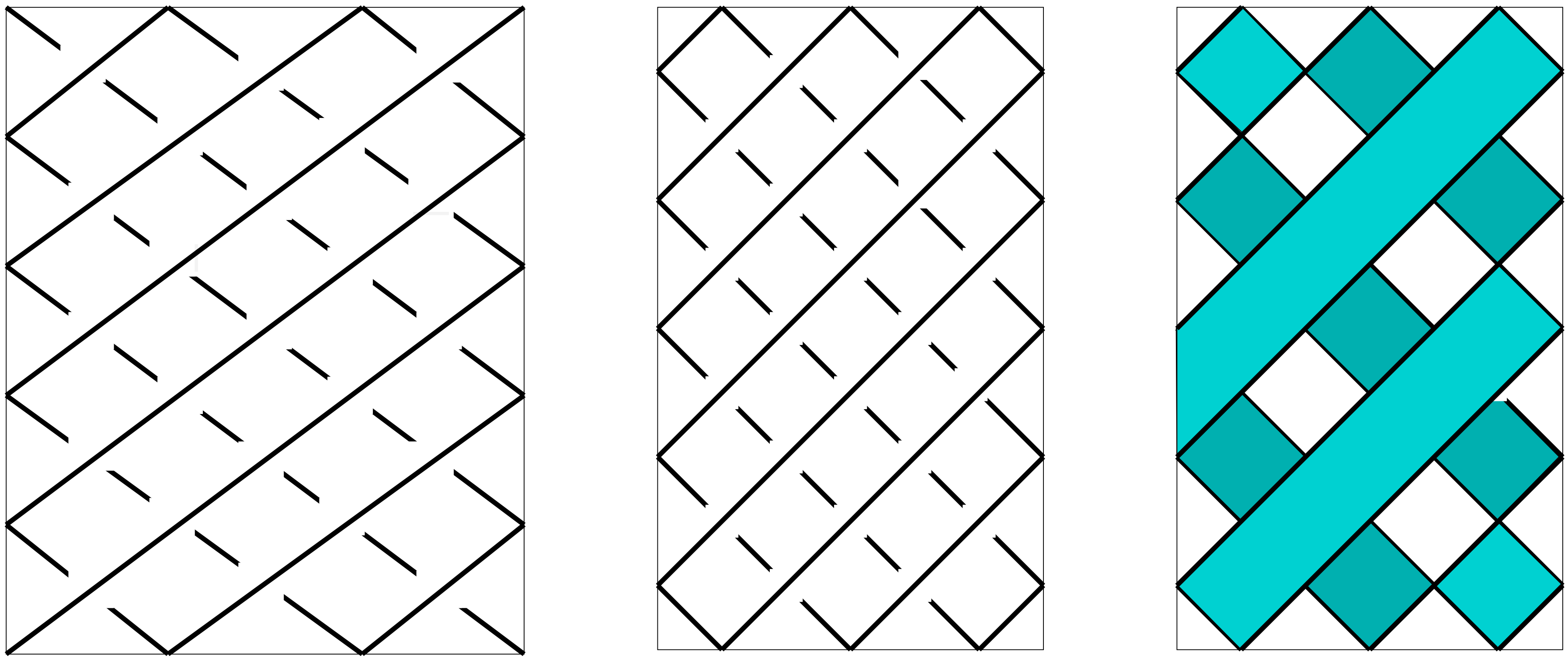
$\varphi^{-1}(k)$ has $(n-1)/2$ 'components' $k_1, k_2, \ldots, k_{\frac{n-1}{2}}$, if n is odd

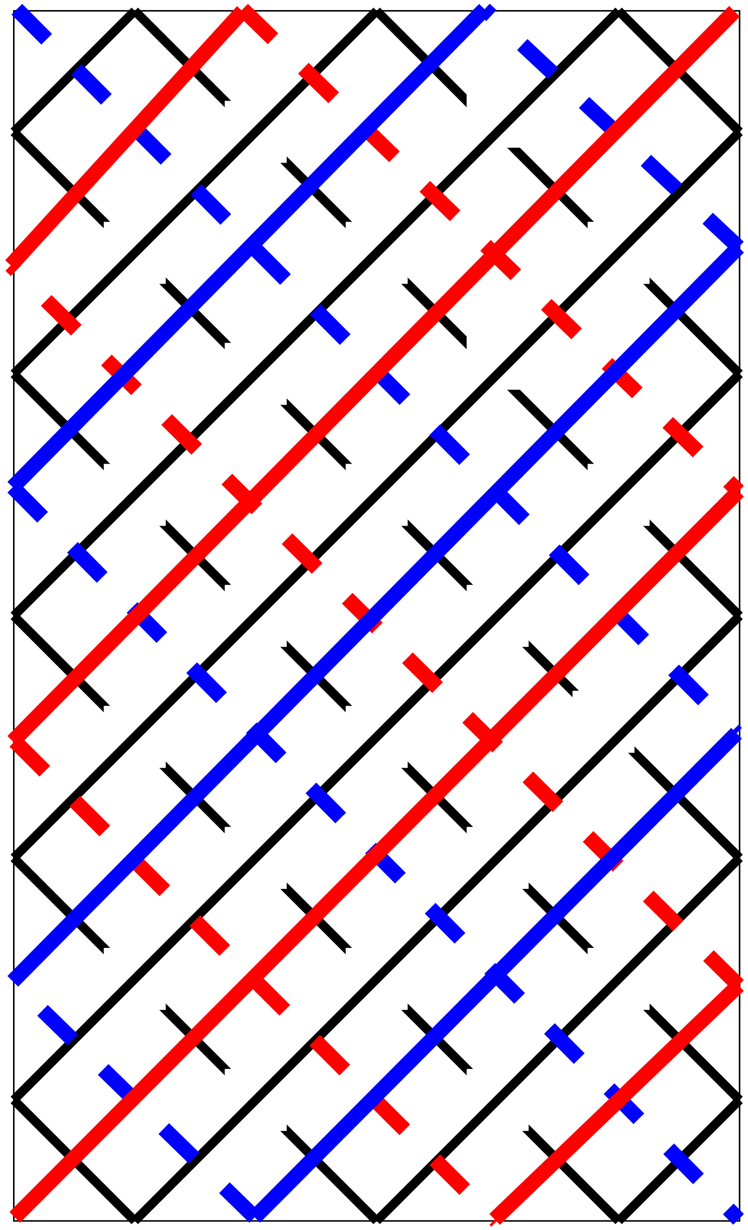
$\varphi^{-1}(k)$ has $n/2$ 'components' $k_1, k_2, \ldots, k_{\frac{n}{2}}$, if n is even.

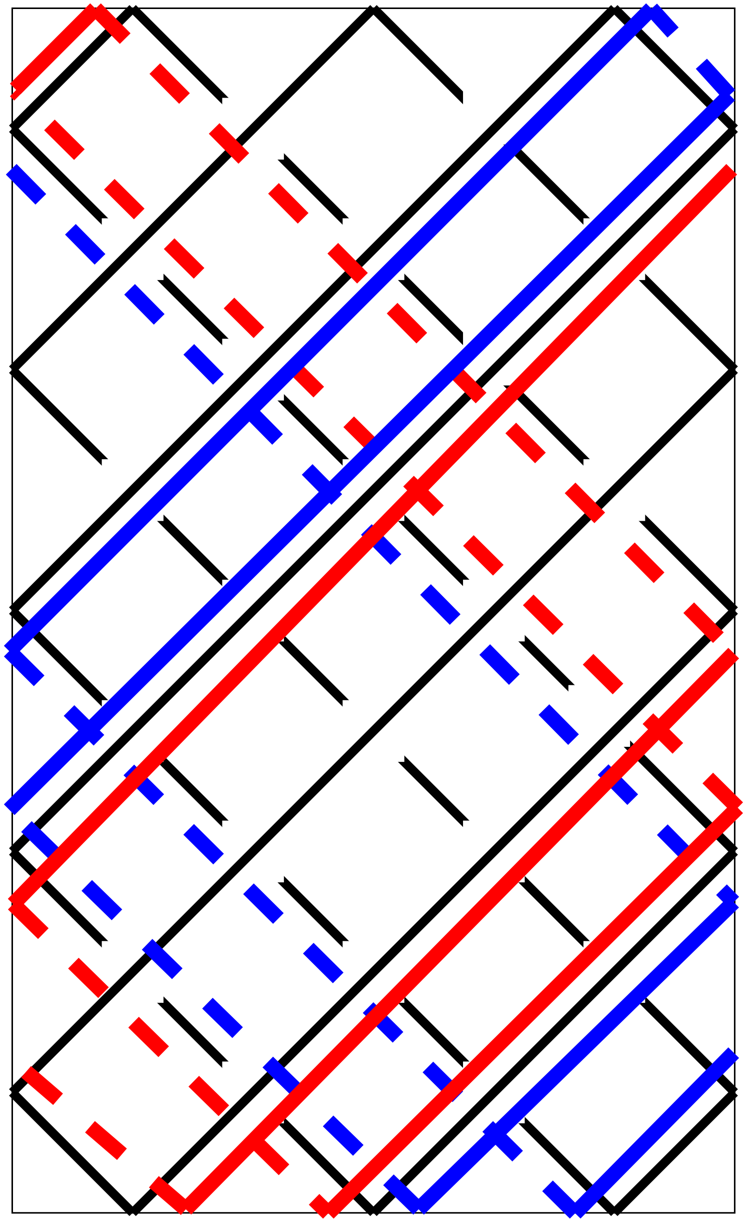
How do those 'components' look like?



The ‘components’ k_1 and k_j of $\varphi^{-1}(k)$.







Algorithm.

Let ε be the sign of the ratio β_i/α_i and draw the oriented meridian d of the rational tangle $|\beta_i/\alpha_i|$.

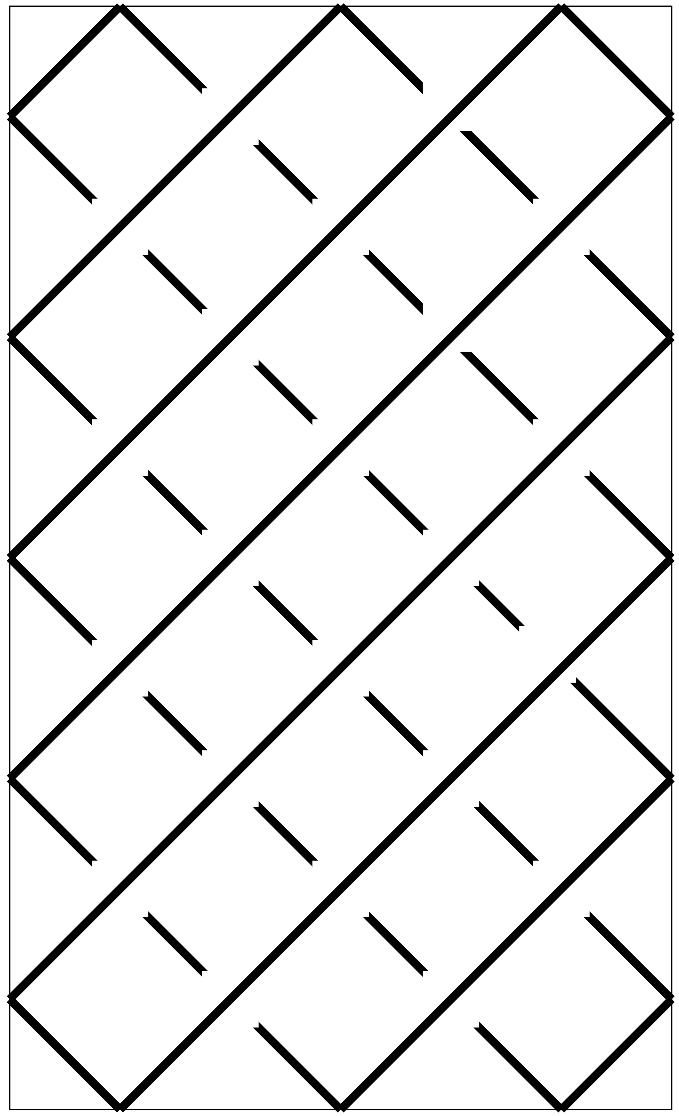
1. Mark the point $p_0 = (0, 1/2\alpha_i)$ with $+1$.

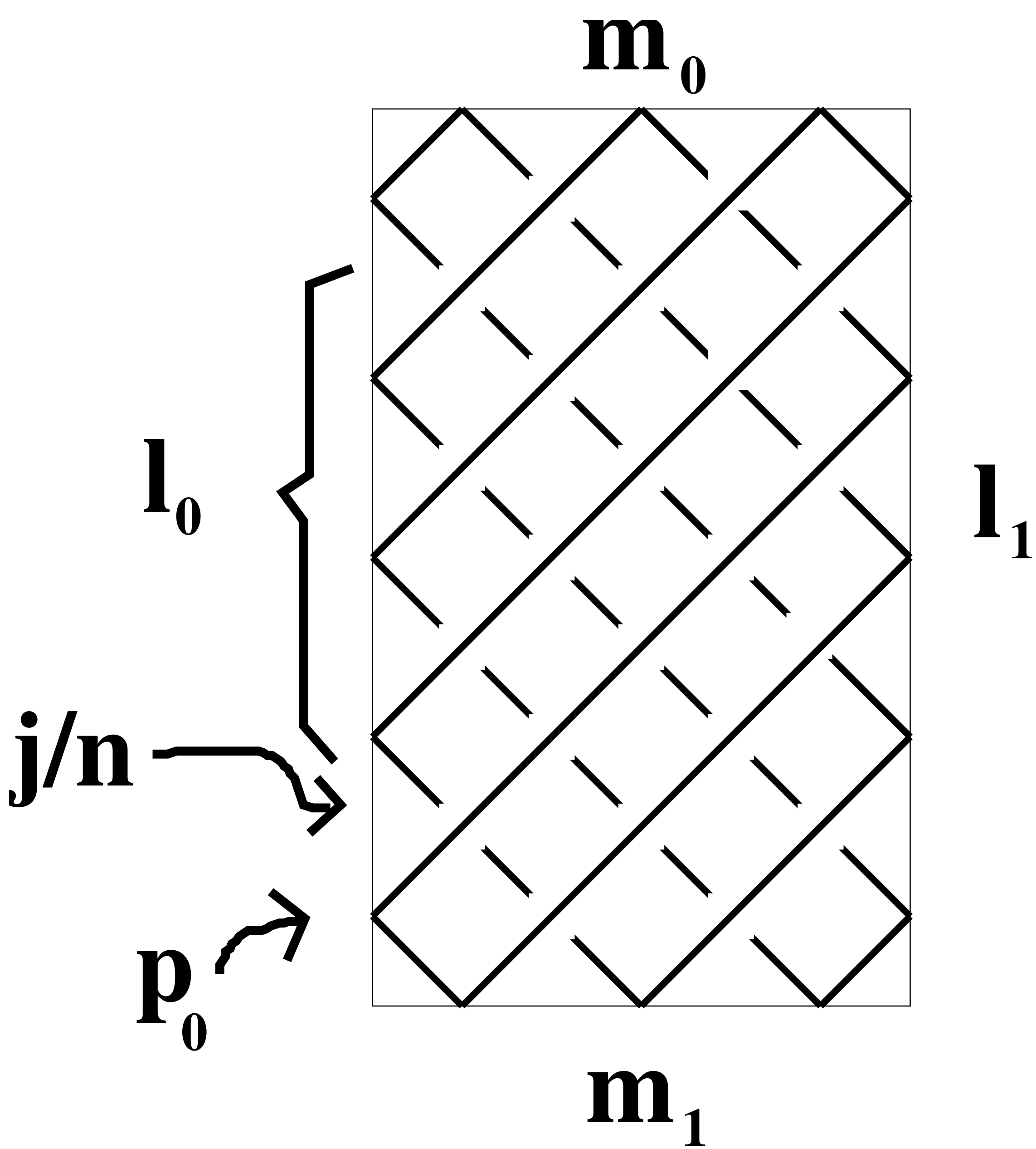
2. If the point $p_u \in d \cap \partial I^2$ is marked with $\varepsilon_u \in \{-1, +1\}$, then the line segment of d which begins at p_u, following the orientation of d, intersects ∂I^2 in the point p_{u+1}.

   (a) If p_{u+1} has no mark, then
   - If ($p_u \in m_0$ and $p_{u+1} \in m_0$) or ($p_u \in m_1$ and $p_{u+1} \in m_1$), then mark p_{u+1} with $\varepsilon_{u+1} = -\varepsilon_u$;
   - otherwise mark p_{u+1} with $\varepsilon_{u+1} = \varepsilon_u$.

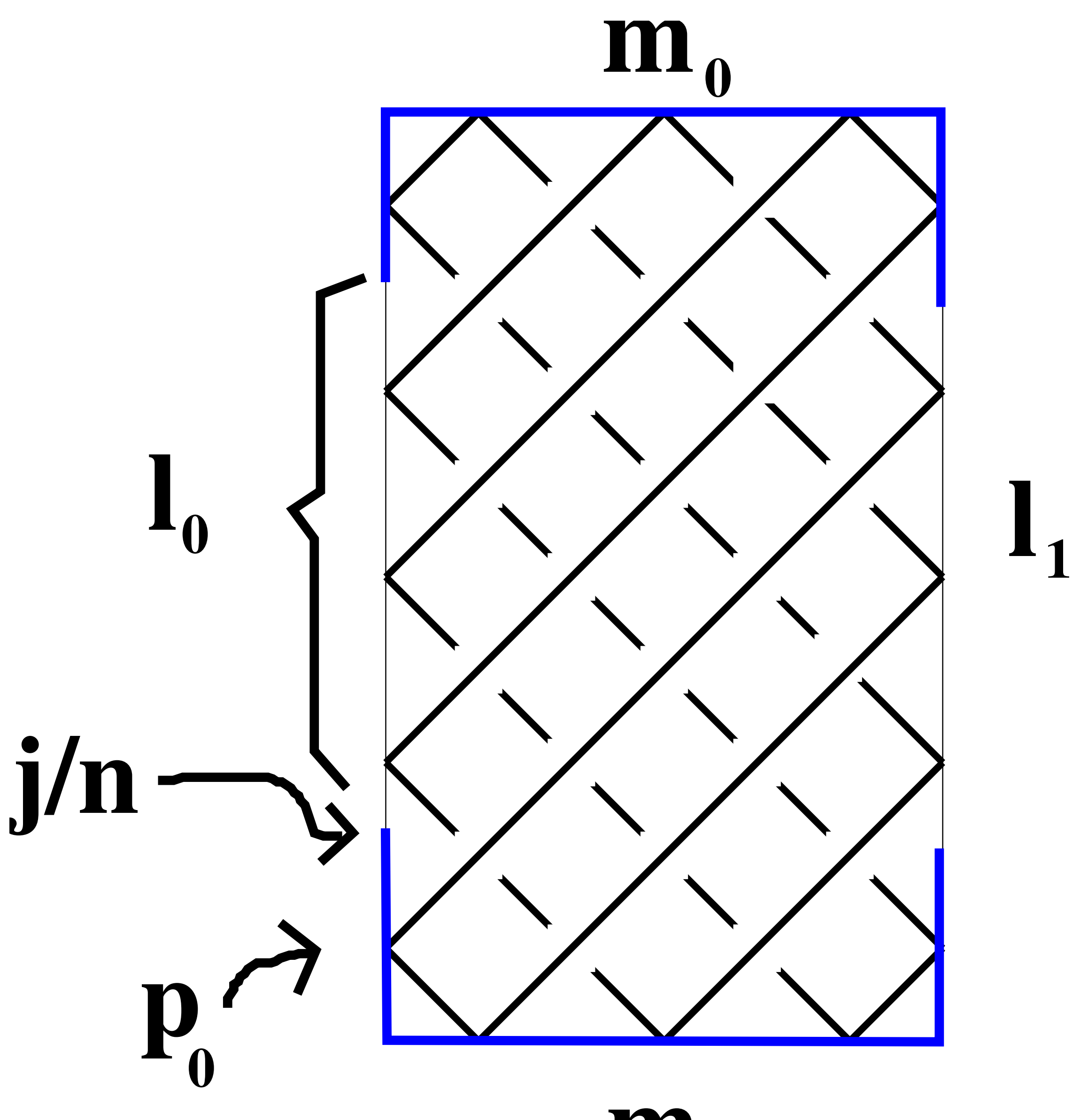
   GOTO 2 with '$u := u + 1$'.

(b) If p_{u+1} is already marked, then write $b =$ the sum of the marks of the points in m_0 and $\alpha =$ the sum of the marks of the points in ℓ_0. Return $b_i^j/\alpha_i^j = \varepsilon b/\alpha$.

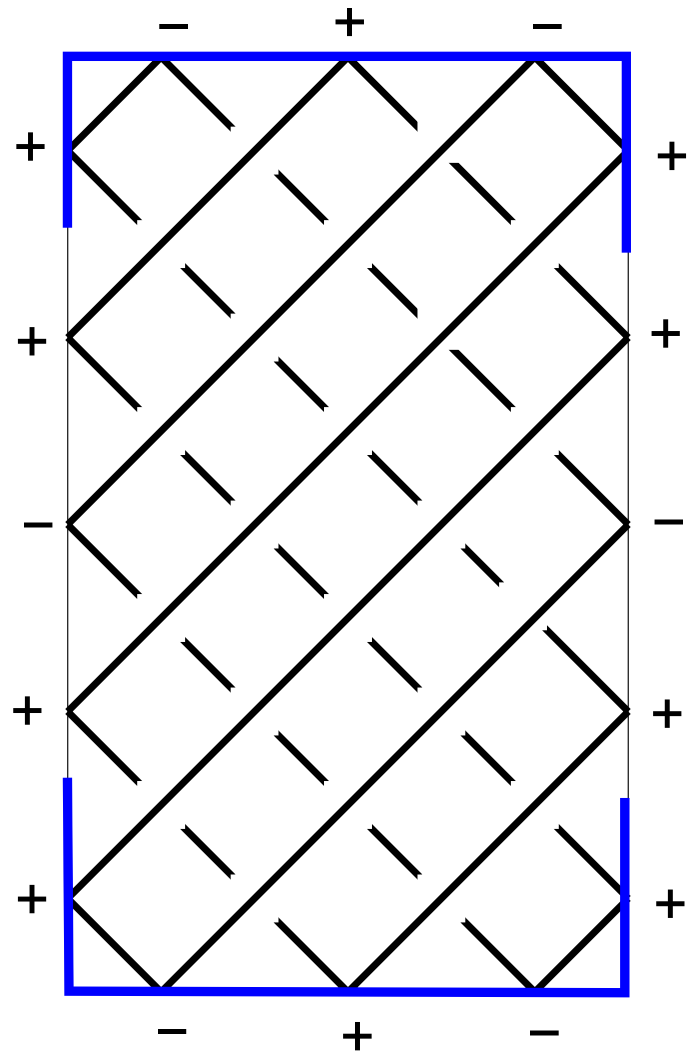




m_0
l_0
l_1
j/n
p_0
m_1



m_0
l_0
l_1
j/n
p_0
m_1



$$b_i^j/\alpha_i^j = 1/1.$$

An application.

Proposition. (J. Rodríguez and V.) Let q be an odd integer, $q \notin \{-11, -7, -5, -3, -1, 1, 3, 5\}$ and let k be the pretzel knot $k = p(2, q, q) = m(1/2, \pm 1/|q|, \pm 1/|q|)$.
Then there exists a $|q+4|$–fold dihedral covering

$$\varphi : S^3 \to (S^3, k)$$

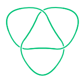
such that

1. If $|q| \equiv 1 \mod 4$, then
   either the Montesinos knot $m(1/2, -1/5, -1/5) \subset \varphi^{-1}(k)$
   or $m(1/2, -2/9, -2/9) \subset \varphi^{-1}(k)$.

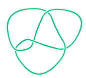
2. If $|q| \equiv -1 \mod 4$, then
   either $m(-1/2, 3/5, 3/5) \subset \varphi^{-1}(k)$
   or $m(-1/2, 2/3, 2/3) \subset \varphi^{-1}(k)$.

How many Montesinos knots are there?

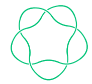
Rational knots.



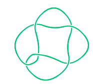
$3_1 = \mathrm{m}(\frac{1}{3})$



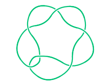
$4_1 = \mathrm{m}(\frac{2}{5})$



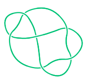
$5_1 = \mathrm{m}(\frac{1}{5})$



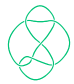
$5_2 = \mathrm{m}(\frac{3}{7})$



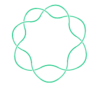
$6_1 = \mathrm{m}(\frac{4}{9})$



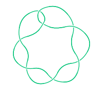
$6_2 = \mathrm{m}(\frac{4}{11})$



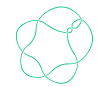
$6_3 = \mathrm{m}(\frac{5}{13})$



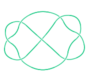
$7_1 = \mathrm{m}(\frac{1}{7})$



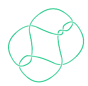
$7_2 = \mathrm{m}(\frac{5}{11})$



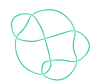
$7_3 = \mathrm{m}(\frac{4}{13})$



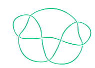
$7_4 = \mathrm{m}(\frac{4}{15})$



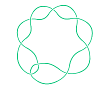
$7_5 = \mathrm{m}(\frac{7}{17})$



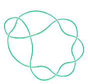
$7_6 = \mathrm{m}(\frac{7}{19})$



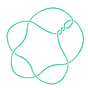
$7_7 = \mathrm{m}(\frac{8}{21})$



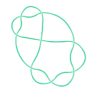
$8_1 = \mathrm{m}(\frac{6}{13})$



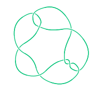
$8_2 = \mathrm{m}(\frac{6}{17})$



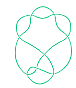
$8_3 = \mathrm{m}(\frac{4}{17})$



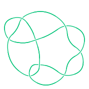
$8_4 = \mathrm{m}(\frac{5}{19})$



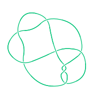
$8_6 = \mathrm{m}(\frac{10}{23})$



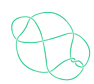
$8_7 = \mathrm{m}(\frac{9}{23})$



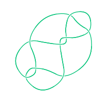
$8_8 = \mathrm{m}(\frac{9}{25})$



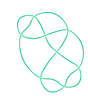
$8_9 = \mathrm{m}(\frac{7}{25})$



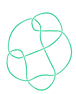
$8_{11} = \mathrm{m}(\frac{10}{27})$



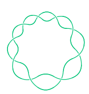
$8_{12} = \mathrm{m}(\frac{12}{29})$



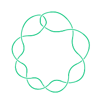
$8_{13} = \mathrm{m}(\frac{11}{29})$



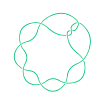
$8_{14} = \mathrm{m}(\frac{12}{31})$



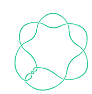
$9_1 = \mathrm{m}(\frac{1}{9})$



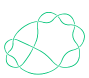
$9_2 = \mathrm{m}(\frac{7}{15})$



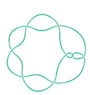
$9_3 = \mathrm{m}(\frac{6}{19})$



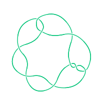
$9_4 = \mathrm{m}(\frac{5}{21})$



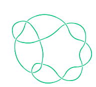
$9_5 = \mathrm{m}(\frac{6}{23})$



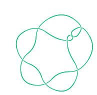
$9_6 = \mathrm{m}(\frac{11}{27})$



$9_7 = \mathrm{m}(\frac{13}{29})$

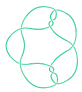


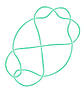
$9_8 = \mathrm{m}(\frac{11}{31})$

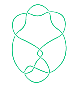


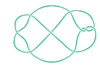
$9_9 = \mathrm{m}(\frac{9}{31})$

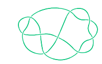
Rational knots.

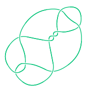

$9_{10} = \mathrm{m}(\frac{10}{33})$

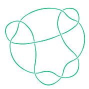

$9_{11} = \mathrm{m}(\frac{14}{33})$

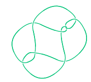

$9_{12} = \mathrm{m}(\frac{13}{35})$

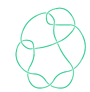

$9_{13} = \mathrm{m}(\frac{10}{37})$

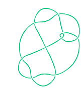

$9_{14} = \mathrm{m}(\frac{14}{37})$

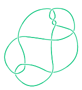

$9_{15} = \mathrm{m}(\frac{16}{39})$

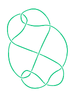

$9_{17} = \mathrm{m}(\frac{14}{39})$

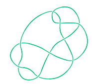

$9_{18} = \mathrm{m}(\frac{17}{41})$

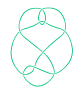

$9_{19} = \mathrm{m}(\frac{16}{41})$

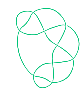

$9_{20} = \mathrm{m}(\frac{15}{41})$

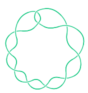

$9_{21} = \mathrm{m}(\frac{18}{43})$

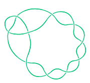

$9_{23} = \mathrm{m}(\frac{19}{45})$

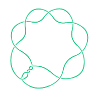

$9_{26} = \mathrm{m}(\frac{18}{47})$

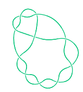

$9_{27} = \mathrm{m}(\frac{19}{49})$

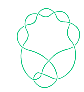

$9_{31} = \mathrm{m}(\frac{21}{55})$

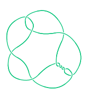

$10_{1} = \mathrm{m}(\frac{8}{17})$

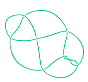

$10_{2} = \mathrm{m}(\frac{8}{23})$

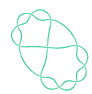

$10_{3} = \mathrm{m}(\frac{6}{25})$

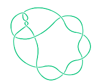

$10_{4} = \mathrm{m}(\frac{7}{27})$

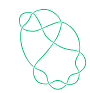

$10_{5} = \mathrm{m}(\frac{13}{33})$

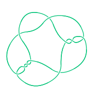

$10_{6} = \mathrm{m}(\frac{16}{37})$

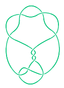

$10_{7} = \mathrm{m}(\frac{16}{43})$

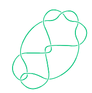

$10_{8} = \mathrm{m}(\frac{6}{29})$

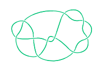

$10_{9} = \mathrm{m}(\frac{11}{39})$

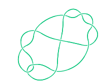

$10_{10} = \mathrm{m}(\frac{17}{45})$

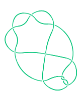

$10_{11} = \mathrm{m}(\frac{13}{43})$

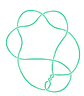

$10_{12} = \mathrm{m}(\frac{17}{47})$

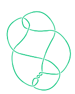

$10_{13} = \mathrm{m}(\frac{22}{53})$

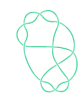

$10_{14} = \mathrm{m}(\frac{22}{57})$

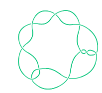

$10_{15} = \mathrm{m}(\frac{19}{43})$


$10_{16} = \mathrm{m}(\frac{14}{47})$

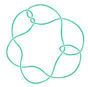

$10_{17} = \mathrm{m}(\frac{9}{41})$


$10_{18} = \mathrm{m}(\frac{23}{55})$

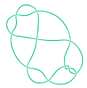

$10_{19} = \mathrm{m}(\frac{14}{51})$


$10_{20} = \mathrm{m}(\frac{16}{35})$

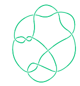
Rational knots.



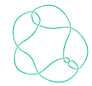
$10_{21} = \mathrm{m}(\frac{16}{45})$



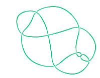
$10_{22} = \mathrm{m}(\frac{13}{49})$



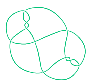
$10_{23} = \mathrm{m}(\frac{23}{59})$



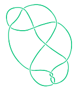
$10_{24} = \mathrm{m}(\frac{24}{55})$



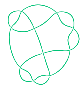
$10_{25} = \mathrm{m}(\frac{24}{65})$



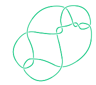
$10_{26} = \mathrm{m}(\frac{17}{61})$



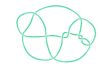
$10_{27} = \mathrm{m}(\frac{27}{71})$



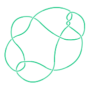
$10_{28} = \mathrm{m}(\frac{19}{53})$



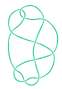
$10_{29} = \mathrm{m}(\frac{26}{63})$



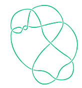
$10_{30} = \mathrm{m}(\frac{26}{67})$



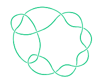
$10_{31} = \mathrm{m}(\frac{25}{57})$



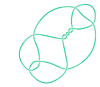
$10_{32} = \mathrm{m}(\frac{29}{69})$



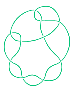
$10_{33} = \mathrm{m}(\frac{18}{65})$



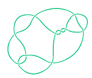
$10_{34} = \mathrm{m}(\frac{13}{37})$



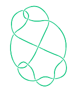
$10_{35} = \mathrm{m}(\frac{20}{49})$



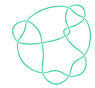
$10_{36} = \mathrm{m}(\frac{20}{51})$



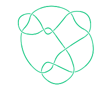
$10_{37} = \mathrm{m}(\frac{23}{53})$



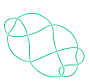
$10_{38} = \mathrm{m}(\frac{25}{59})$



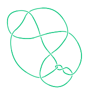
$10_{39} = \mathrm{m}(\frac{22}{61})$



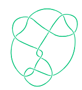
$10_{40} = \mathrm{m}(\frac{29}{75})$



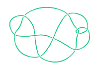
$10_{41} = \mathrm{m}(\frac{26}{71})$



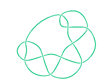
$10_{42} = \mathrm{m}(\frac{31}{81})$



$10_{43} = \mathrm{m}(\frac{27}{73})$

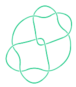


$10_{44} = \mathrm{m}(\frac{30}{79})$

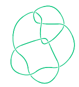


$10_{45} = \mathrm{m}(\frac{34}{89})$

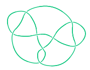
Montesinos knots.



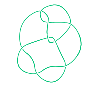
$8_5 = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{2})$



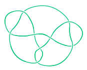
$8_{10} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{1}{2})$



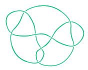
$8_{15} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{1}{2})$



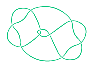
$8_{19} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{-1}{2})$



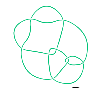
$8_{20} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{-1}{2})$



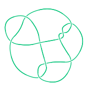
$8_{21} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{2})$



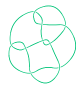
$9_{16} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{3}{2})$



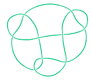
$9_{22} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{2})$



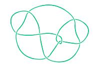
$9_{24} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{3}{2})$



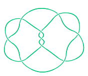
$9_{25} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{1}{2})$



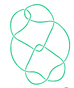
$9_{28} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{3}{2})$



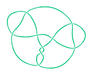
$9_{30} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{1}{2})$



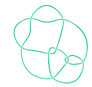
$9_{35} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



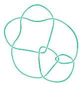
$9_{36} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{1}{2})$



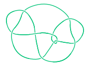
$9_{37} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$



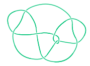
$9_{42} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-1}{2})$



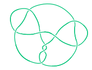
$9_{43} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{2})$



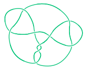
$9_{44} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{2})$



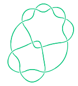
$9_{45} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{-1}{2})$



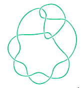
$9_{46} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{-1}{3})$



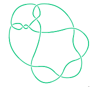
$9_{48} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3})$



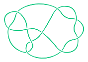
$10_{46} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{1}{2})$



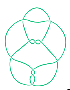
$10_{47} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{1}{2})$



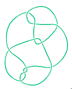
$10_{48} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{1}{2})$



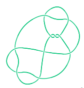
$10_{49} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{1}{2})$



$10_{50} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{1}{2})$

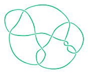


$10_{51} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{1}{2})$

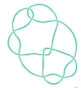


$10_{52} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{1}{2})$

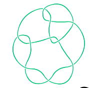
Montesinos knots.



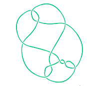
$10_{53} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{1}{2})$



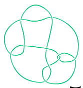
$10_{54} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{1}{2})$



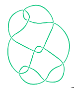
$10_{55} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{1}{2})$



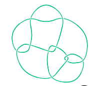
$10_{56} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{1}{2})$



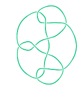
$10_{57} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{1}{2})$



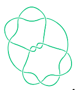
$10_{58} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{1}{2})$



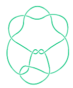
$10_{59} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{1}{2})$



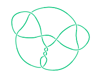
$10_{60} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{1}{2})$



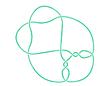
$10_{61} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{1}{3})$



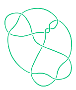
$10_{62} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{2}{3})$



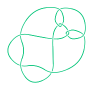
$10_{63} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{2}{3})$



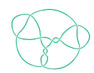
$10_{64} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{1}{3})$



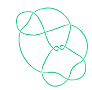
$10_{65} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{2}{3})$



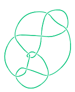
$10_{66} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{2}{3})$



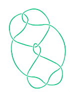
$10_{67} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{2}{3})$



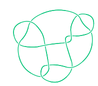
$10_{68} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{3})$



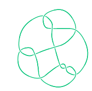
$10_{69} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{2}{3})$



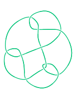
$10_{70} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{3}{2})$



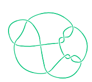
$10_{71} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{3}{2})$



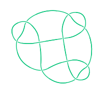
$10_{72} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{3}{2})$



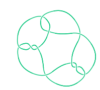
$10_{73} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{3}{2})$



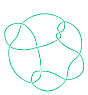
$10_{74} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{3})$



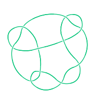
$10_{75} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{3})$



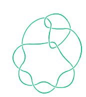
$10_{76} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{2})$



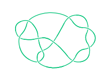
$10_{77} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{5}{2})$



$10_{78} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{2})$

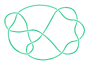


$10_{124} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{-1}{2})$

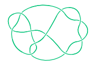


$10_{125} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{-1}{2})$

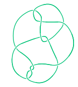
Montesinos knots.



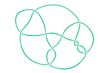
$10_{126} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{-1}{2})$



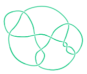
$10_{127} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{-1}{2})$



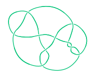
$10_{128} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{-1}{2})$



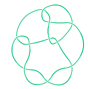
$10_{129} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{-1}{2})$



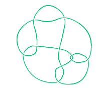
$10_{130} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{-1}{2})$



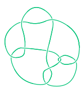
$10_{131} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{-1}{2})$



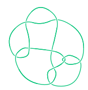
$10_{132} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{-1}{2})$



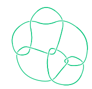
$10_{133} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{-1}{2})$



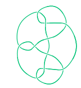
$10_{134} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{-1}{2})$



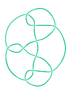
$10_{135} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{-1}{2})$



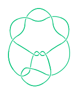
$10_{136} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{-1}{2})$



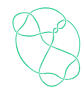
$10_{137} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{-1}{2})$



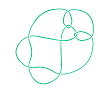
$10_{138} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{-1}{2})$



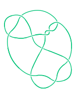
$10_{139} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-2}{3})$



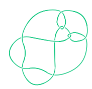
$10_{140} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-1}{3})$



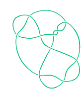
$10_{141} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{-1}{3})$



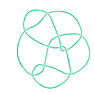
$10_{142} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-2}{3})$



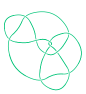
$10_{143} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-1}{3})$



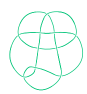
$10_{144} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{-1}{3})$



$10_{145} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-2}{3})$



$10_{146} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{3})$

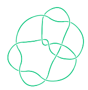


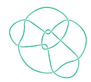
$10_{147} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{3})$

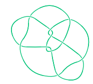
The others.

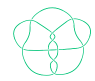
8_{16} 8_{17} 8_{18} 9_{29} 9_{32} 9_{33} 9_{34}
9_{38} 9_{39} 9_{40} 9_{41} 9_{47} 9_{49} 10_{79}
10_{80} 10_{81} 10_{82} 10_{83} 10_{84} 10_{85} 10_{86}
10_{87} 10_{88} 10_{89} 10_{90} 10_{91} 10_{92} 10_{93}
10_{94} 10_{95} 10_{96} 10_{97} 10_{98} 10_{99} 10_{100}
10_{101} 10_{102} 10_{103} 10_{104} 10_{105} 10_{106} 10_{107}

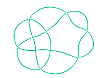
The others.

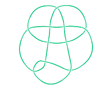

10_{108}

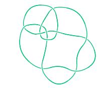

10_{109}

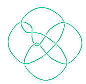

10_{110}

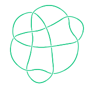

10_{111}

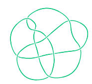

10_{112}

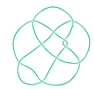

10_{113}

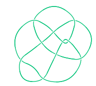

10_{114}

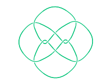

10_{115}

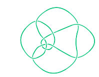

10_{116}

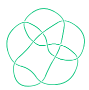

10_{117}

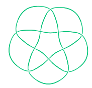

10_{118}

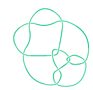

10_{119}

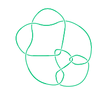

10_{120}

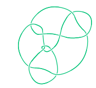

10_{121}

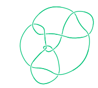

10_{122}

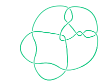

10_{123}

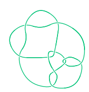

10_{148}

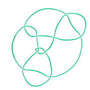

10_{149}

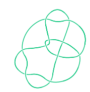

10_{150}

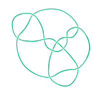

10_{151}

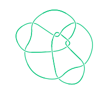

10_{152}

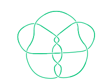

10_{153}

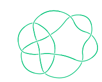

10_{154}

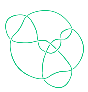

10_{155}

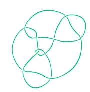

10_{156}

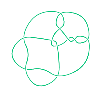

10_{157}

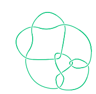

10_{158}

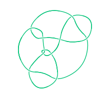

10_{159}

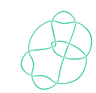

10_{160}

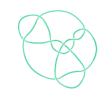

10_{151}


10_{152}


10_{153}


10_{154}


10_{155}


10_{156}

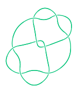
We need some specific universal knots.

Dihedral like coverings

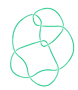
$$\begin{array}{ccc} & \tilde{M} & \\ {}^{q}\swarrow & & \searrow^{\psi} \\ M & & B_2(k) \\ {}_{\varphi}\searrow & & \swarrow_{p} \\ & (S^3, k) & \end{array}$$

ψ is any covering space.

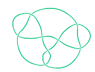
1. $k = m(\beta_1/2, \beta_2/3, \beta_3/3)$ is universal $\Leftrightarrow \Delta(k) \neq \pm 3$.



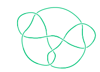
$8_5 = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{2})$



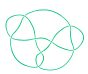
$8_{10} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{1}{2})$



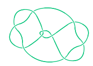
$8_{15} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{1}{2})$



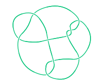
$8_{20} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{-1}{2})$



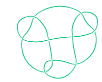
$8_{21} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{2})$



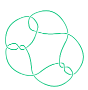
$9_{16} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{3}{2})$



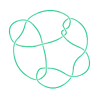
$9_{24} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{3}{2})$



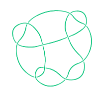
$9_{28} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{3}{2})$



$10_{76} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{2})$

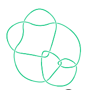


$10_{77} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{5}{2})$

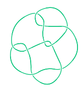


$10_{78} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{2})$

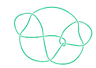
2. $k = m(\beta_1/2, \beta_2/3, \beta_3/5)$ is universal $\Leftrightarrow \Delta(k) \neq \pm 1$.



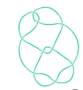
$9_{22} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{2})$



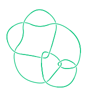
$9_{25} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{1}{2})$



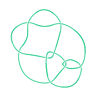
$9_{30} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{1}{2})$



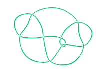
$9_{36} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{1}{2})$



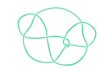
$9_{42} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-1}{2})$



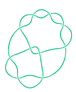
$9_{43} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{2})$



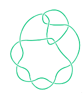
$9_{44} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{2})$



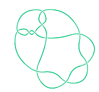
$9_{45} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{-1}{2})$



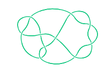
$10_{46} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{1}{2})$



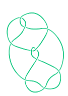
$10_{47} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{1}{2})$



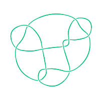
$10_{48} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{1}{2})$



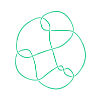
$10_{49} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{1}{2})$



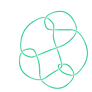
$10_{70} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{3}{2})$



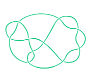
$10_{71} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{3}{2})$



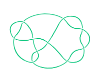
$10_{72} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{3}{2})$



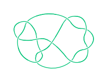
$10_{73} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{3}{2})$



$10_{125} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{-1}{2})$

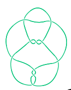


$10_{126} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{-1}{2})$

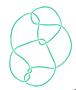


$10_{127} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{-1}{2})$

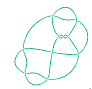
3. $k = m(\beta_1/2, \beta_2/3, \beta_3/7)$ is universal.



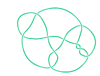
$10_{50} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{1}{2})$



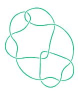
$10_{51} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{1}{2})$



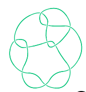
$10_{52} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{1}{2})$



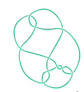
$10_{53} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{1}{2})$



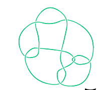
$10_{54} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{1}{2})$



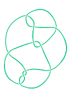
$10_{55} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{1}{2})$



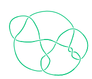
$10_{56} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{1}{2})$



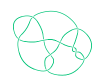
$10_{57} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{1}{2})$



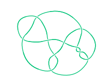
$10_{128} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{-1}{2})$



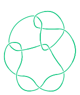
$10_{129} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{-1}{2})$



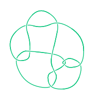
$10_{130} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{-1}{2})$



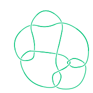
$10_{131} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{-1}{2})$



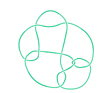
$10_{132} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{-1}{2})$



$10_{133} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{-1}{2})$



$10_{134} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{-1}{2})$

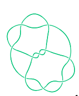


$10_{135} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{-1}{2})$

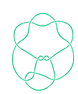
4. (a) $|x| > 1 \Rightarrow k = p(e; 2x, 3y, 3z)$ is universal.

(a.1) $|y| > 1$ o $|z| > 1 \Rightarrow k = p(2, 3y, 3z)$ is universal.

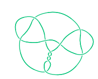
(a.2) $|y| > 1$ o $|z| > 1$ y $\beta_2 \equiv \pm 1 \pmod{y}$ y $\beta_3 \equiv \pm 1 \pmod{z}$
$\Rightarrow k = m(1/2, \beta_2/3y, \beta_3/3z)$ is universal.



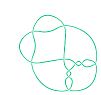
$10_{61} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{1}{3})$



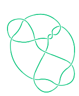
$10_{62} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{2}{3})$



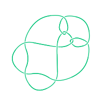
$10_{63} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{2}{3})$



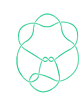
$10_{64} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{1}{3})$



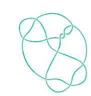
$10_{65} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{2}{3})$



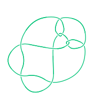
$10_{66} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{2}{3})$



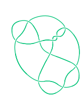
$10_{139} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-2}{3})$



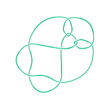
$10_{140} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-1}{3})$



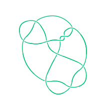
$10_{141} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{-1}{3})$



$10_{142} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-2}{3})$



$10_{143} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-1}{3})$

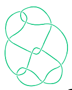


$10_{144} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{-1}{3})$

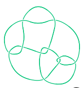
(b) $|x| > 1 \Rightarrow k = p(\pm 2, \pm 3y, \pm 5z)$ is universal.

(c) $z > 0 \Rightarrow k = p(\pm 2, \pm 3, \pm 7)$ is universal.

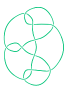
5. $y, z \neq 0 \Rightarrow p(\pm 2, 5y, 5z)$ is universal.



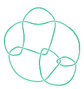
$10_{58} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{1}{2})$



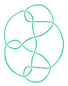
$10_{59} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{1}{2})$



$10_{60} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{1}{2})$



$10_{136} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{-1}{2})$



$10_{138} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{-1}{2})$

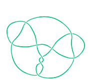
Theorem.

If $p(b; \alpha_1, \ldots, \alpha_t)$ is an Uchida universal link and $(n, \alpha_i) = 1 \quad \forall i$
$\Rightarrow m(nb/1, n/\alpha_1, \ldots, n/\alpha_t)$ is universal.

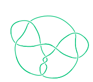
Theorem.

If $|p| > 1$ and $(n, p) = 1$ and p is odd $\Rightarrow m(n/p, n/p, -n/p)$ is universal.

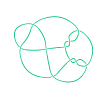
If $p \neq 2$ and $(n, p) = 1$ and p is even $\Rightarrow m(n/3, n/3, n/p)$ is universal.



$9_{37} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$



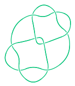
$9_{46} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{-1}{3})$



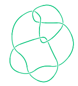
$10_{74} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{3})$

Sixty six universal Montesinos knots!

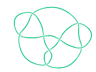
Universal Montesinos knots.



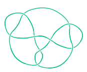
$8_5 = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{2})$



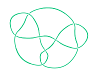
$8_{10} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{1}{2})$



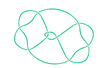
$8_{15} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{1}{2})$



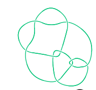
$8_{20} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{-1}{2})$



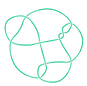
$8_{21} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{2})$



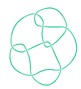
$9_{16} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{3}{2})$



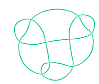
$9_{22} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{2})$



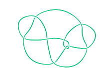
$9_{24} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{3}{2})$



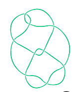
$9_{25} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{1}{2})$



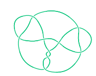
$9_{28} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{3}{2})$



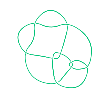
$9_{30} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{1}{2})$



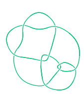
$9_{36} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{1}{2})$



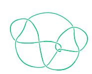
$9_{37} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$



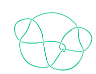
$9_{42} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-1}{2})$



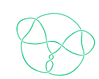
$9_{43} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{2})$



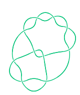
$9_{44} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{2})$



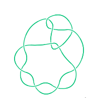
$9_{45} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{-1}{2})$



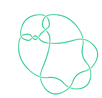
$9_{46} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{-1}{3})$



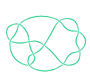
$10_{46} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{1}{2})$



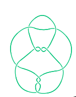
$10_{47} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{1}{2})$



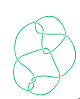
$10_{48} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{1}{2})$



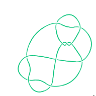
$10_{49} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{1}{2})$



$10_{50} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{1}{2})$

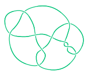


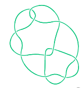
$10_{51} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{1}{2})$

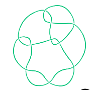


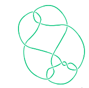
$10_{52} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{1}{2})$

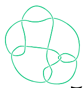
Universal Montesinos knots.

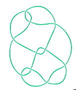

$10_{53} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{1}{2})$

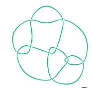

$10_{54} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{1}{2})$

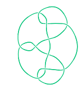

$10_{55} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{1}{2})$

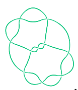

$10_{56} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{1}{2})$

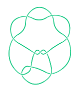

$10_{57} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{1}{2})$

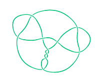

$10_{58} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{1}{2})$

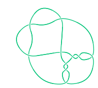

$10_{59} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{1}{2})$

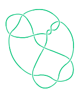

$10_{60} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{1}{2})$

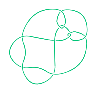

$10_{61} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{1}{3})$

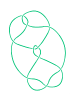

$10_{62} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{2}{3})$

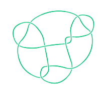

$10_{63} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{2}{3})$

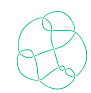

$10_{64} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{1}{3})$

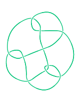

$10_{65} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{2}{3})$

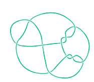

$10_{66} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{2}{3})$

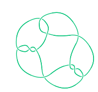

$10_{70} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{3}{2})$

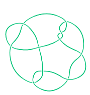

$10_{71} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{3}{2})$

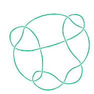

$10_{72} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{3}{2})$

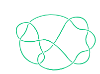

$10_{73} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{3}{2})$


$10_{74} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{3})$

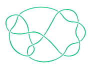

$10_{76} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{2})$


$10_{77} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{5}{2})$

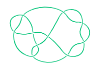

$10_{78} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{2})$


$10_{125} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{-1}{2})$

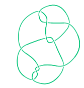
Universal Montesinos knots.



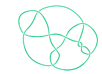
$10_{126} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{-1}{2})$



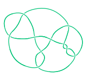
$10_{127} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{-1}{2})$



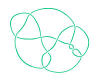
$10_{128} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{-1}{2})$



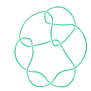
$10_{129} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{-1}{2})$



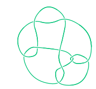
$10_{130} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{-1}{2})$



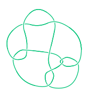
$10_{131} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{-1}{2})$



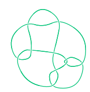
$10_{132} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{-1}{2})$



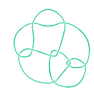
$10_{133} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{-1}{2})$



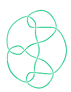
$10_{134} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{-1}{2})$



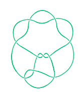
$10_{135} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{-1}{2})$



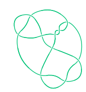
$10_{136} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{-1}{2})$



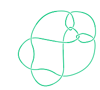
$10_{138} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{-1}{2})$



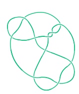
$10_{139} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-2}{3})$



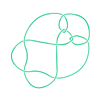
$10_{140} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-1}{3})$



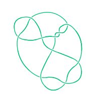
$10_{141} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{-1}{3})$



$10_{142} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-2}{3})$

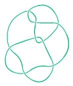


$10_{143} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-1}{3})$

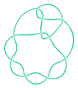


$10_{144} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{-1}{3})$

Torus knots.

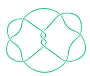


$8_{19} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{-1}{2})$

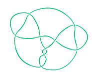


$10_{124} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{-1}{2})$

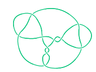
Undecided.



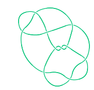
$9_{35} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



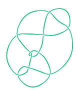
$9_{48} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3})$



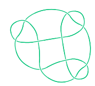
$10_{67} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{2}{3})$



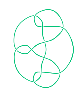
$10_{68} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{3})$



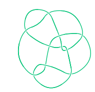
$10_{69} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{2}{3})$



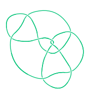
$10_{75} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{3})$



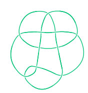
$10_{137} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{-1}{2})$



$10_{145} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-2}{3})$



$10_{146} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{3})$



$10_{147} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{3})$

More applications.

Theorem. Let q be an odd integer, $q \notin \{-1, -3, -7, -11\}$. Then $p(2, q, q)$ is universal.

Note that $p(2, -1, -1) =$ trefoil knot, and $p(2, -3, -3) = \tau_{3,4}$ are not universal knots.

Question: The knots $p(2, -7, -7)$ and $p(2, -11, -11)$, are universal knots?

Conjecture. Let q be an odd integer. The knot $p(2, q, q)$ is universal if and only if $q \neq -1, -3$.

Example.

$k = m(1/3, 3/5, -3/4, -2/7, 3/11, -5/13)$, $\Delta(k) = 12869$.

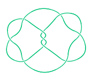
In the 12869-fold dihedral branched covering the ‘component’

$$k_{2758} = m(1/1, -1/2, 1/1, 1/1) = m(7/2)$$

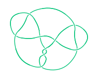
Thus k is universal.

Coverings of pillowcases (again).

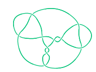
Undecided.



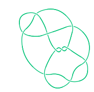
$9_{35} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



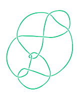
$9_{48} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3})$



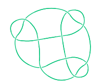
$10_{67} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{2}{3})$



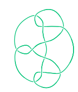
$10_{68} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{3})$



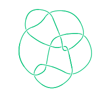
$10_{69} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{2}{3})$



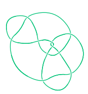
$10_{75} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{3})$



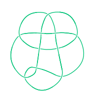
$10_{137} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{-1}{2})$



$10_{145} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-2}{3})$

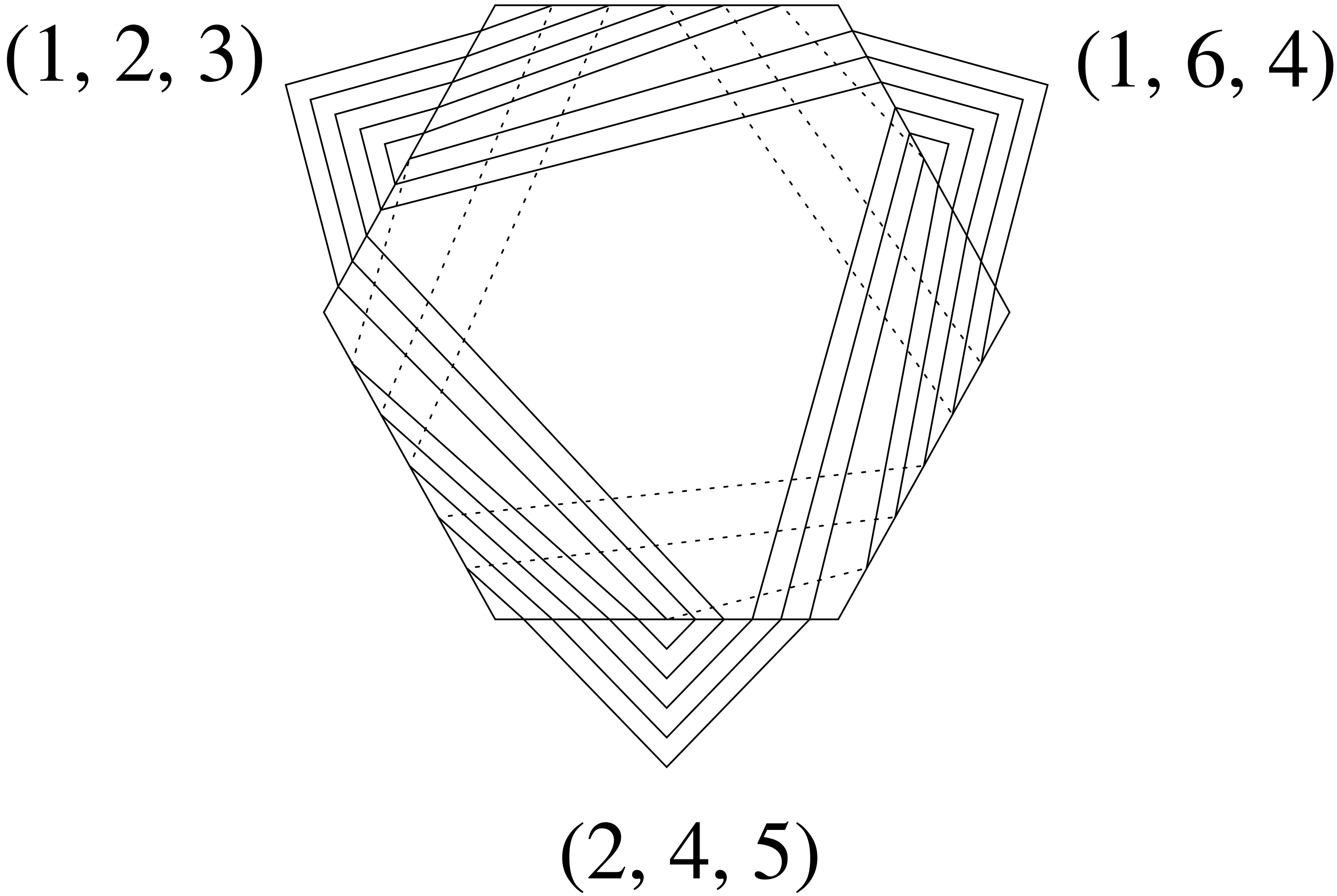


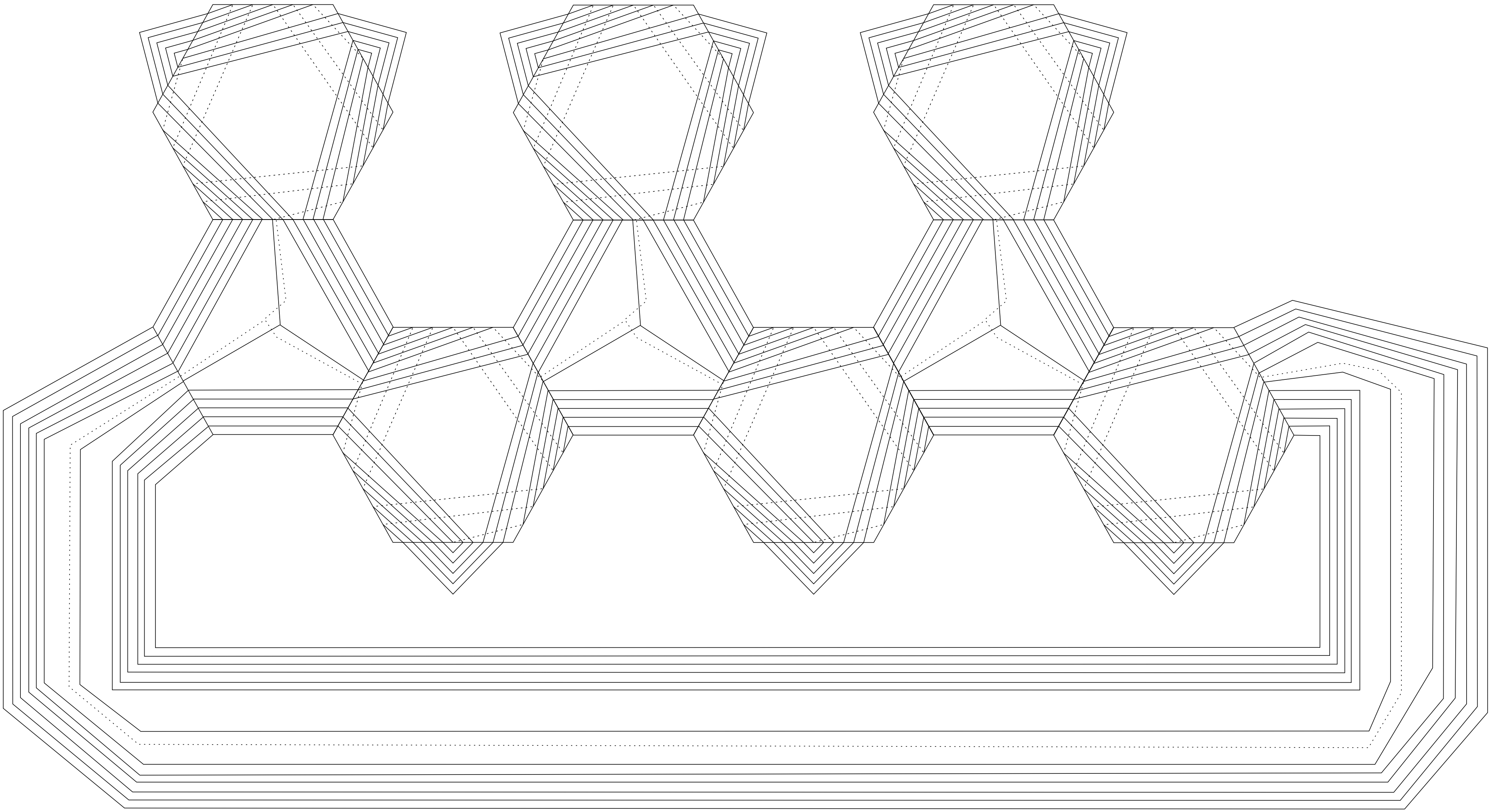
$10_{146} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{3})$

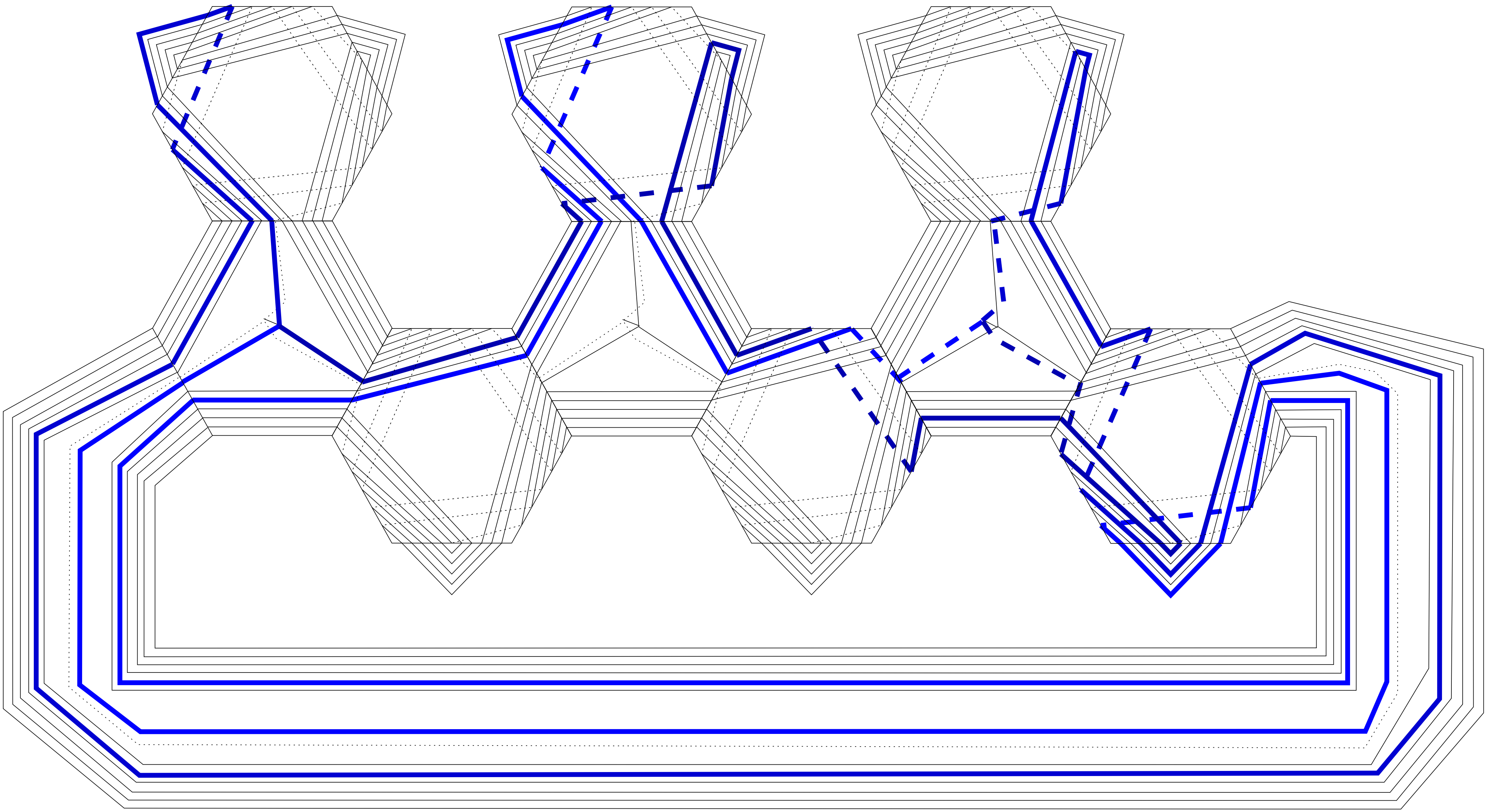


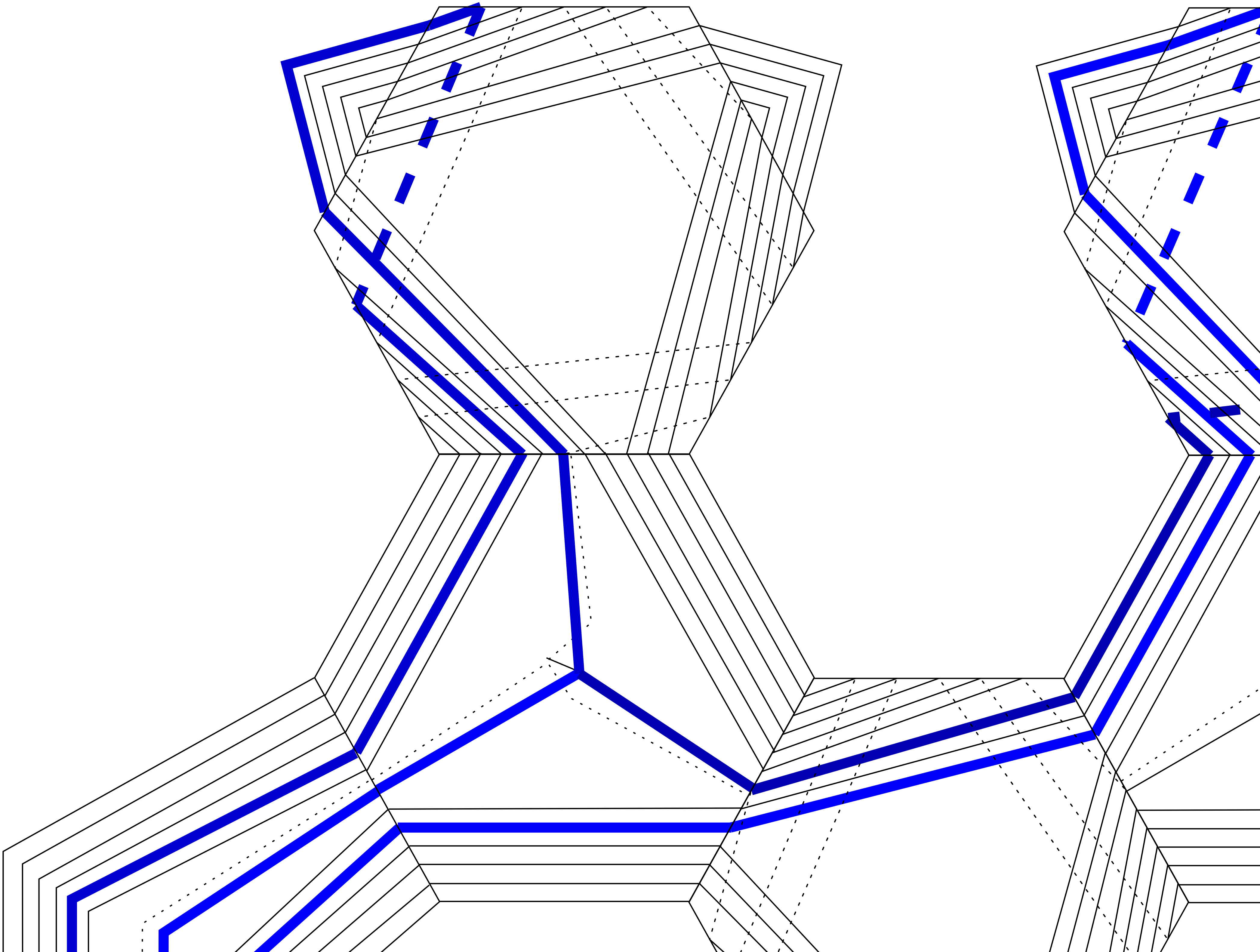
$10_{147} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{3})$

Specification of the problem.

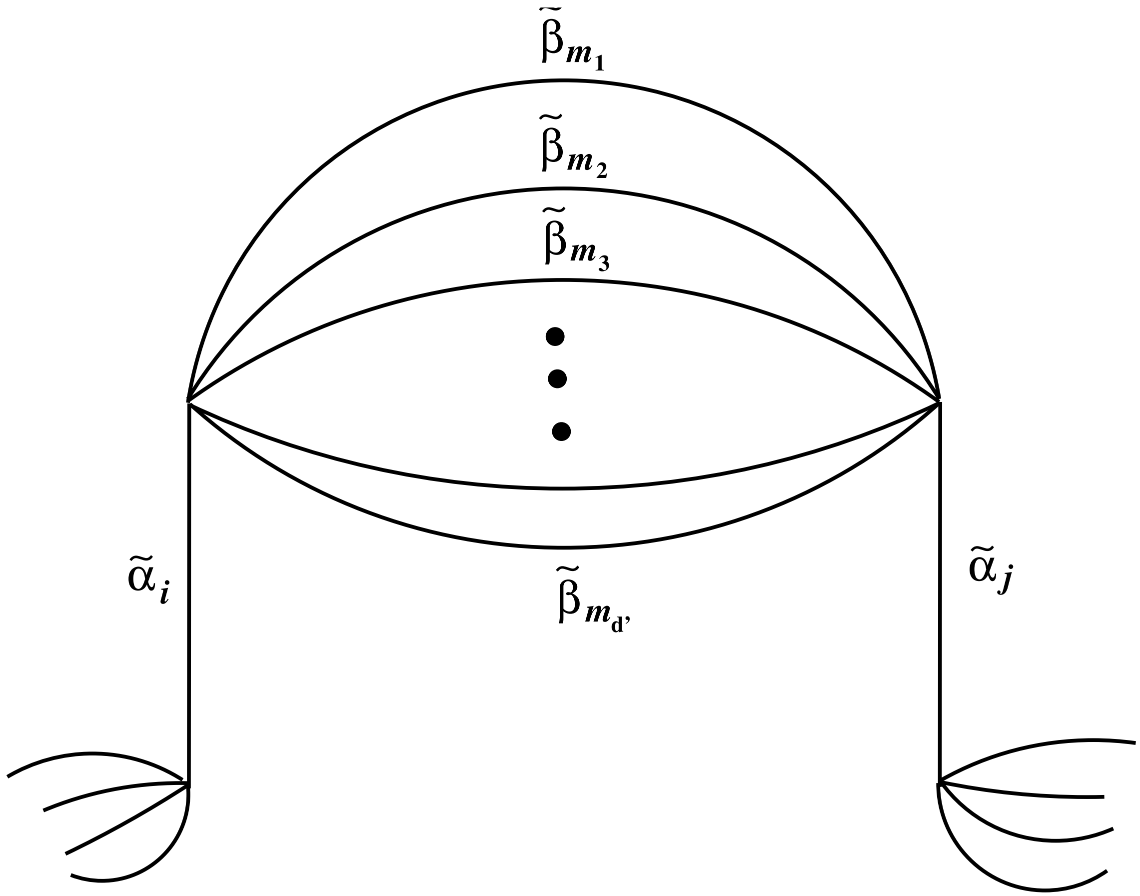

(1, 2, 3)
(1, 6, 4)
(2, 4, 5)





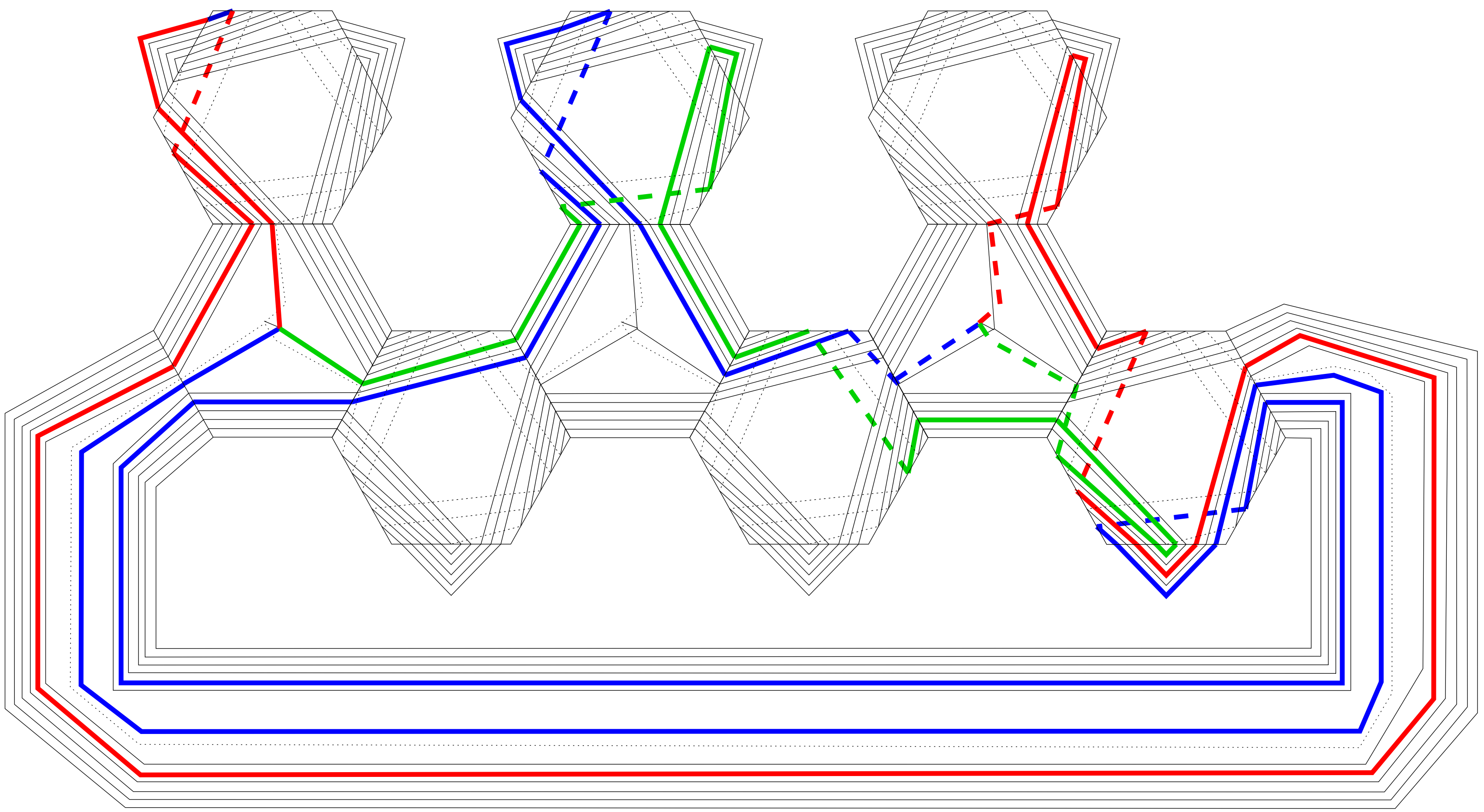


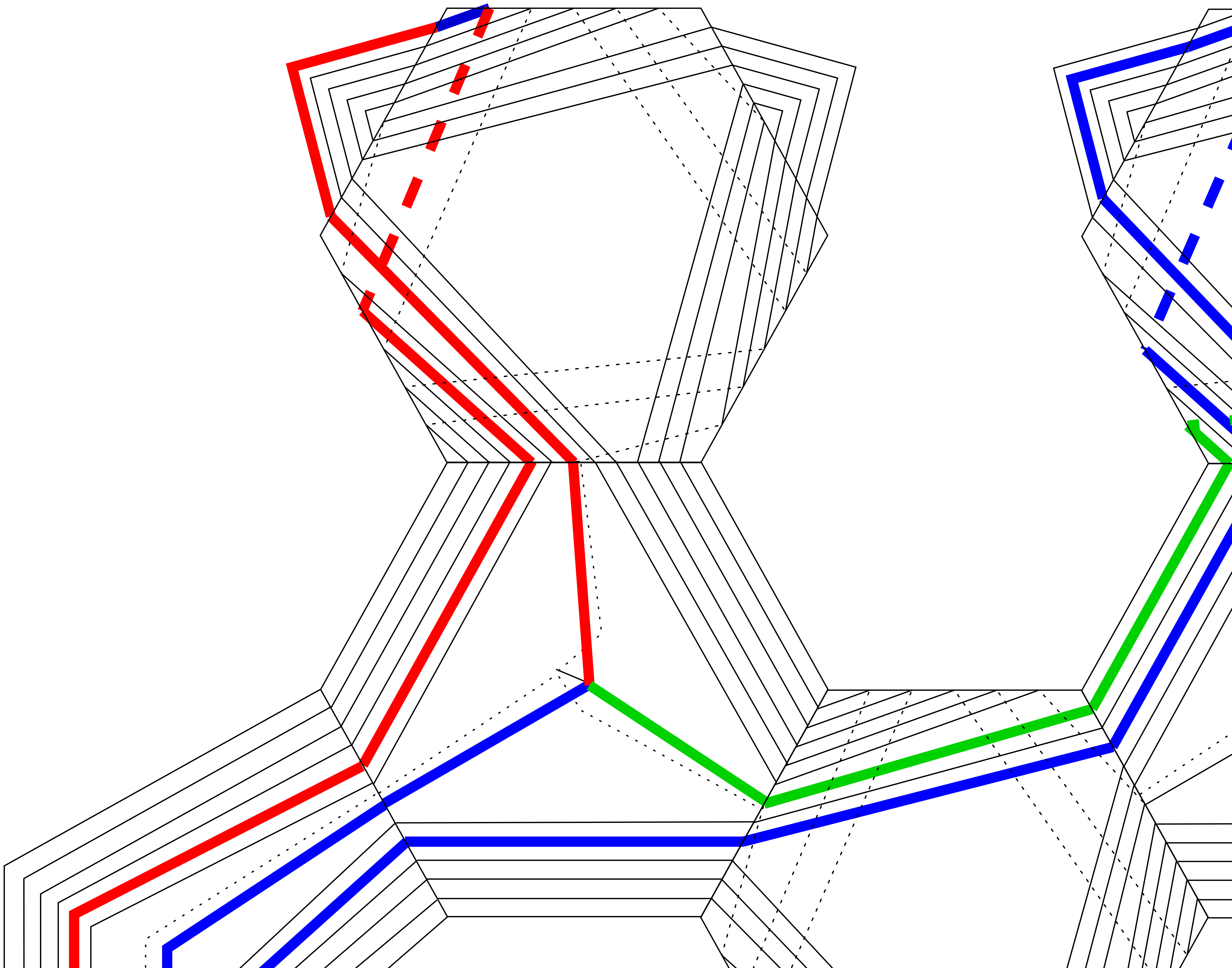
In general



$\varphi^{-1}(\beta_m)$ is a union of θ-graphs

Given an arc $\beta_m \subset \partial B$, a pair of consecutive arcs in $\varphi^{-1}(\beta_m)$ is called a ramification cycle.





Delete all arcs, except one, in each of the ramification cycles of $\varphi^{-1}(k)$.

The result is called

a cleansing of $\varphi^{-1}(k)$ on $\varphi^{-1}(B) \cong B_\omega$

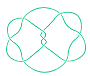
Theorem. (M. Jordán and V.)
Let $k \subset S^3$ be a link in an n–bridge position and let (B, ℓ) be a $2n$-gonal pillowcase for k. Let $\omega : \pi_1(S^3 - k) \to S_d$ be a transitive representation and let $\varphi : M \to (S^3, k)$ and $\psi : B_\omega \to (B, B \cap k)$ the d–fold branched coverings associated tor ω.

If there exists an embedding $\varepsilon : B_\omega \hookrightarrow M$ such that the ramification cycles on $\varepsilon(\partial B_\omega)$ bound disjoint 2-cells in $\overline{M - \varepsilon(B_\omega)}$, then any homeomorphism $\varepsilon(B_\omega) \cong \varphi^{-1}(B)$ can be extended to a homeomorphism of pairs $(M, \tilde{\ell}) \cong (M, \varphi^{-1}(k))$ where $\tilde{\ell}$ is a cleansing of $\varepsilon(\psi^{-1}(\ell))$.

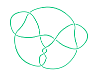
Note that the pair $(\partial B_\omega, \mathrm{ramification\ cycles})$ induces a Heegaard diagram for M. [1]

[1] This helps to identify what manifold is M

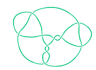
Undecided.



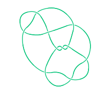
$9_{35} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



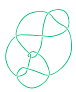
$9_{48} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3})$



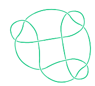
$10_{67} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{2}{3})$



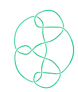
$10_{68} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{3})$



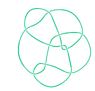
$10_{69} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{2}{3})$



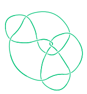
$10_{75} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{3})$



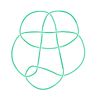
$10_{137} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{-1}{2})$



$10_{145} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-2}{3})$

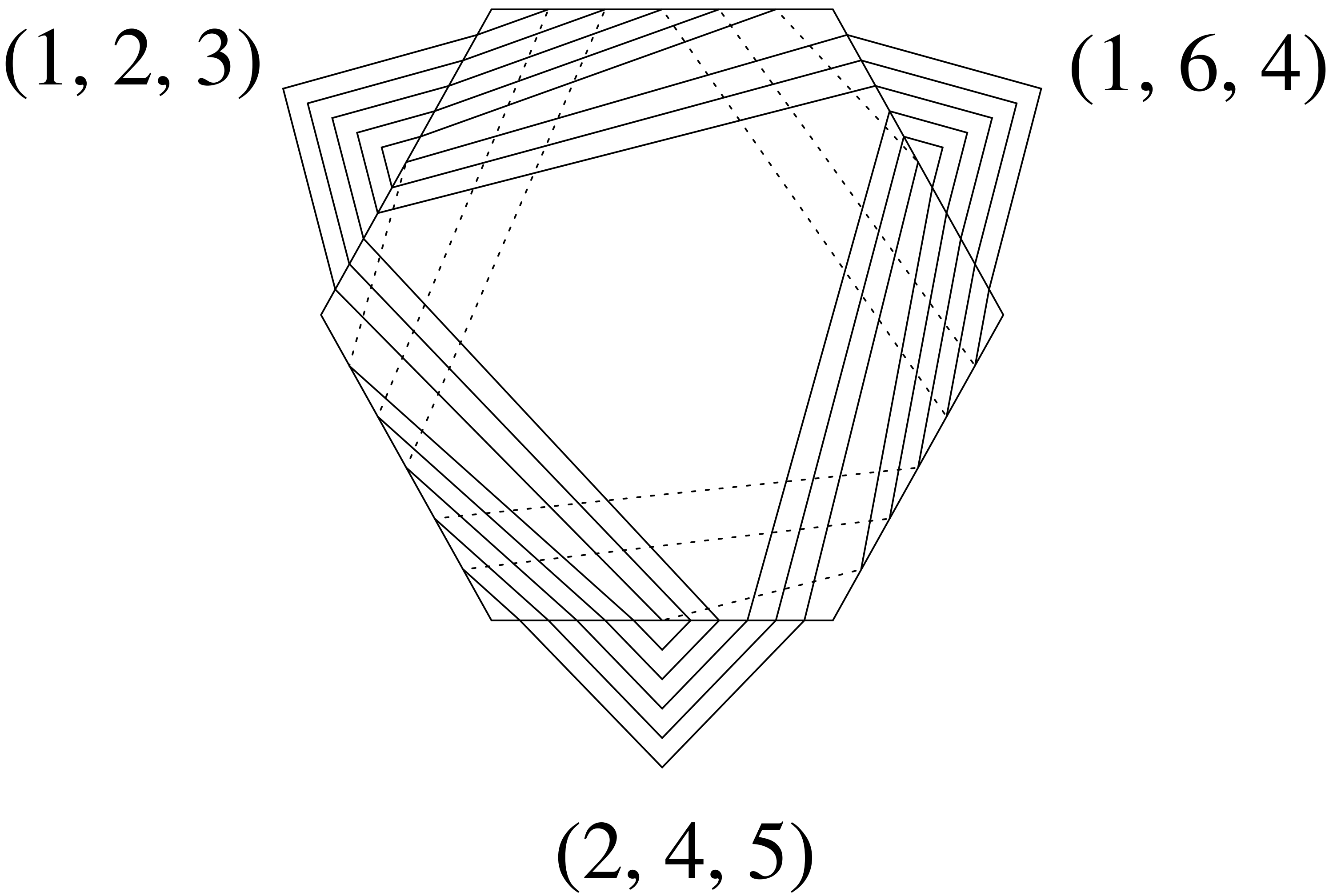


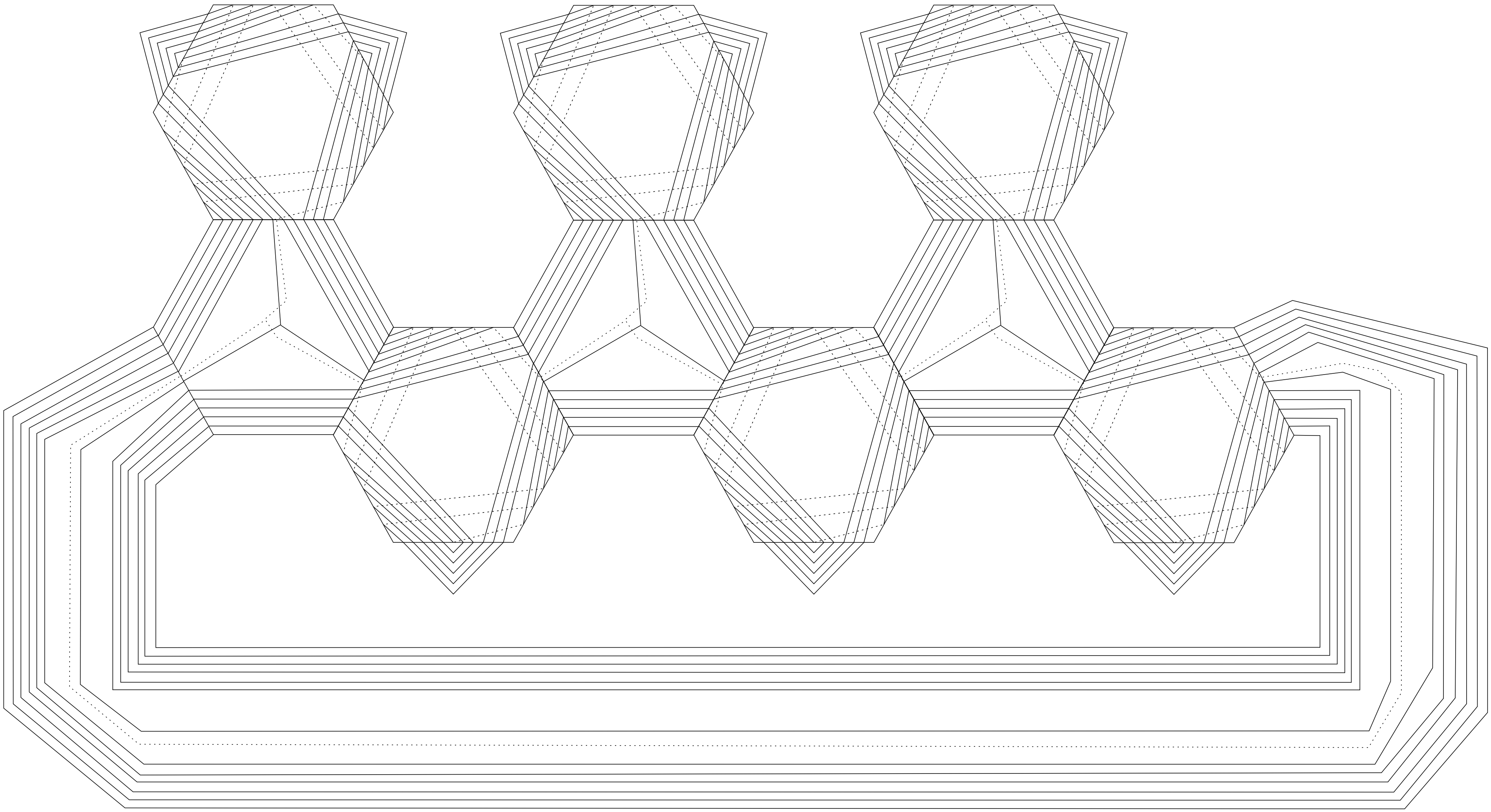
$10_{146} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{3})$

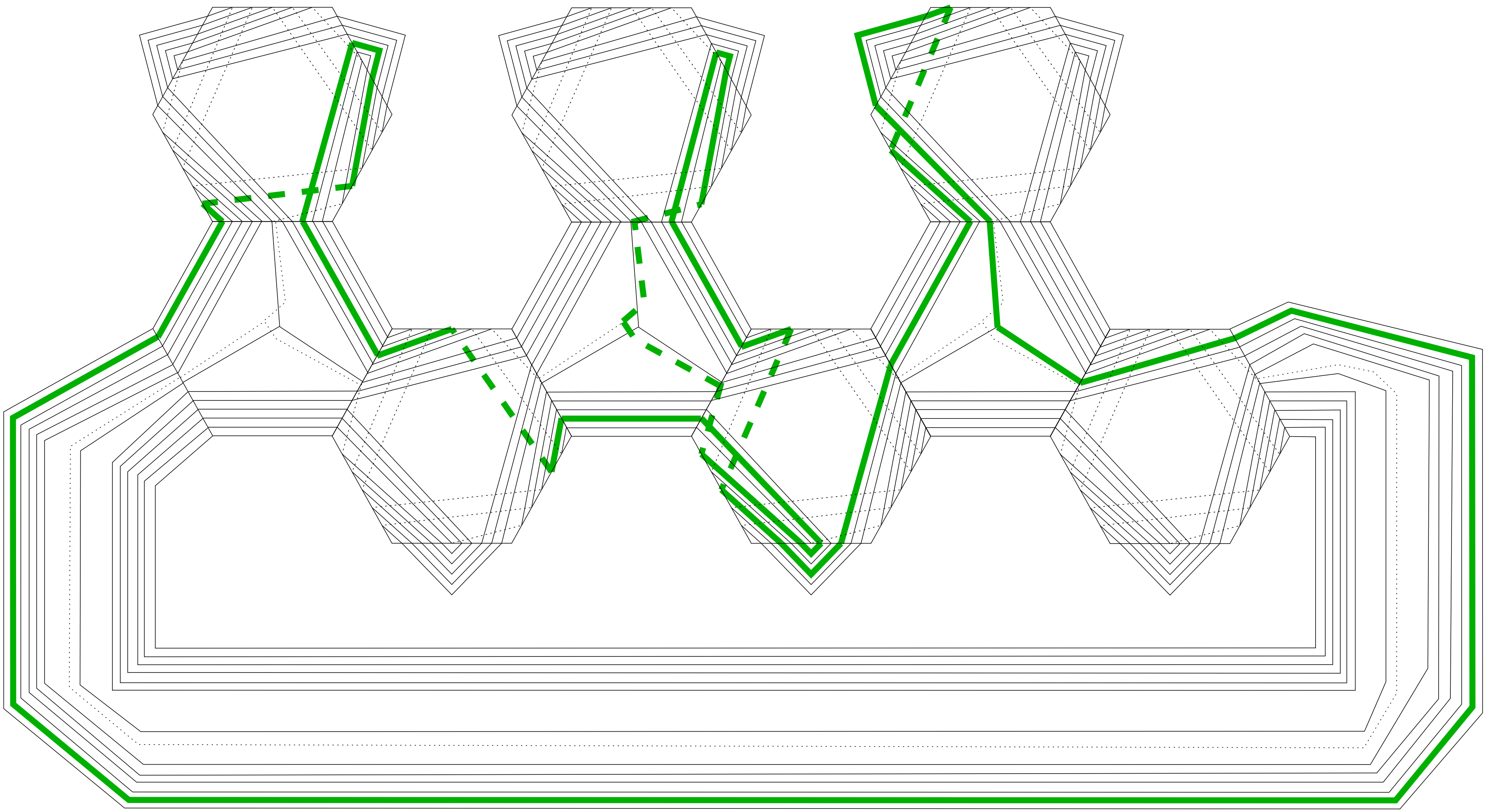


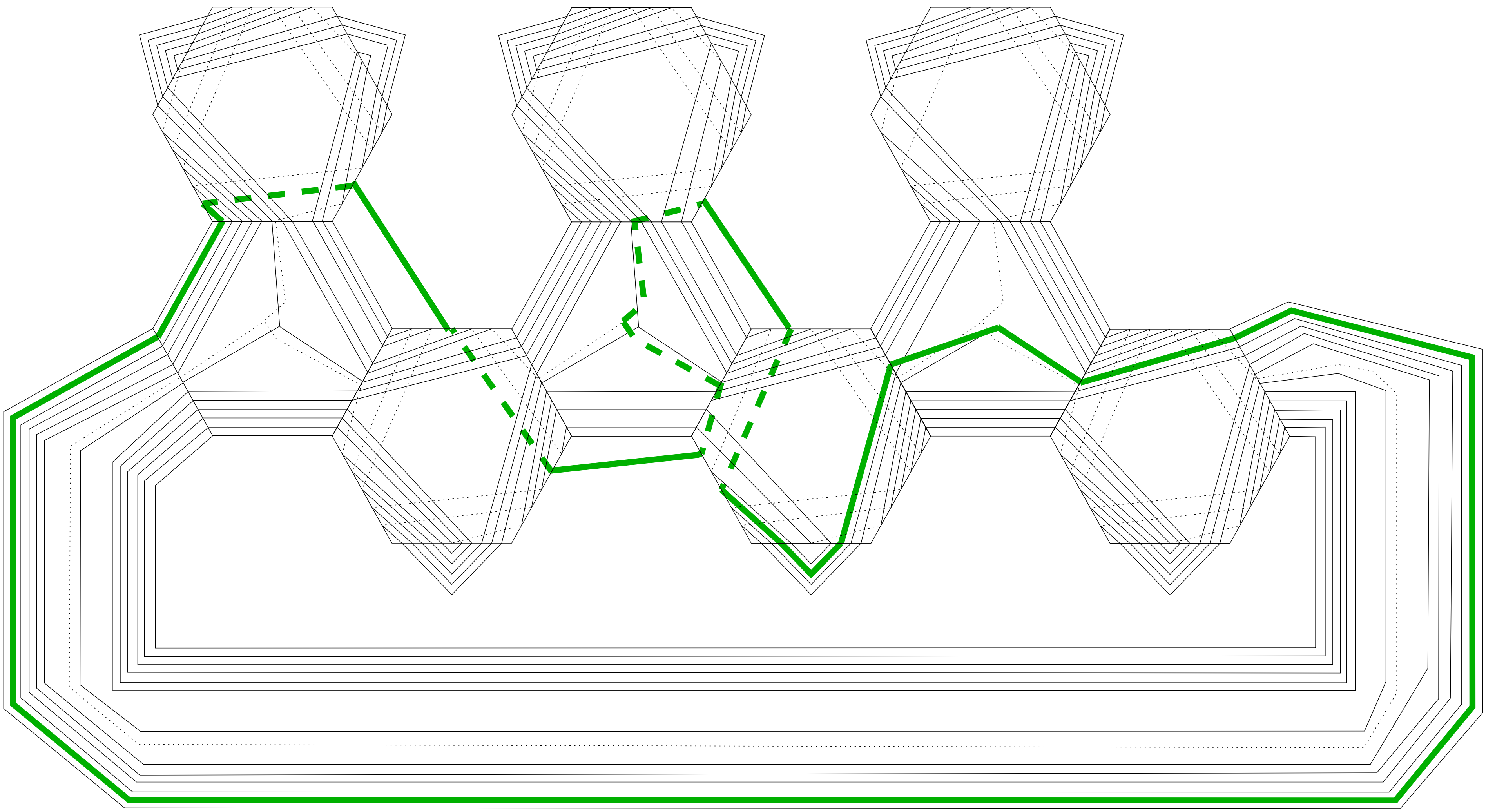
$10_{147} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{3})$

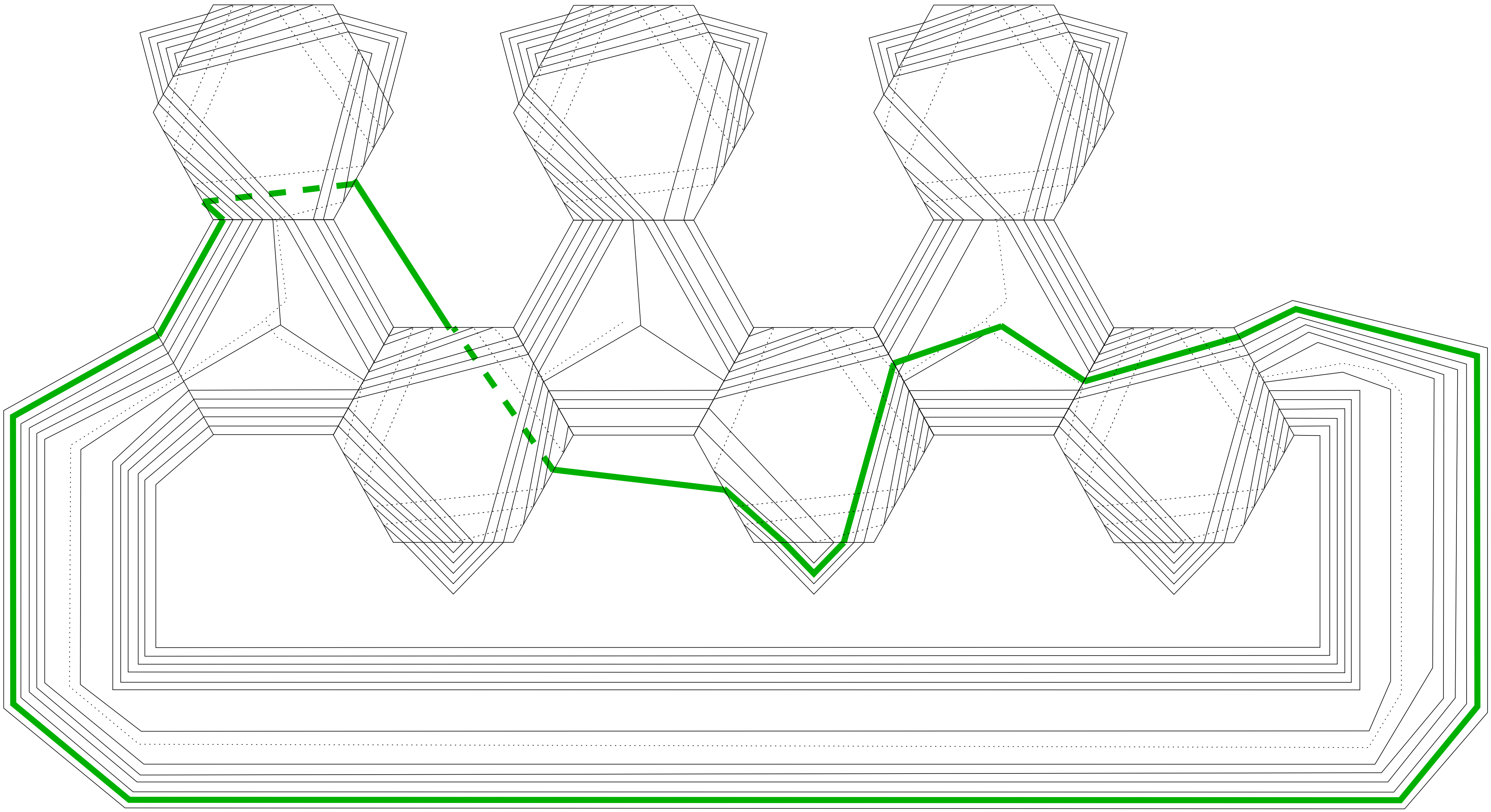
$$9_{35} = m(1/3, 1/3, 1/3)$$

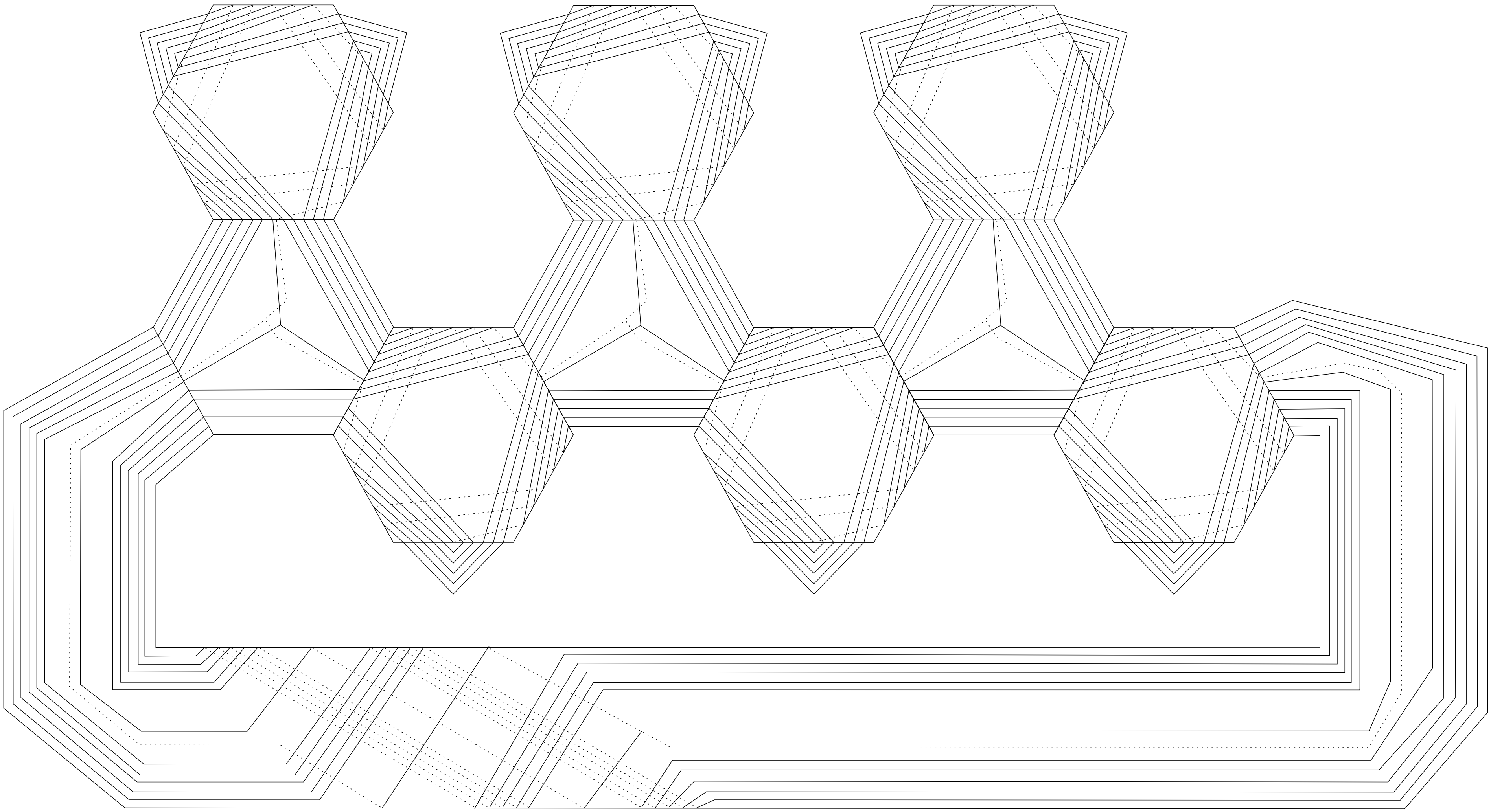


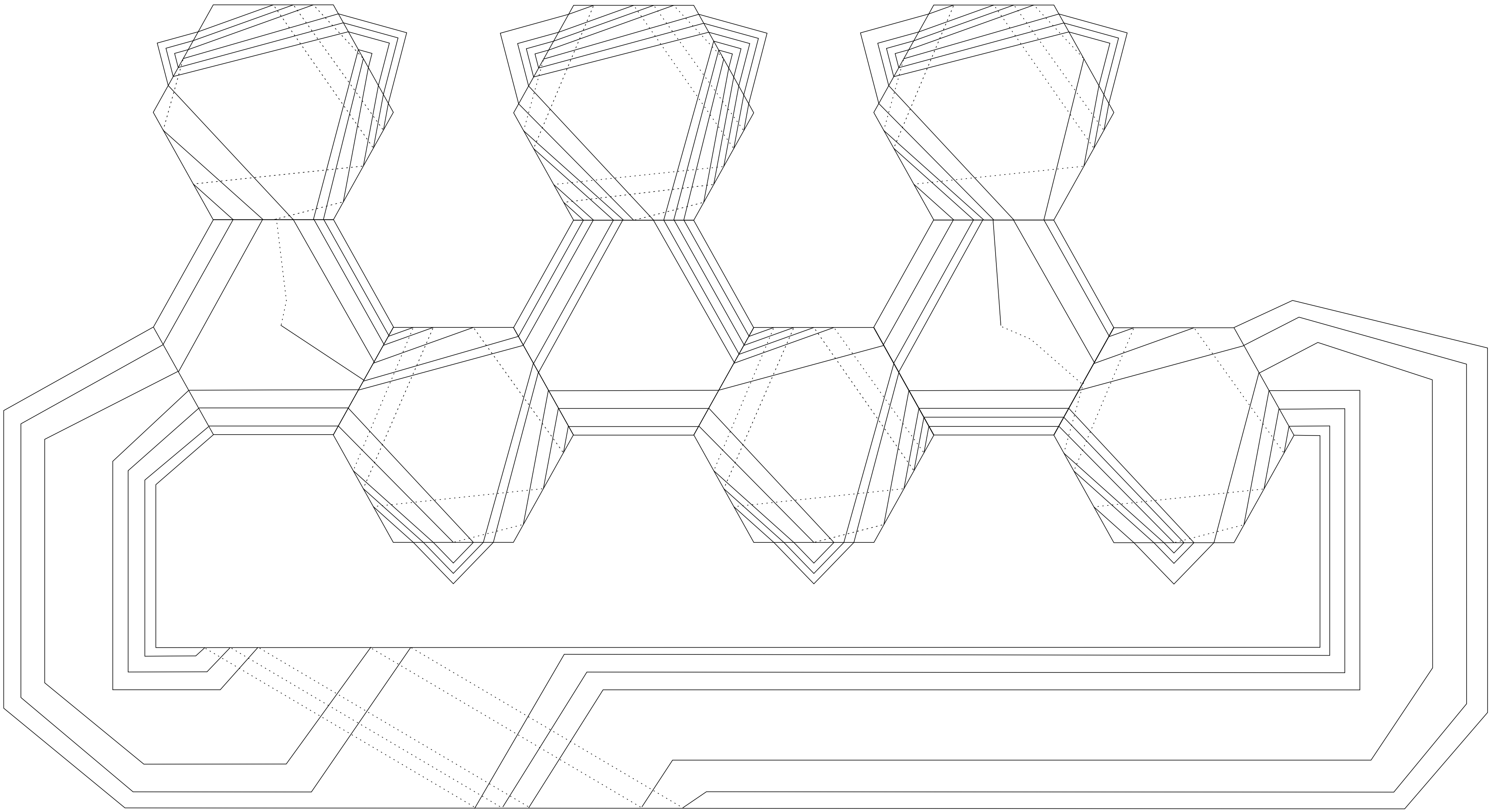


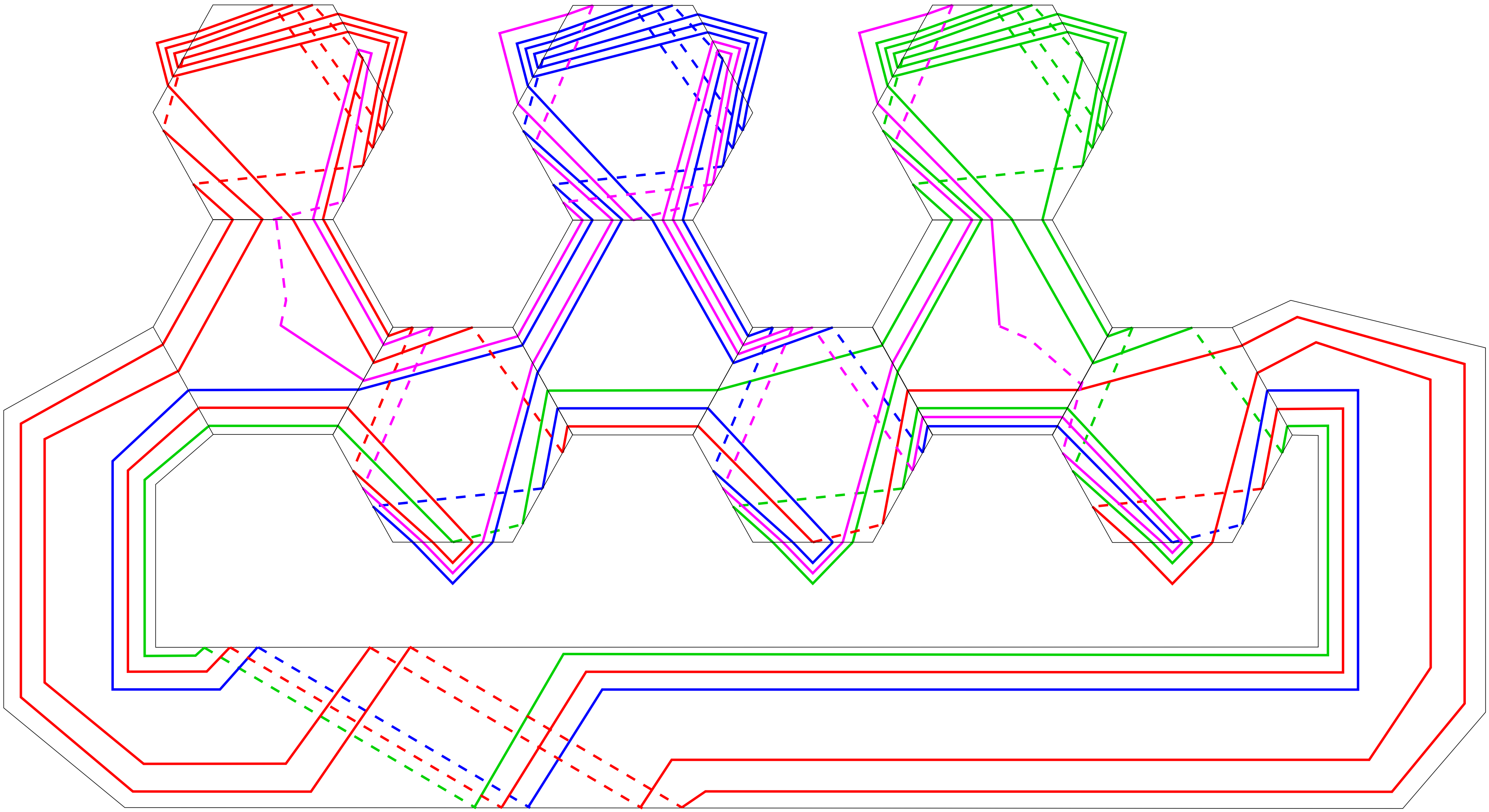


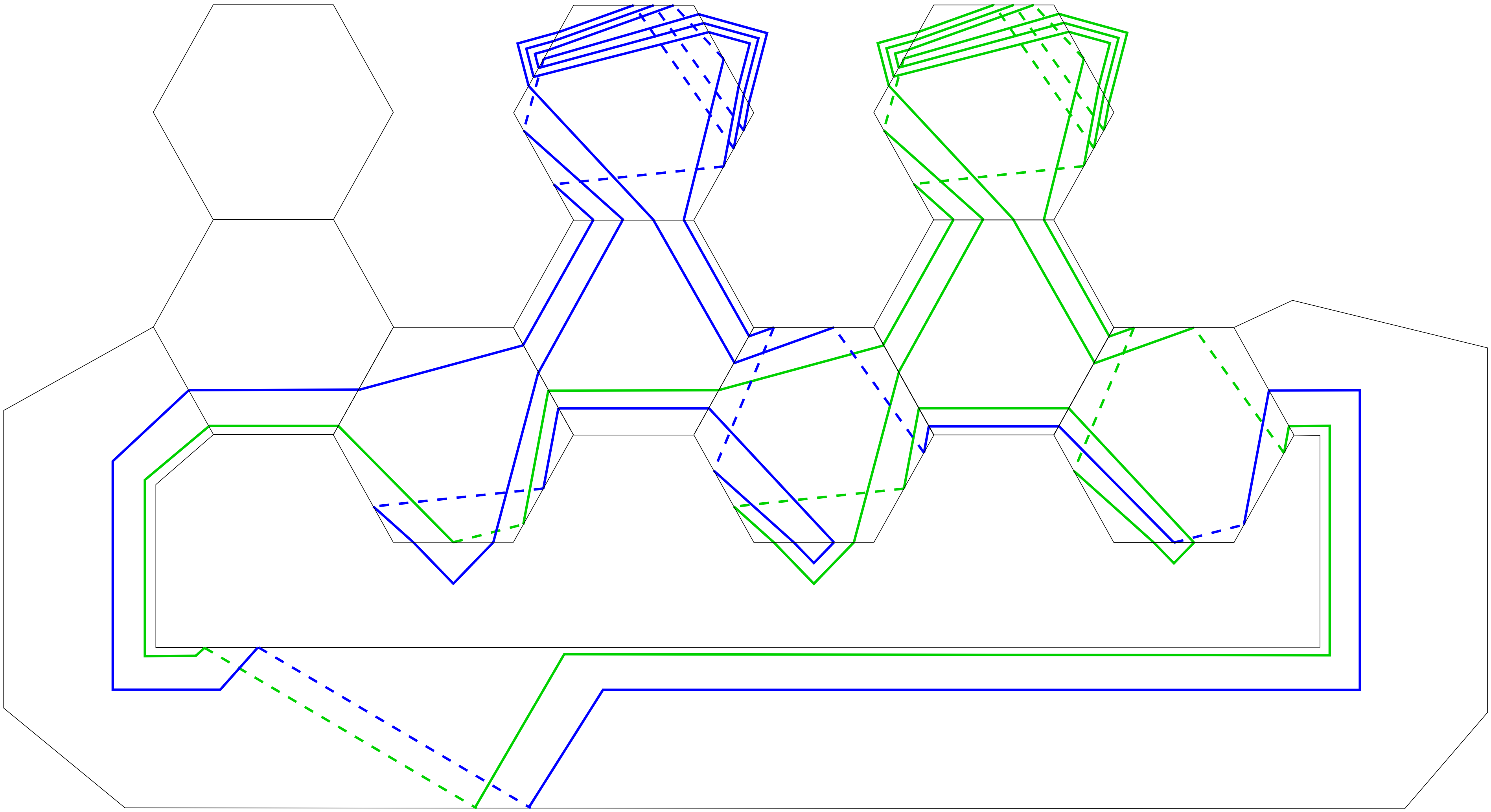


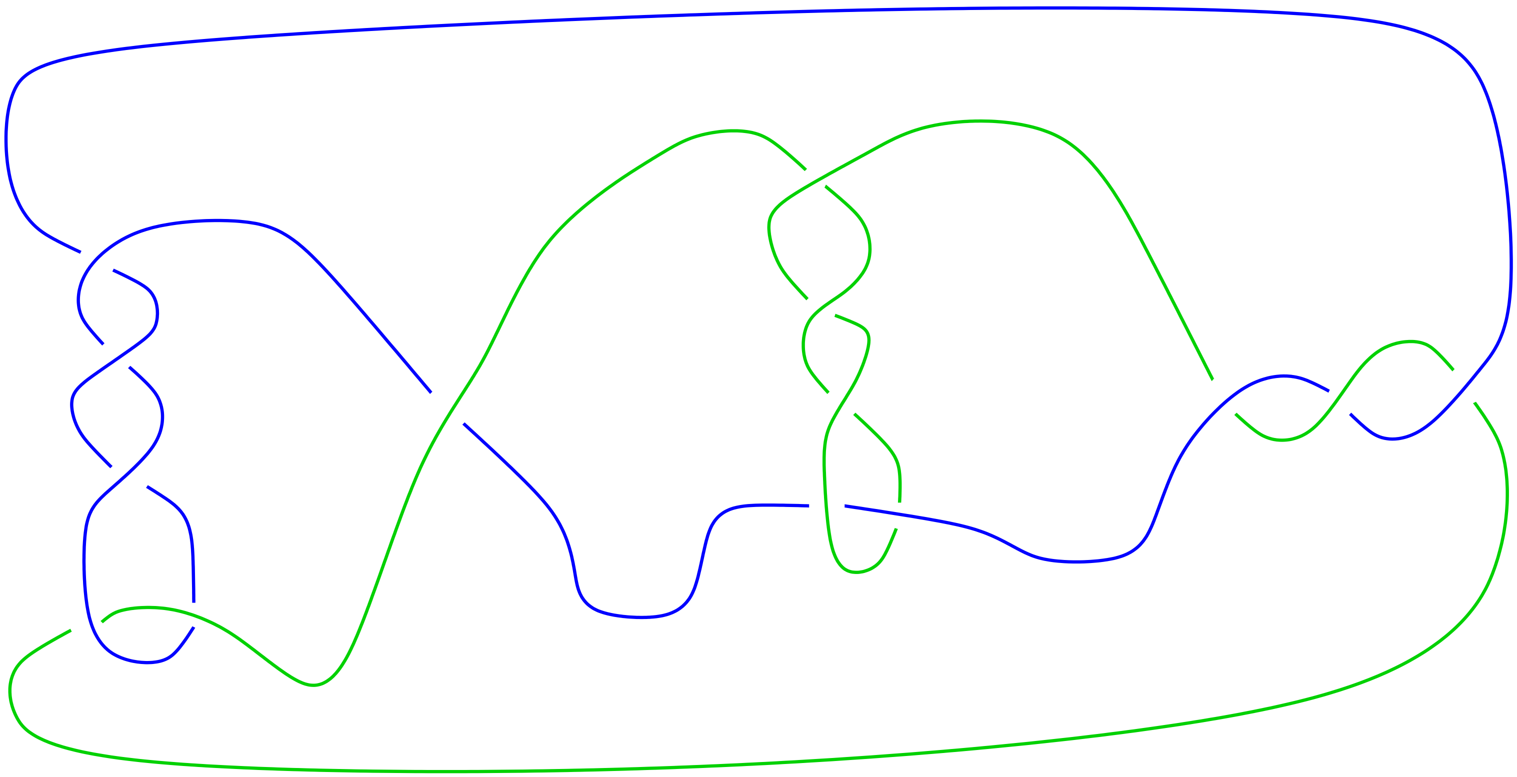








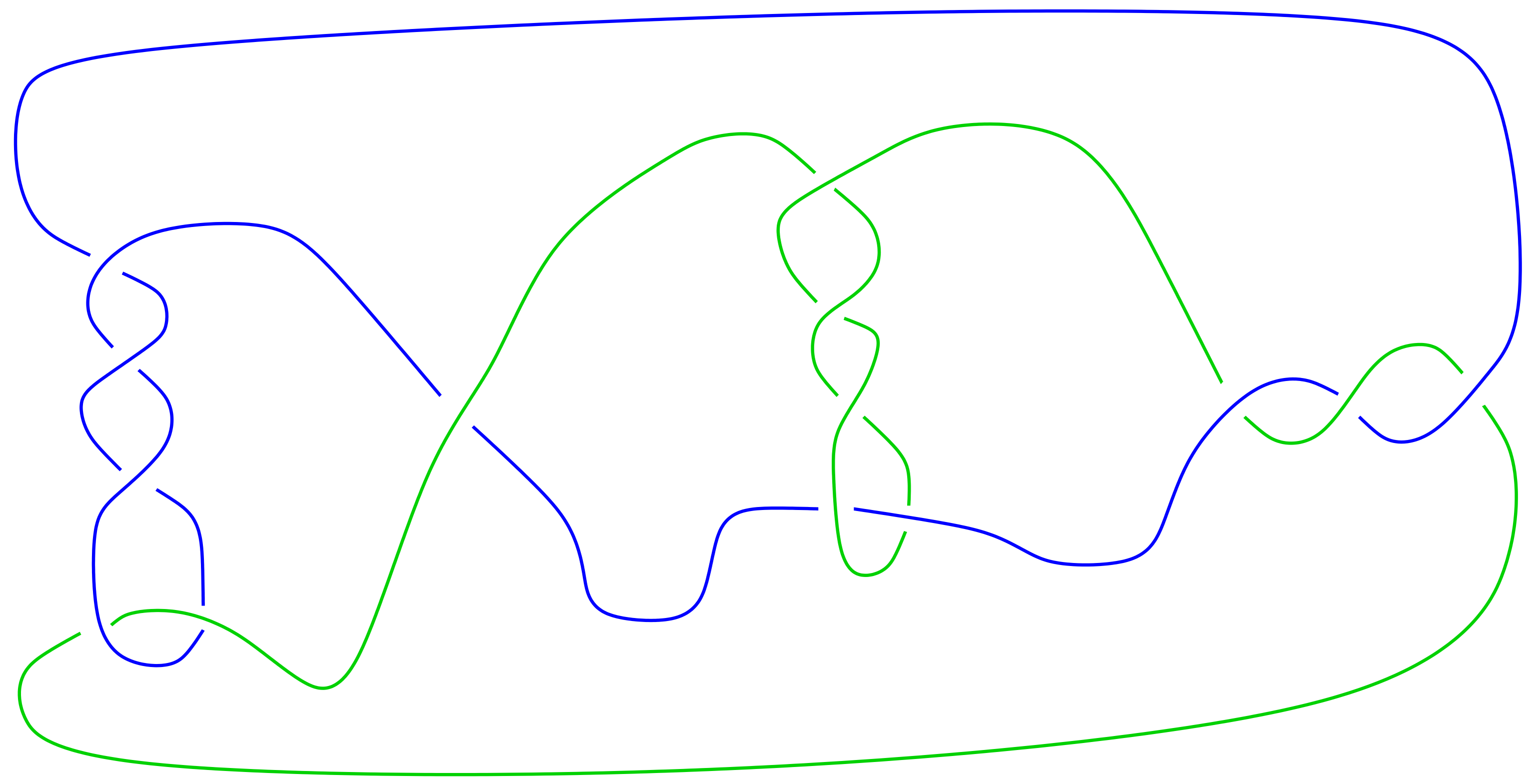




$$m(2/7, 1, 2/7, 3) \sim m(9/7, 23/7)$$

$$m(\frac{\beta_1}{\alpha_1}, \frac{\beta_2}{\alpha_2}) \sim m(-\frac{\alpha_1\beta_2 + \alpha_2\beta_1}{\alpha_2 r_1 + \beta_2 s_1})$$

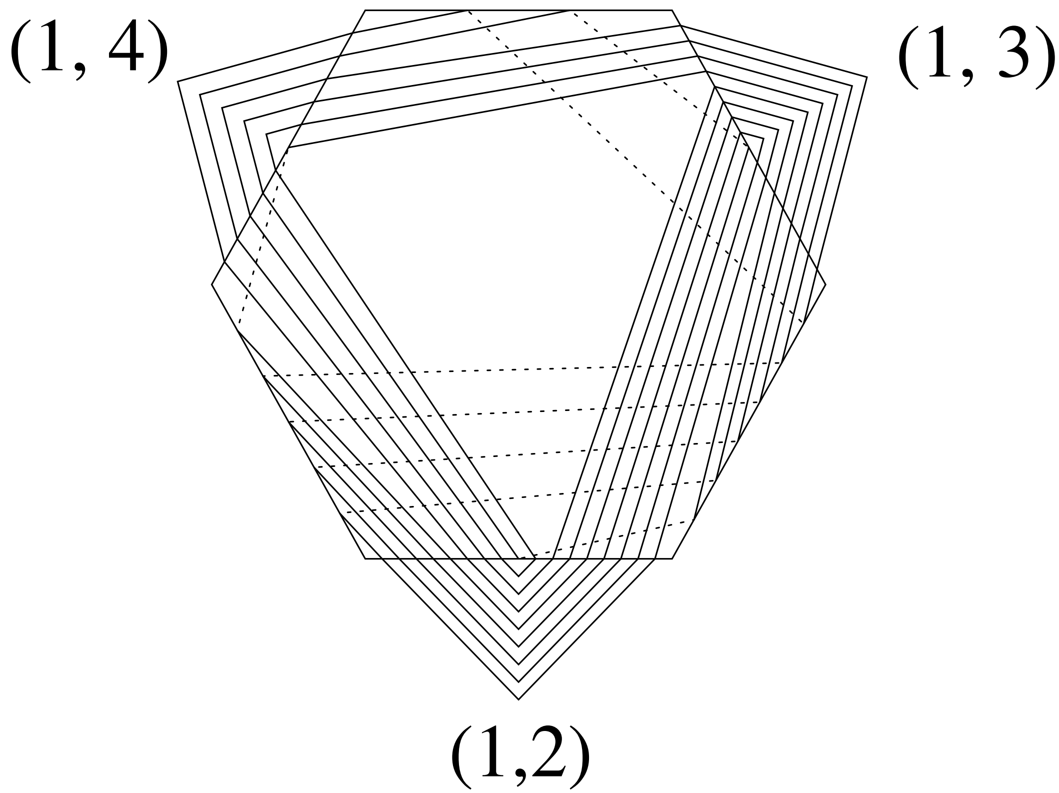
where $\alpha_1 r_1 - \beta_1 s_1 = 1$.

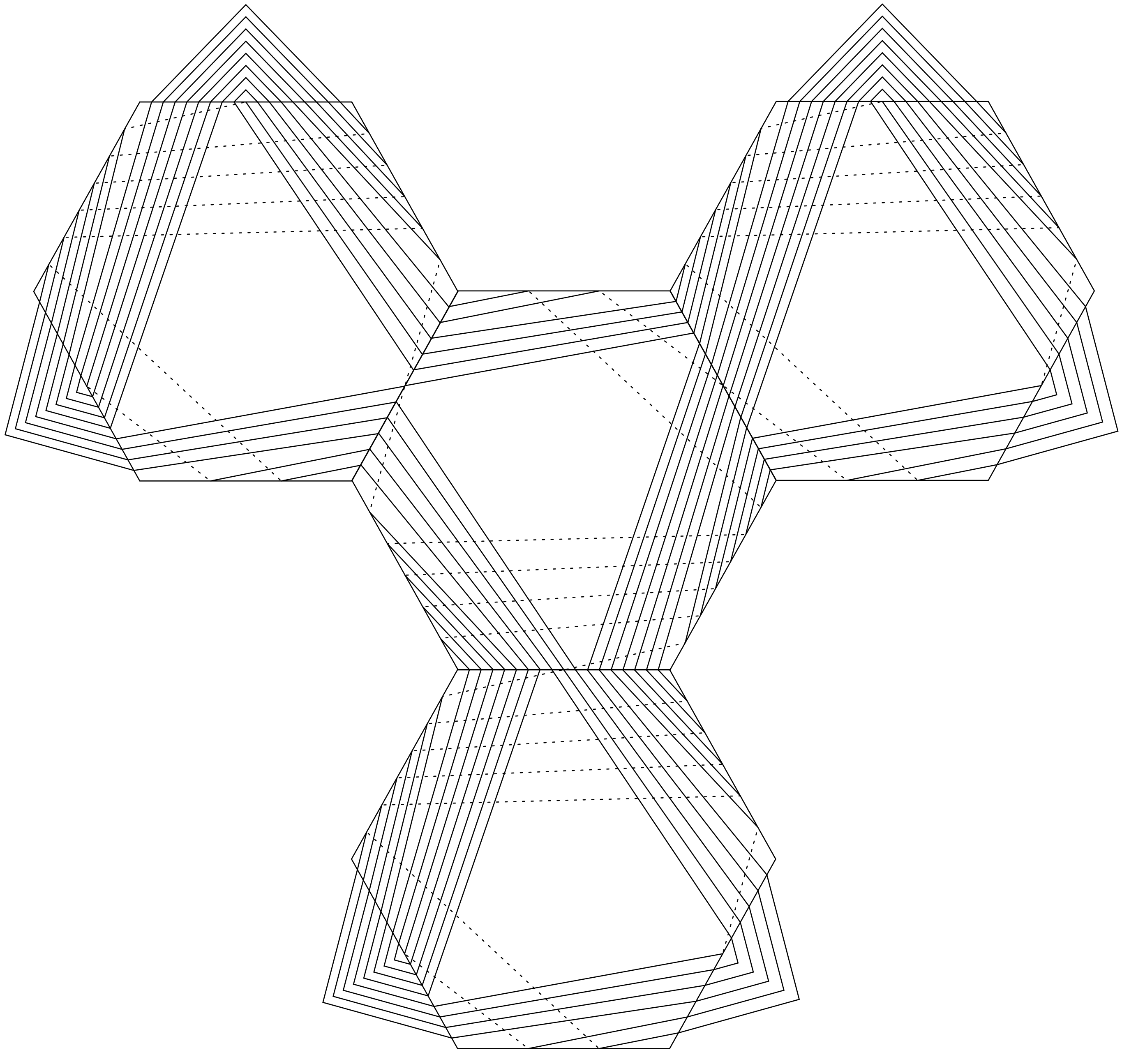


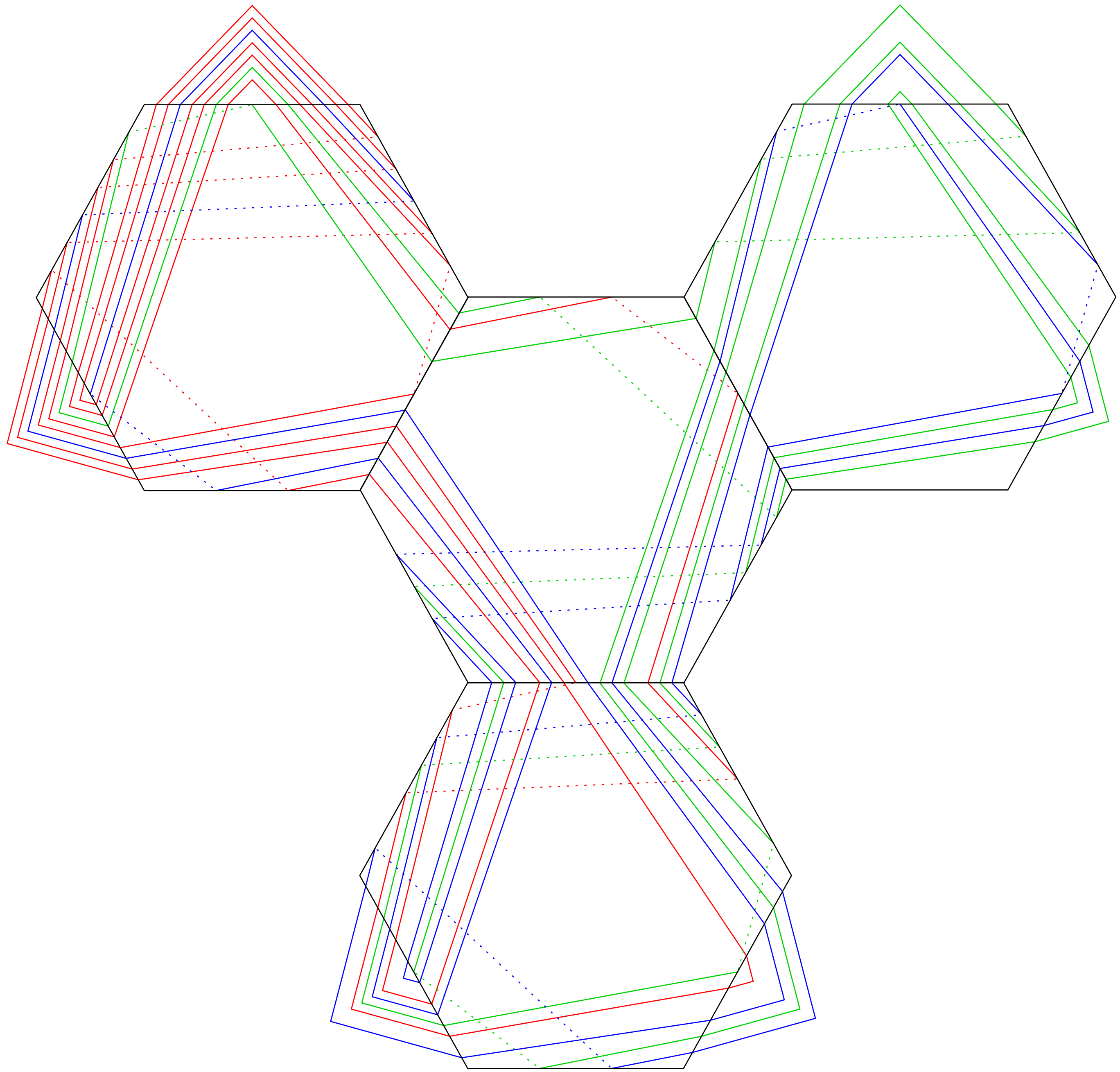
$$m(2/7, 1, 2/7, 3) \sim m(9/7, 23/7) \sim m(-224/97)$$

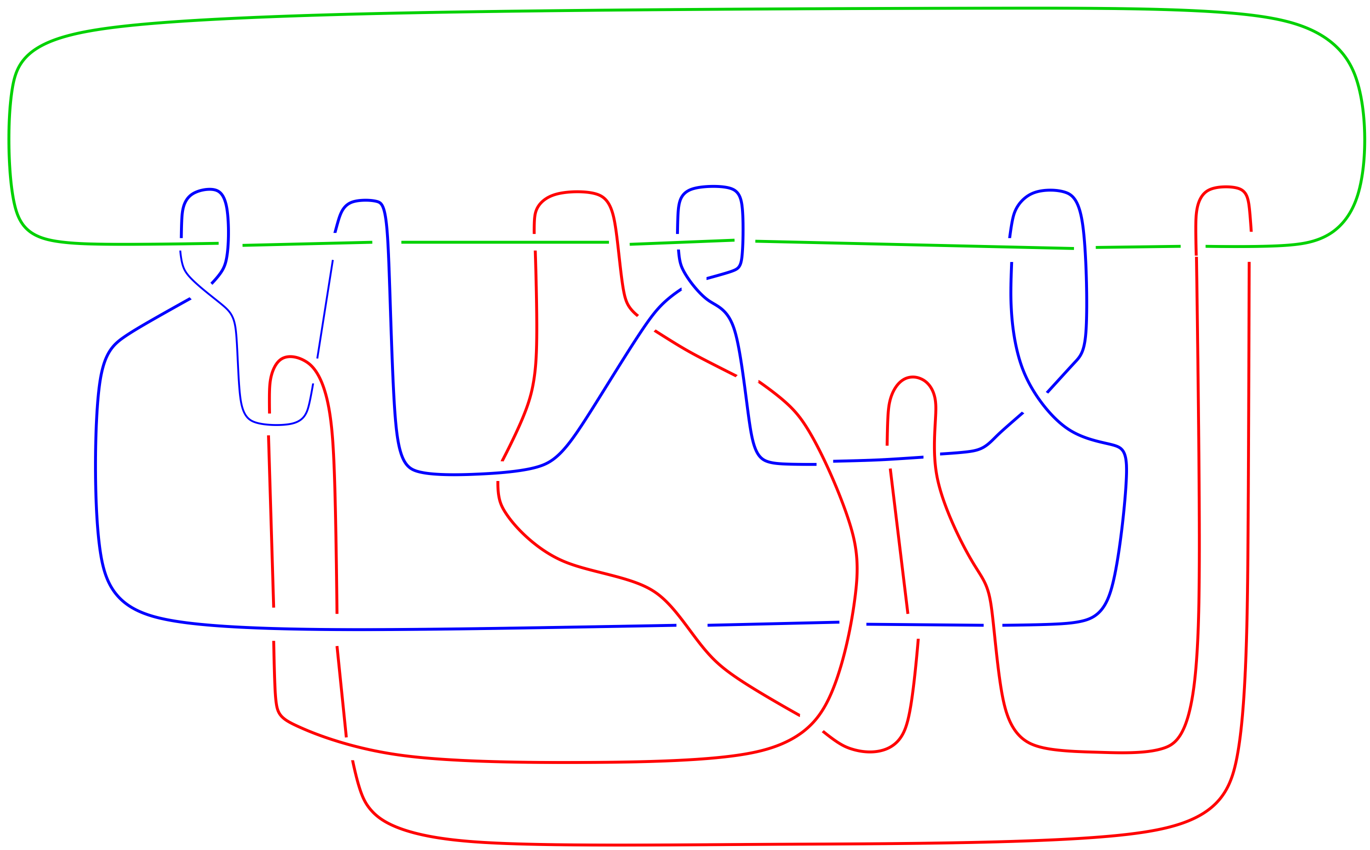
The knot 9_{35} is universal.

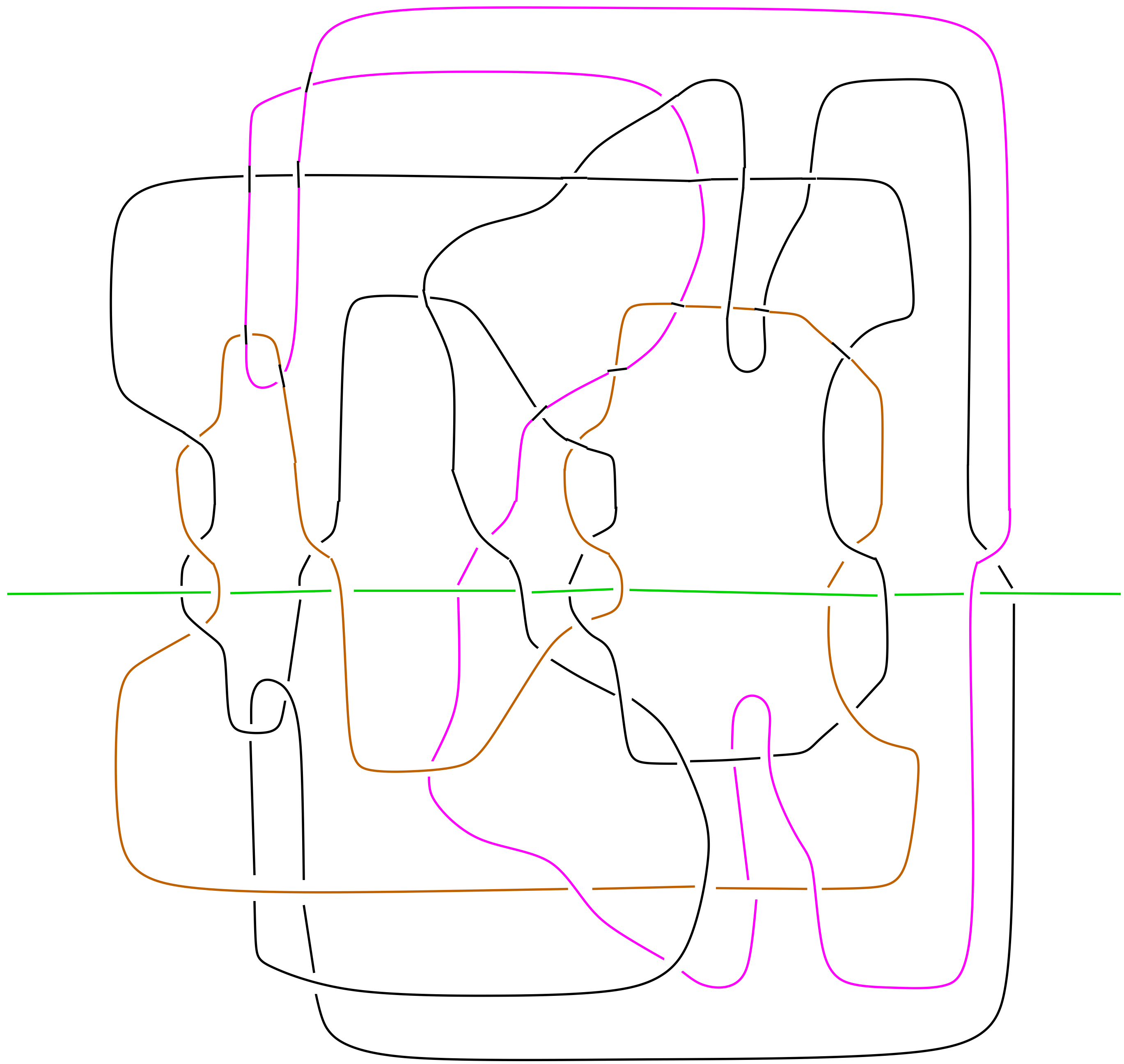
$$9_{48} = m(2/3, 2/3, -1/3)$$

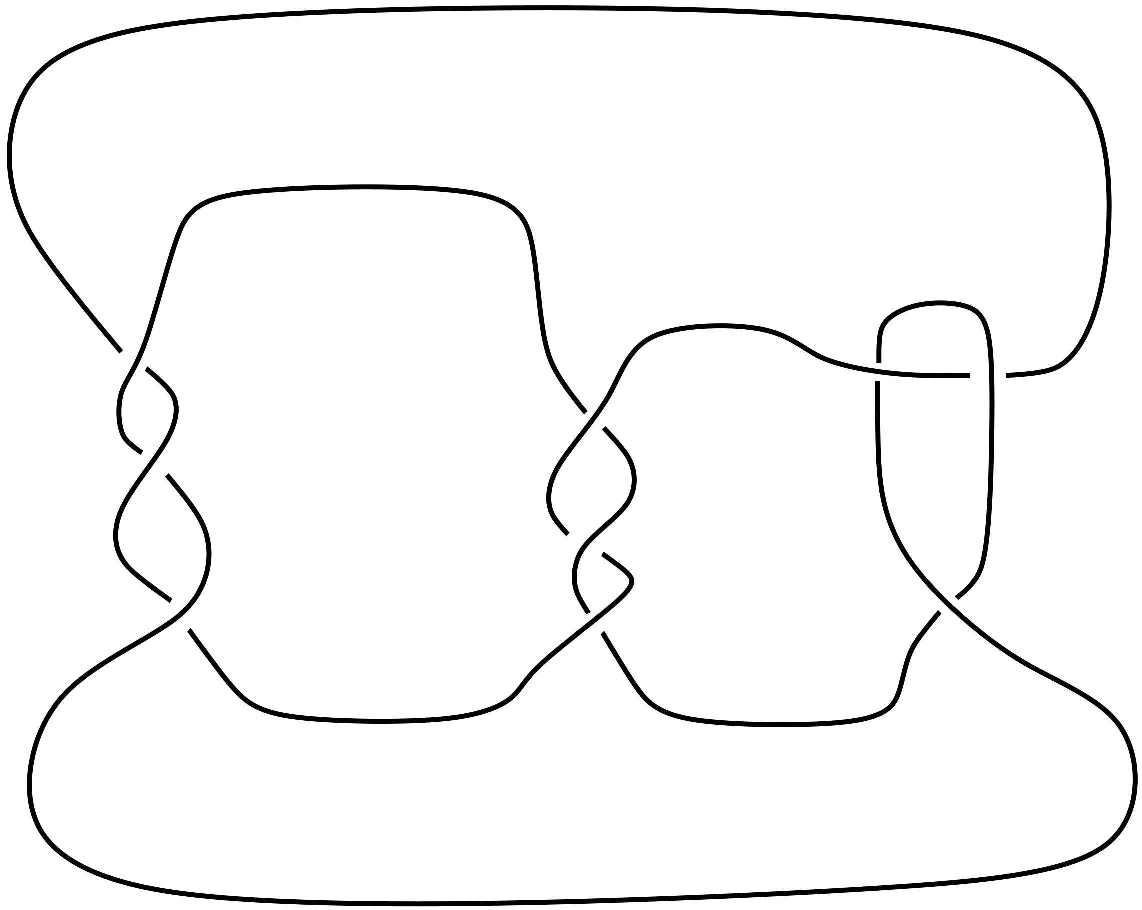








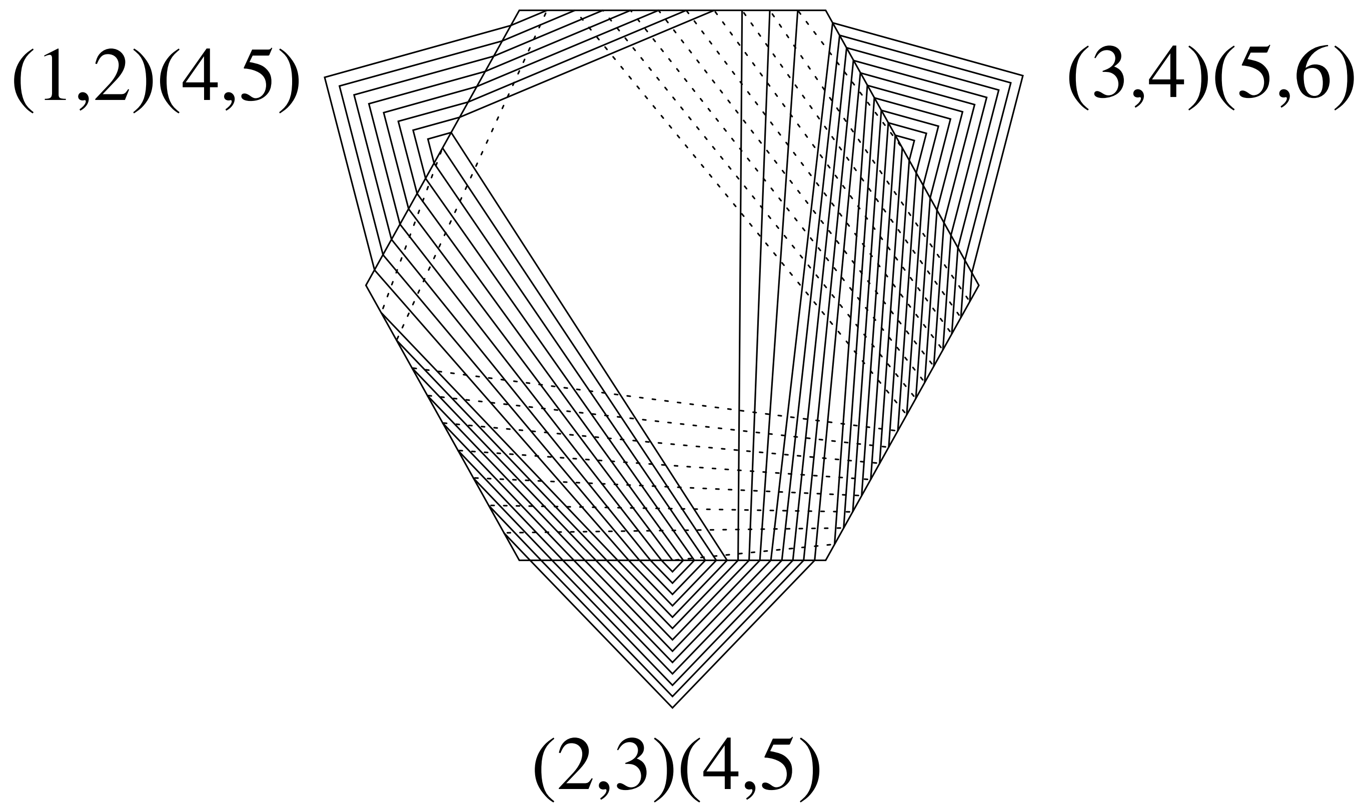


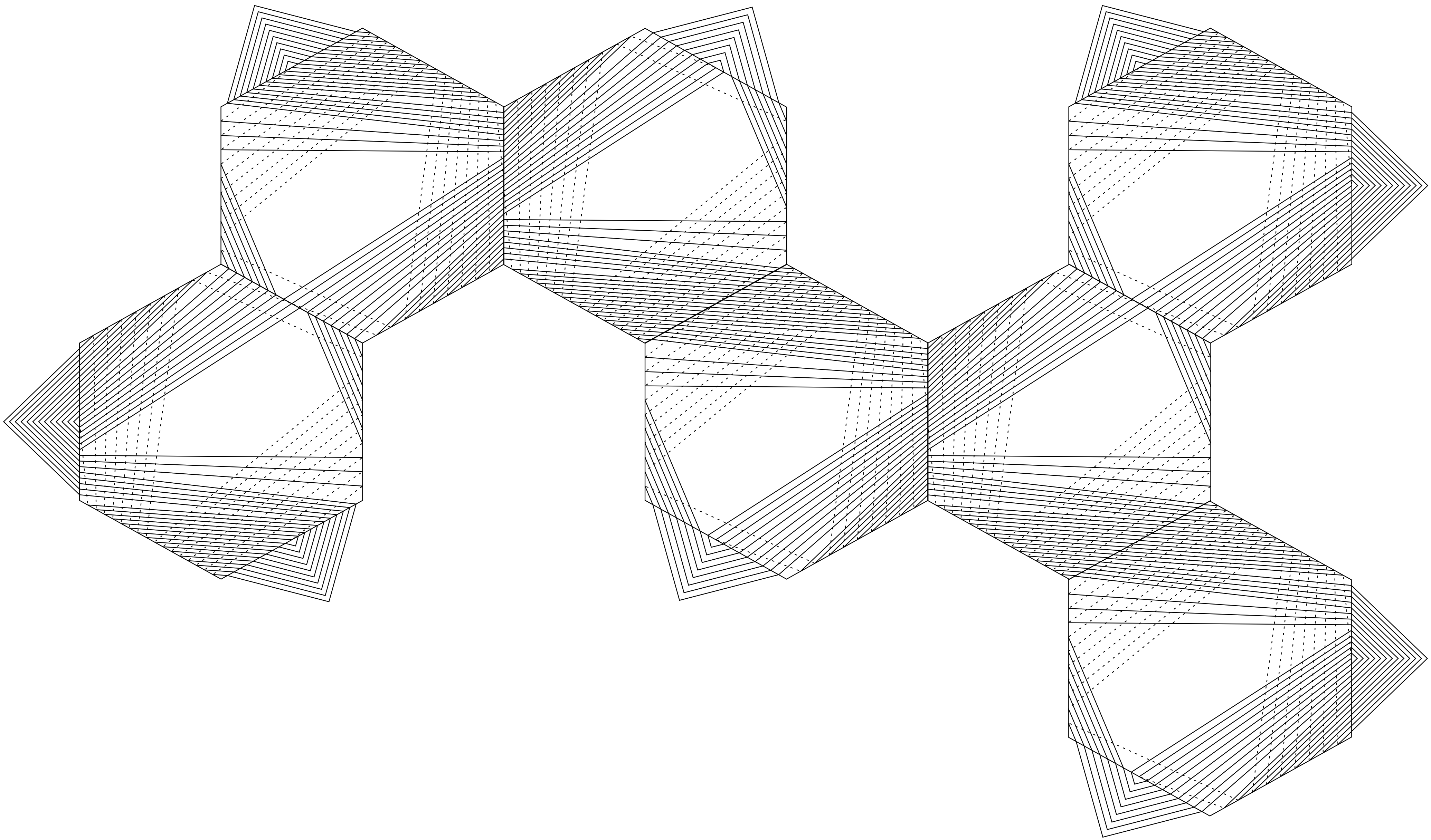


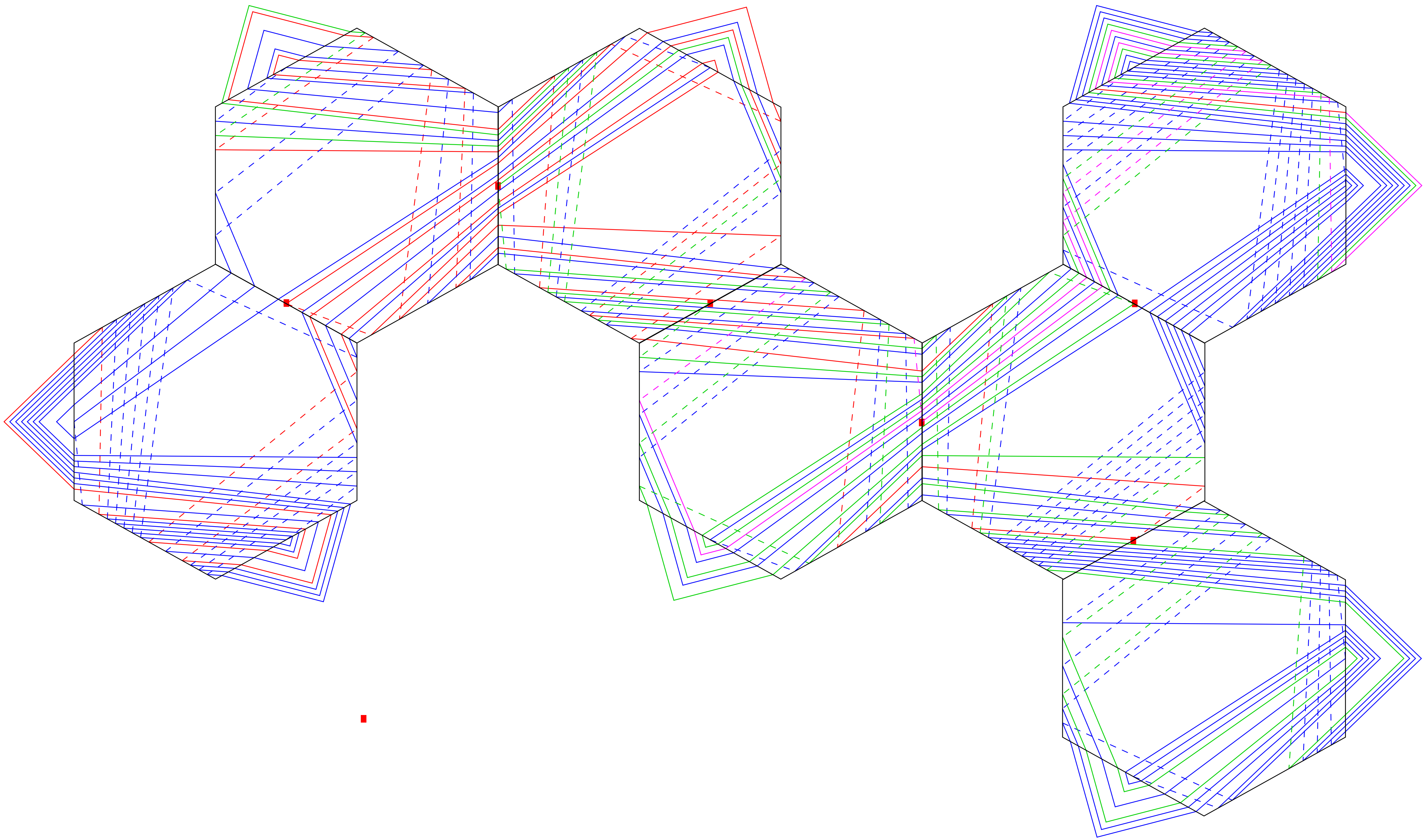
$$m(1/3, 1/3, 2/3) \sim m(4/3, 4/3, -4/3) \leftarrow m(1/3, 1/3, -1/3)$$

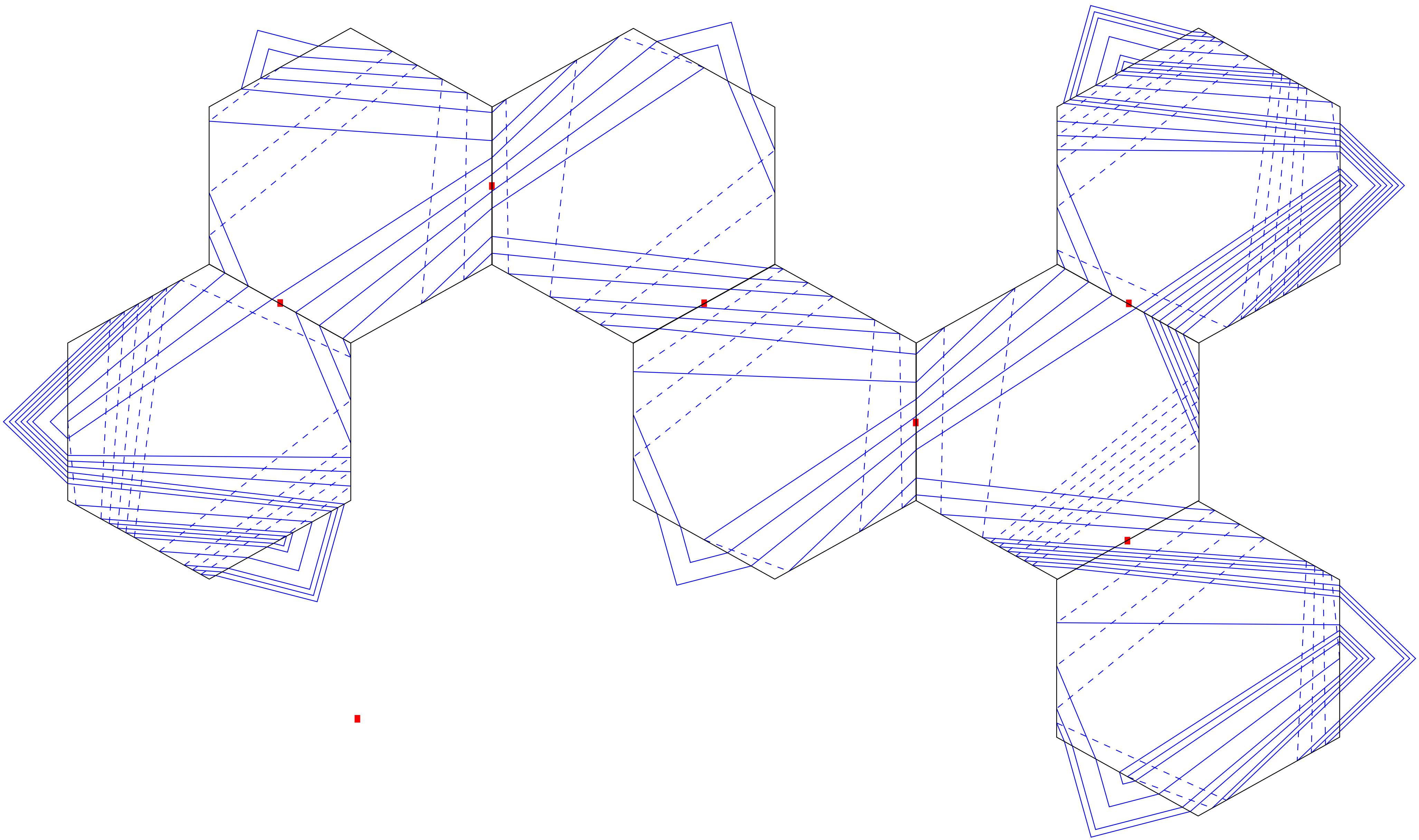
The knot 9_{48} *is universal*

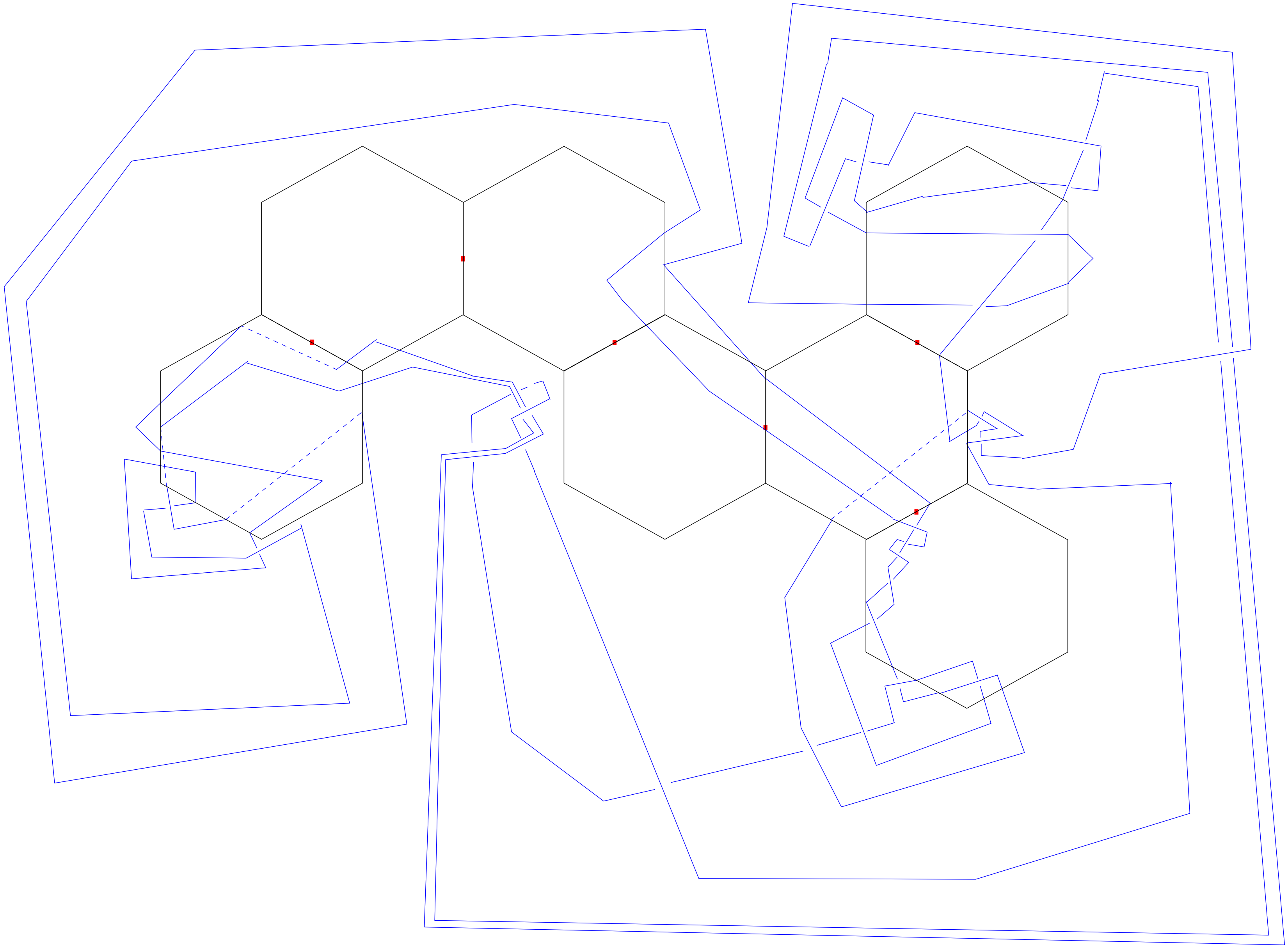
$$10_{67} = m(2/5, 1/3, 1/3)$$

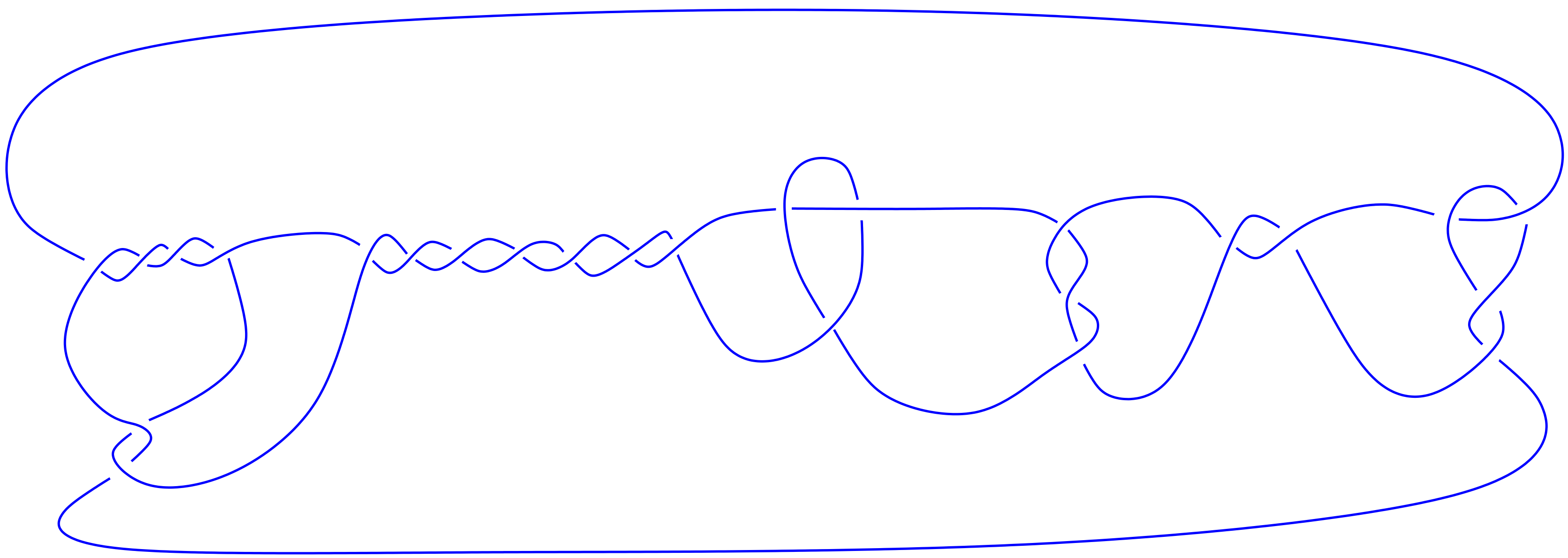








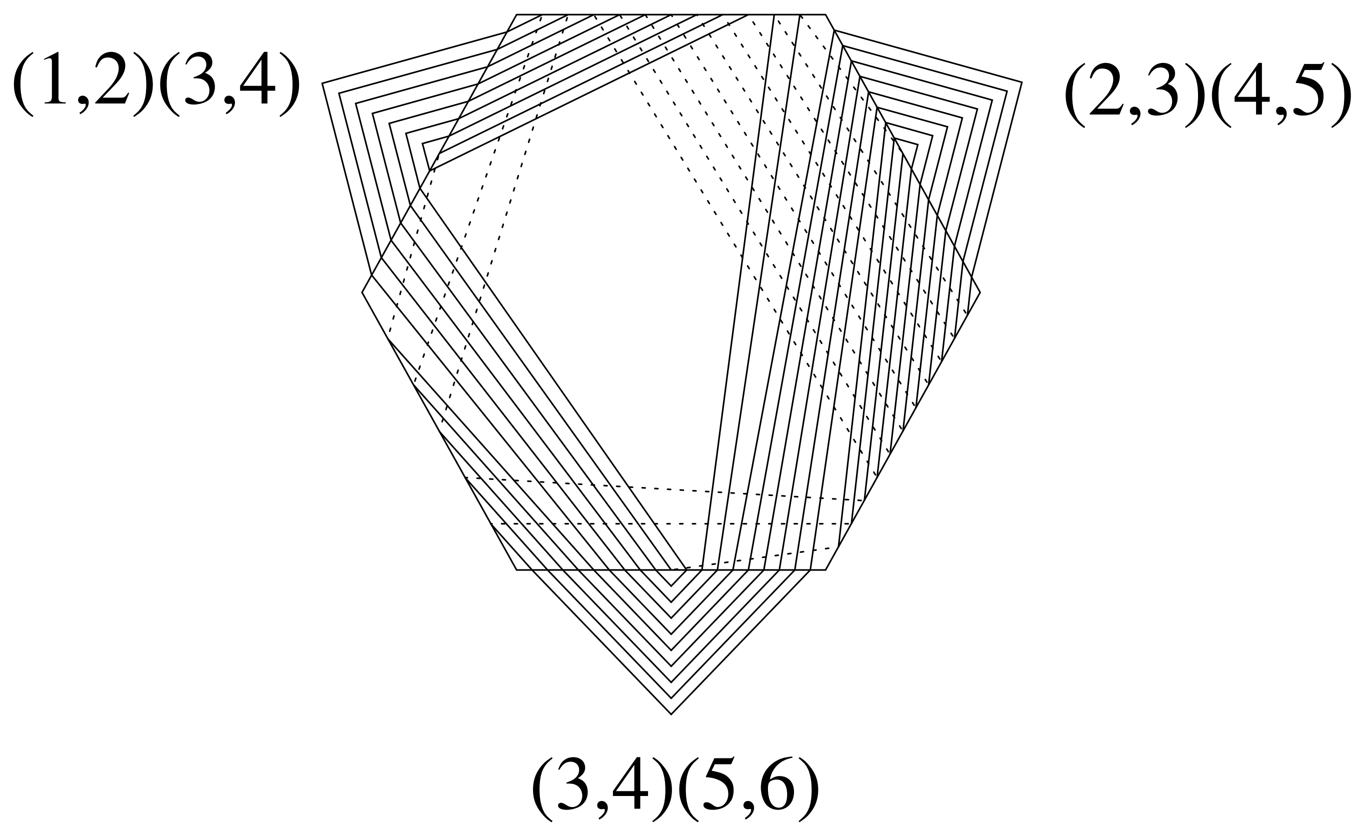


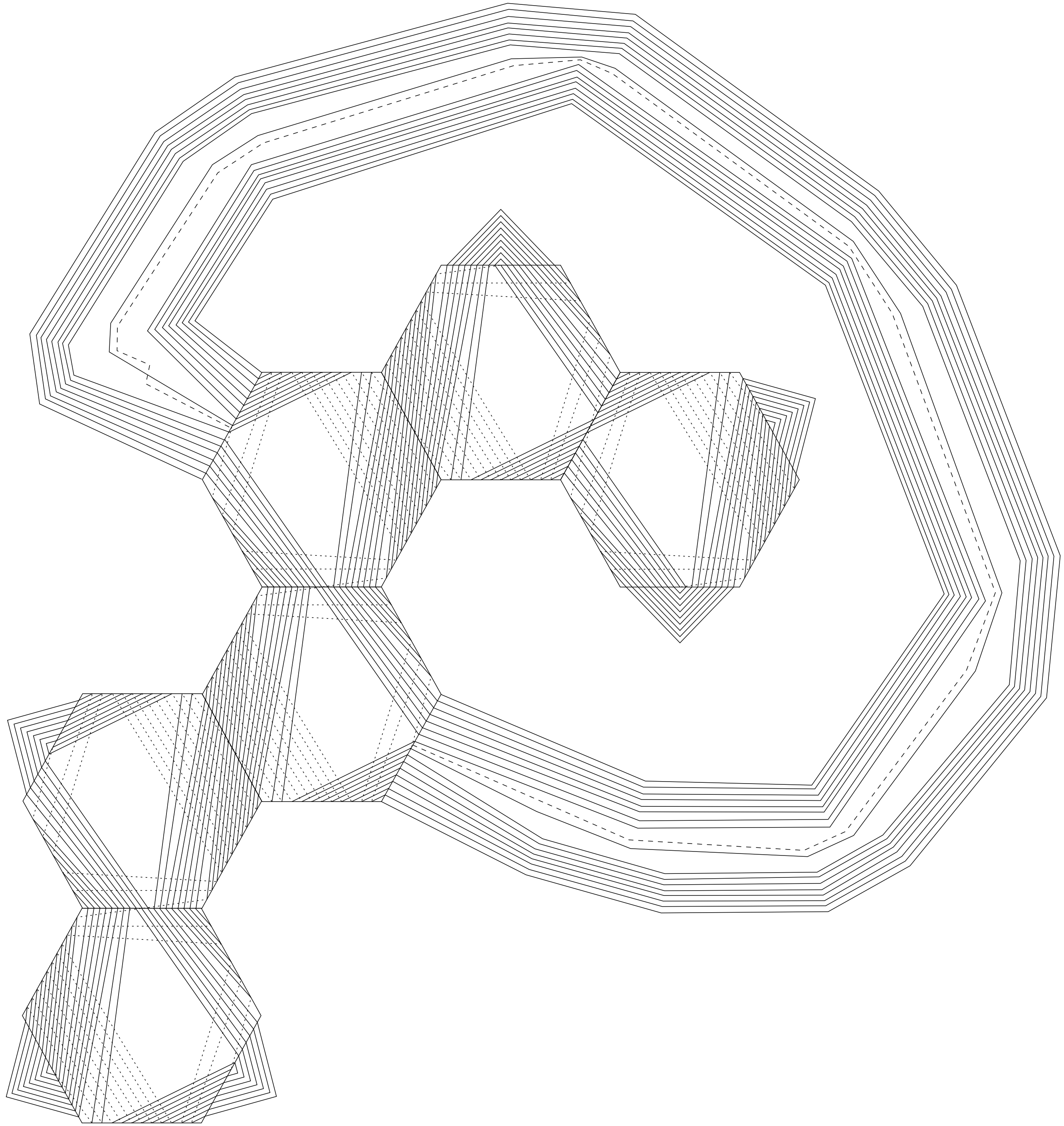


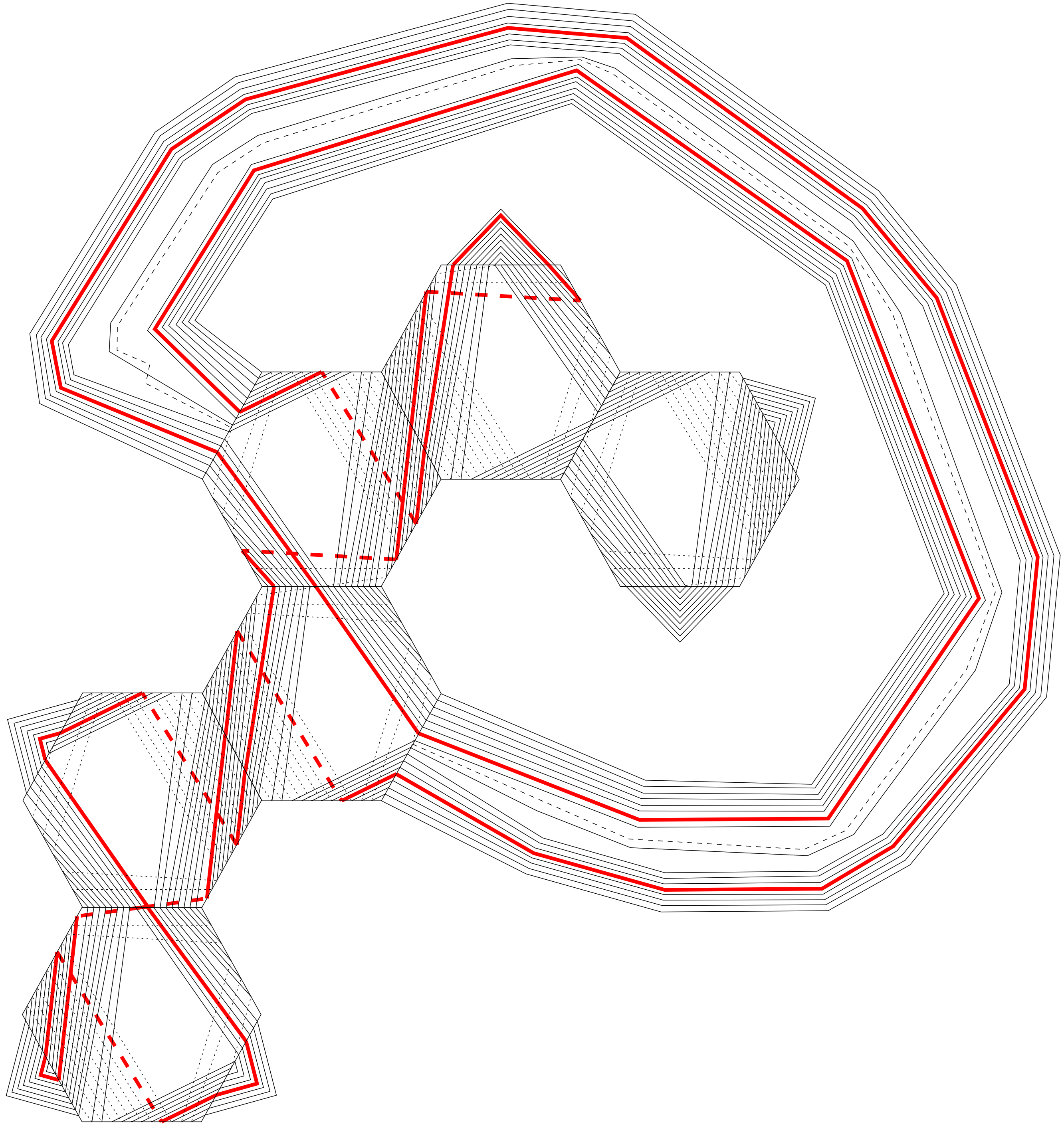
$$m(-\frac{4}{7}, 7, \frac{2}{3}, \frac{1}{3}, 2, \frac{2}{5}) \sim m(\frac{8\cdot 3}{7}, \frac{8}{3}, \frac{8\cdot 4}{3}, -\frac{8}{5}) \leftarrow m(1, -9, \frac{29}{3}) \sim m(\frac{5}{3})$$

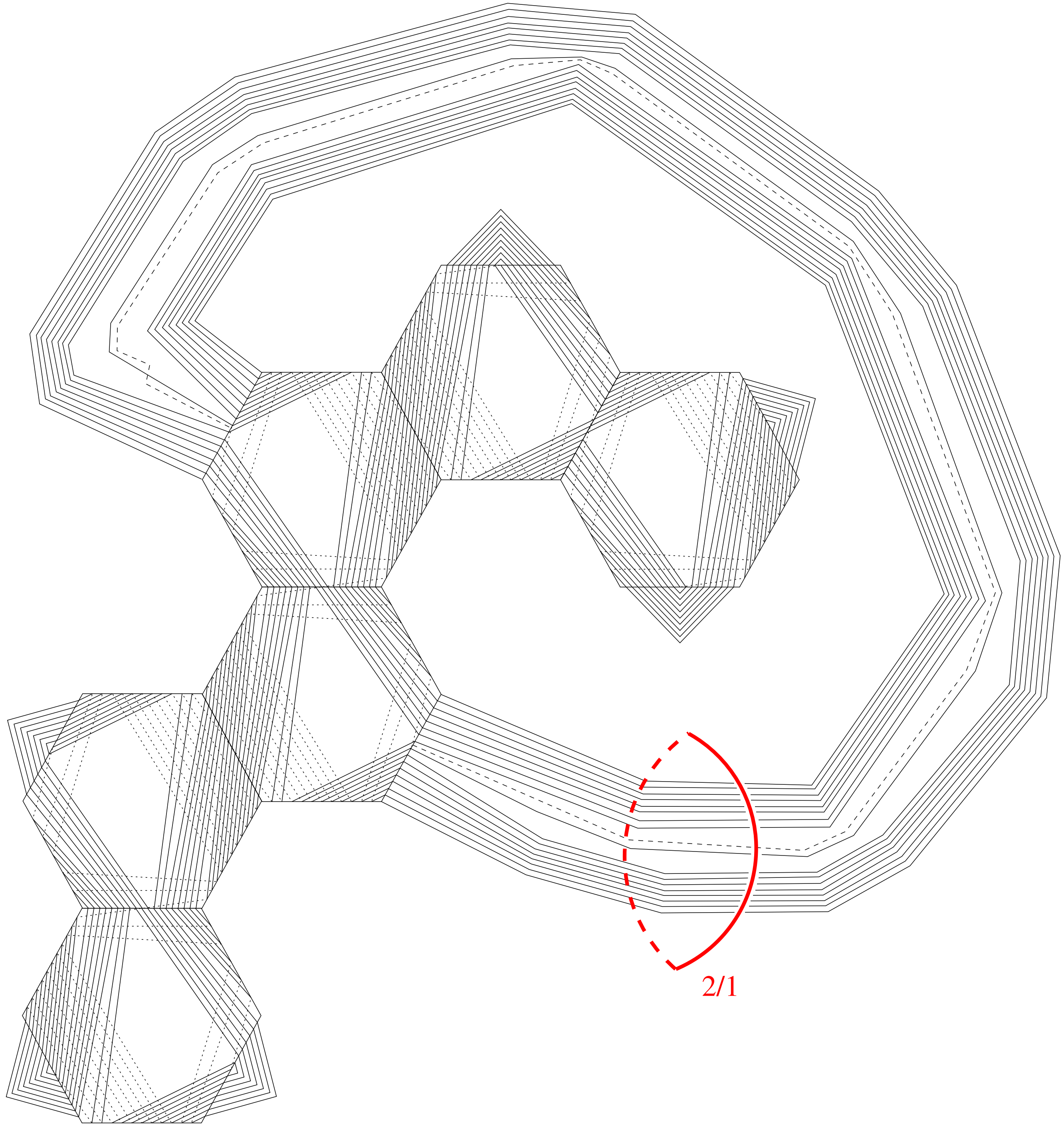
The knot 10_{67} *is universal*

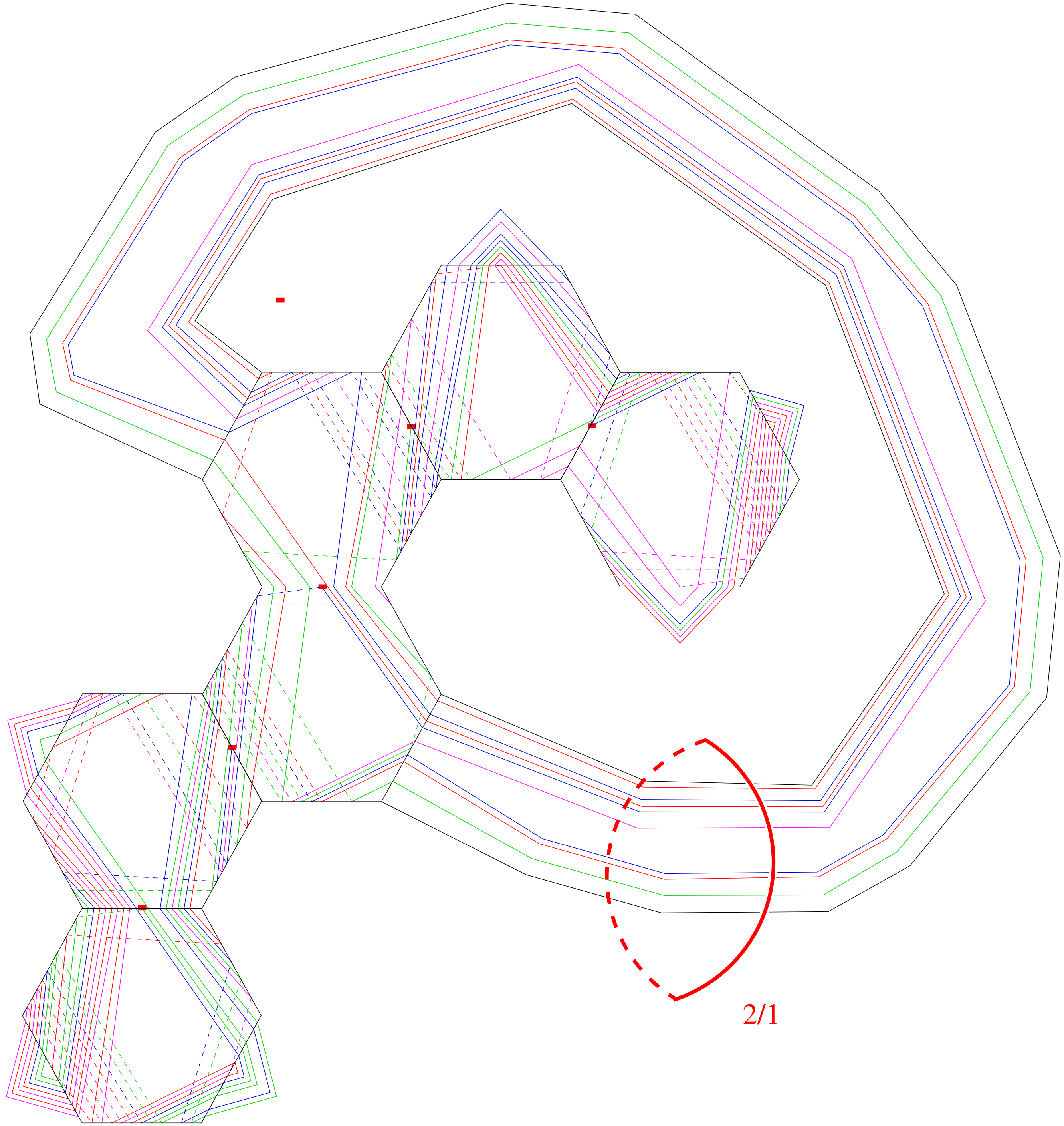
$$10_{137} = m(2/5, 3/5, -1/2)$$

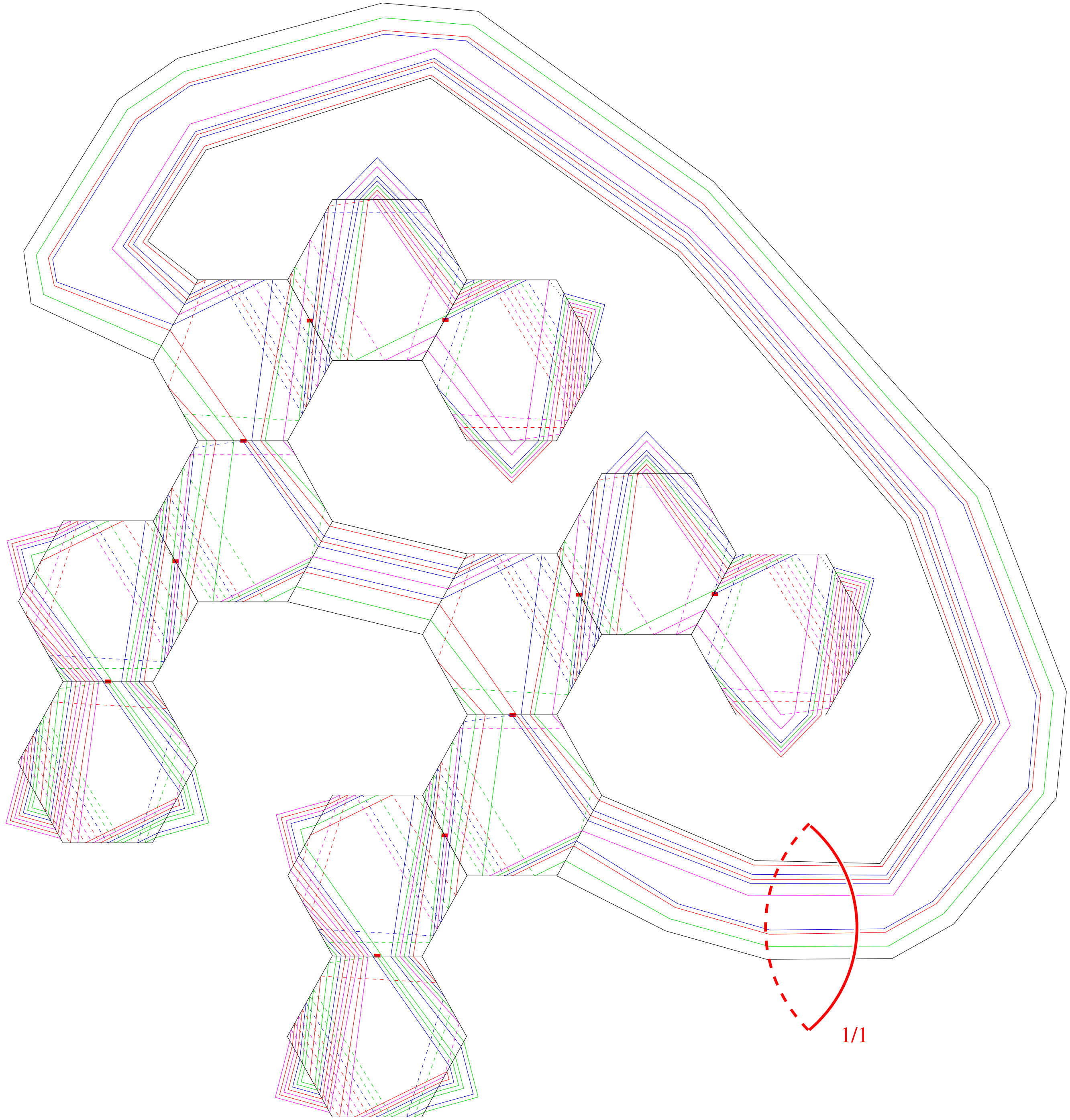


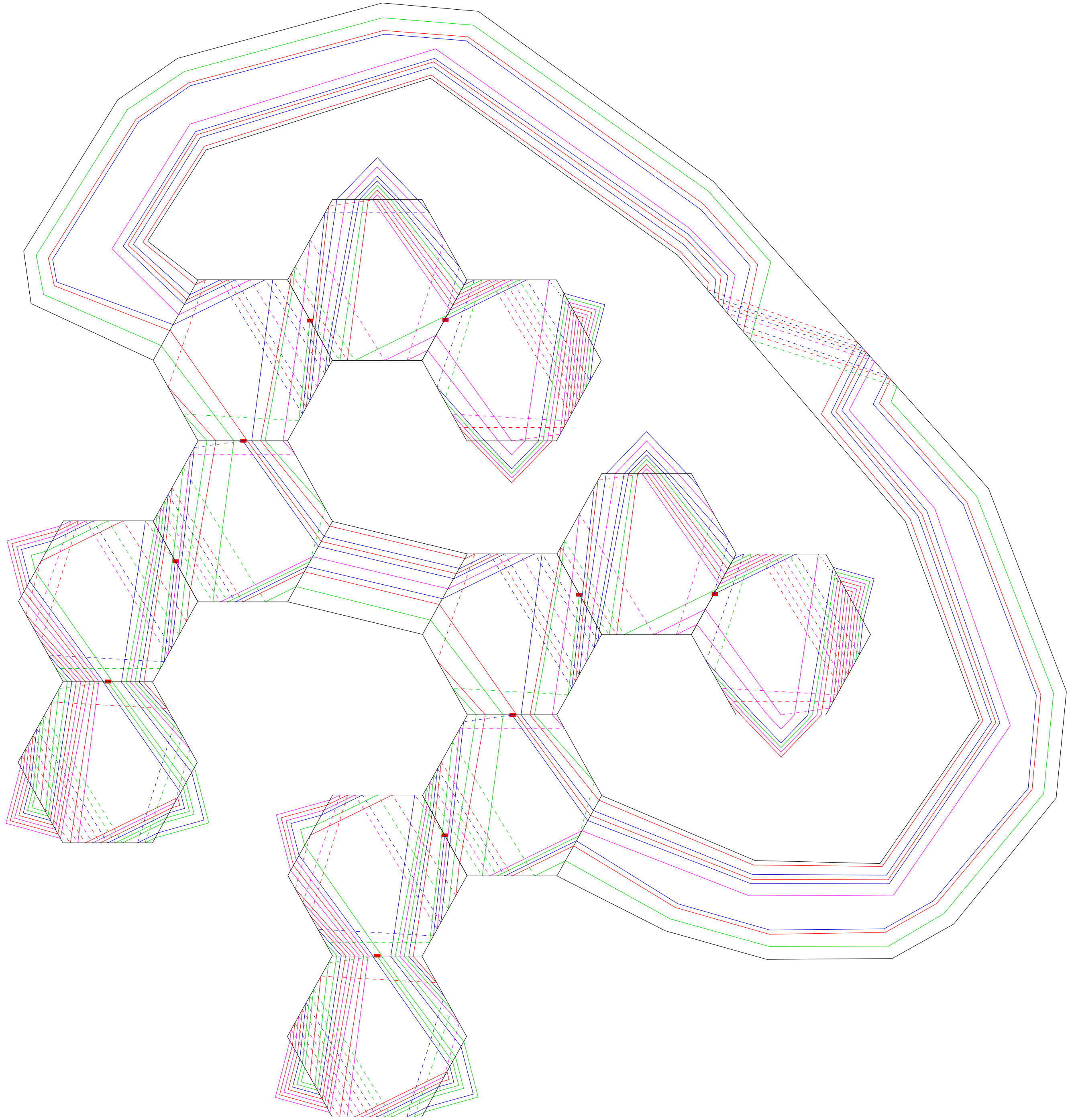


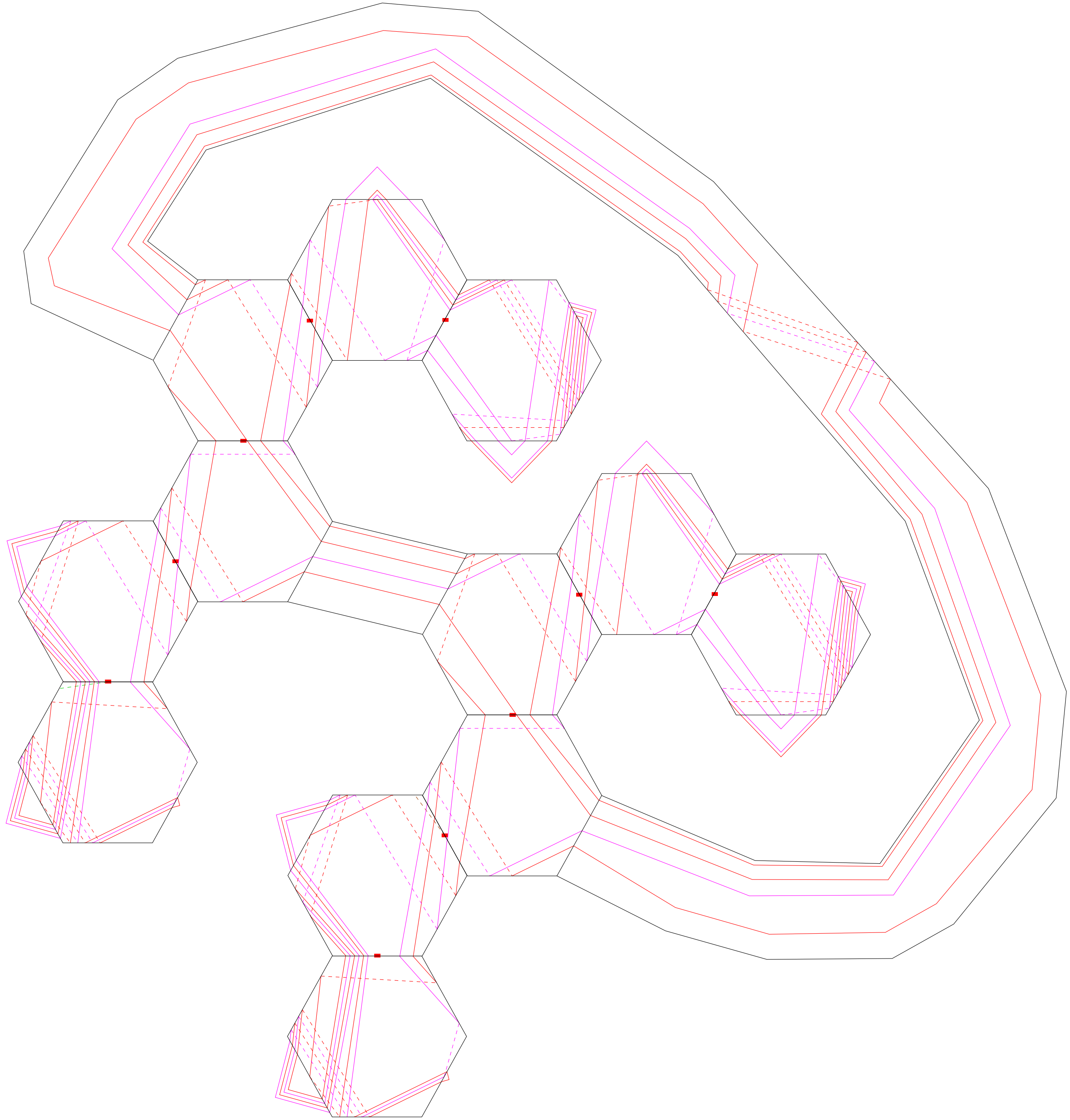


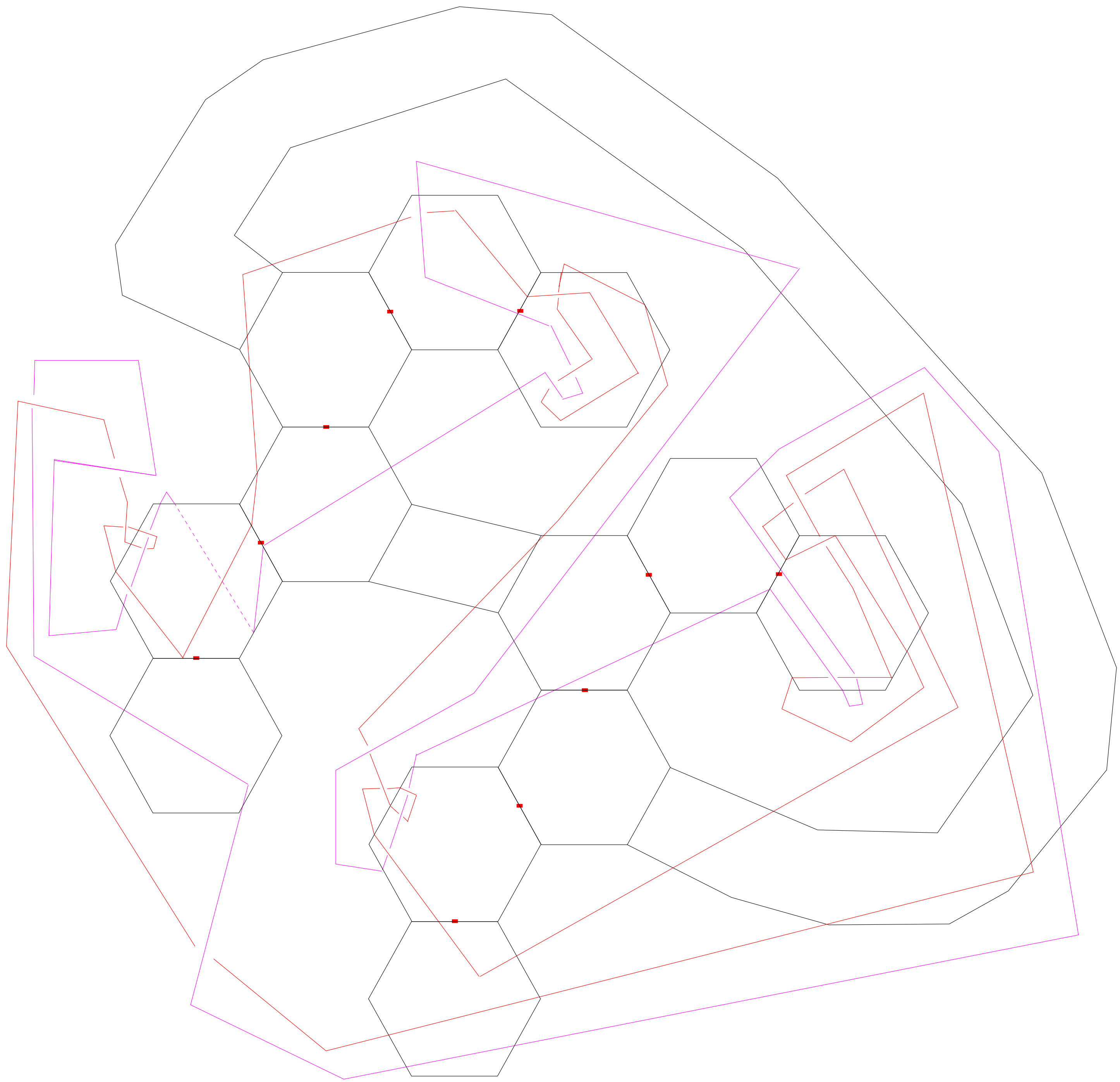

2/1

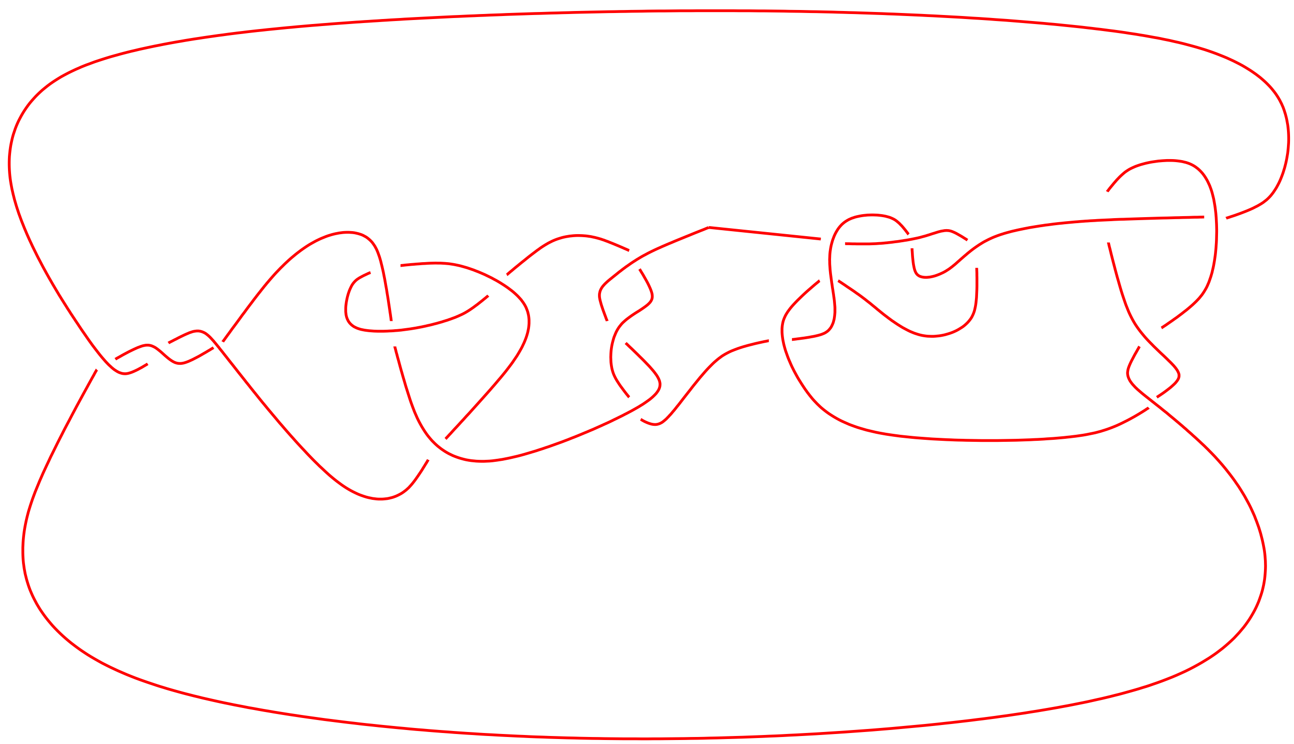

2/1


1/1









$$m(-3, -\frac{3}{5}, \frac{1}{3}, -\frac{3}{5}, -\frac{2}{5}) \sim m(\frac{37}{5}, -\frac{37}{3}, \frac{37}{5}, -\frac{37}{5}) \leftarrow m(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3})$$

The knot 10_{137} *is universal*

- $10_{68} = m(\frac{3}{5}, \frac{1}{3}, \frac{1}{3}) \sim m(-\frac{19 \cdot 3}{5}, \frac{19}{3}, \frac{19}{3}) \leftarrow m(-\frac{3}{5}, \frac{1}{3}, \frac{1}{3}) \sim 10_{145}$

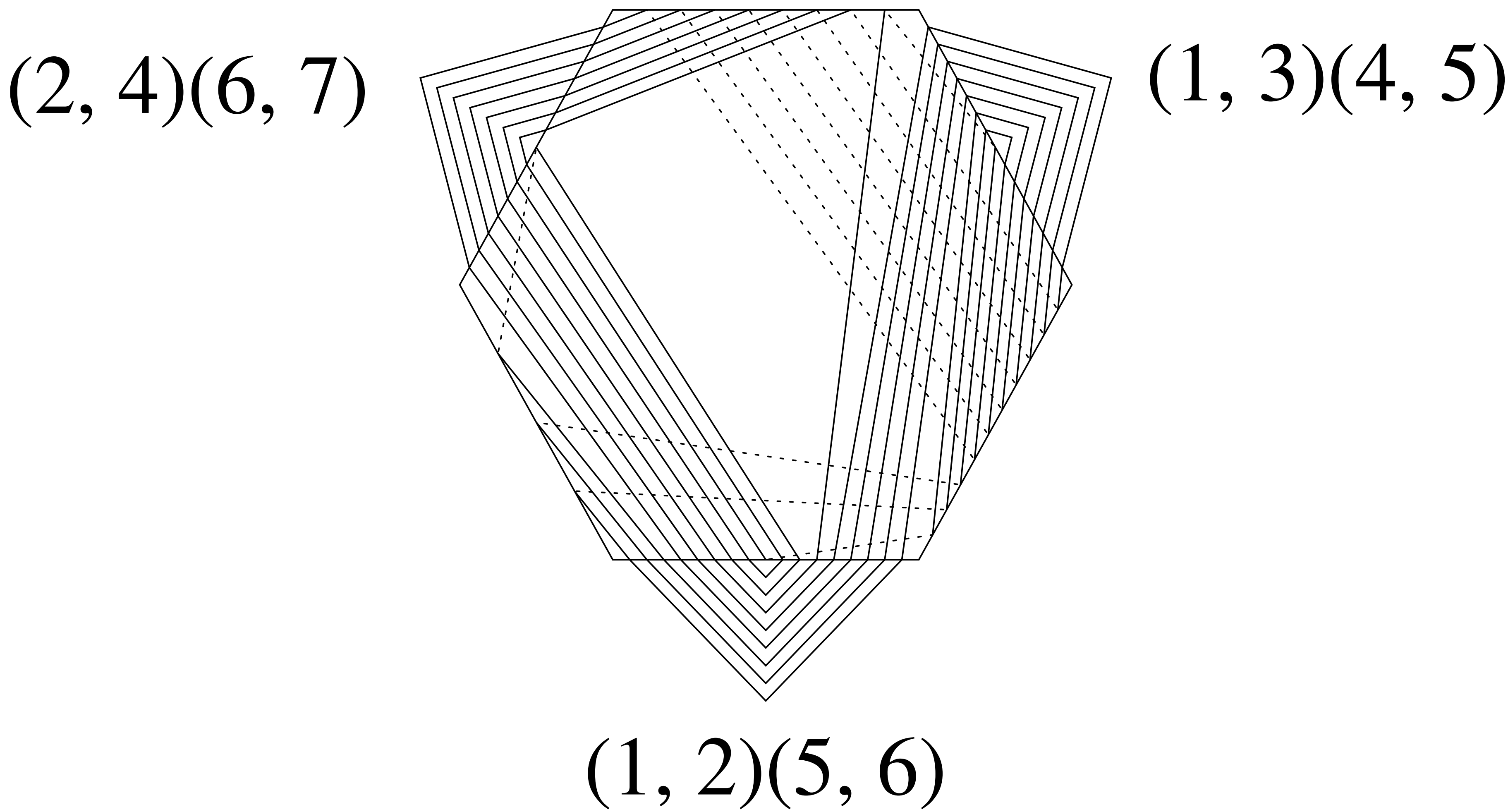
- $10_{69} = m(\frac{3}{5}, \frac{2}{3}, \frac{3}{3}) \sim m(-\frac{29 \cdot 3}{5}, \frac{29}{3}, \frac{29}{3}) \leftarrow m(-\frac{3}{5}, \frac{1}{3}, \frac{1}{3}) \sim 10_{145}$

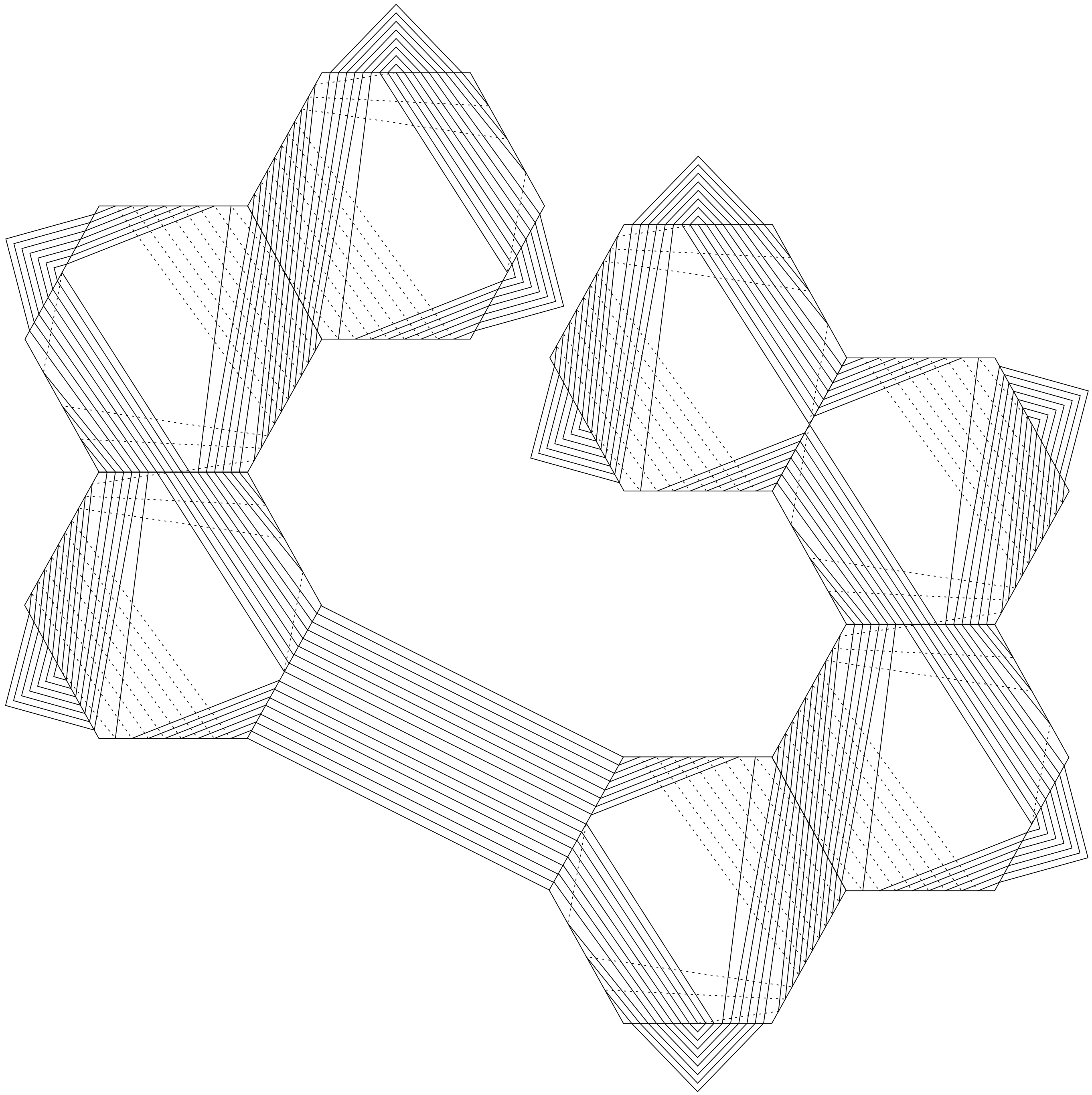
- $10_{146} = m(\frac{2}{5}, \frac{2}{3}, -\frac{1}{3}) \sim m(-\frac{11 \cdot 3}{5}, \frac{11}{3}, \frac{11}{3}) \leftarrow m(-\frac{3}{5}, \frac{1}{3}, \frac{1}{3}) \sim 10_{145}$

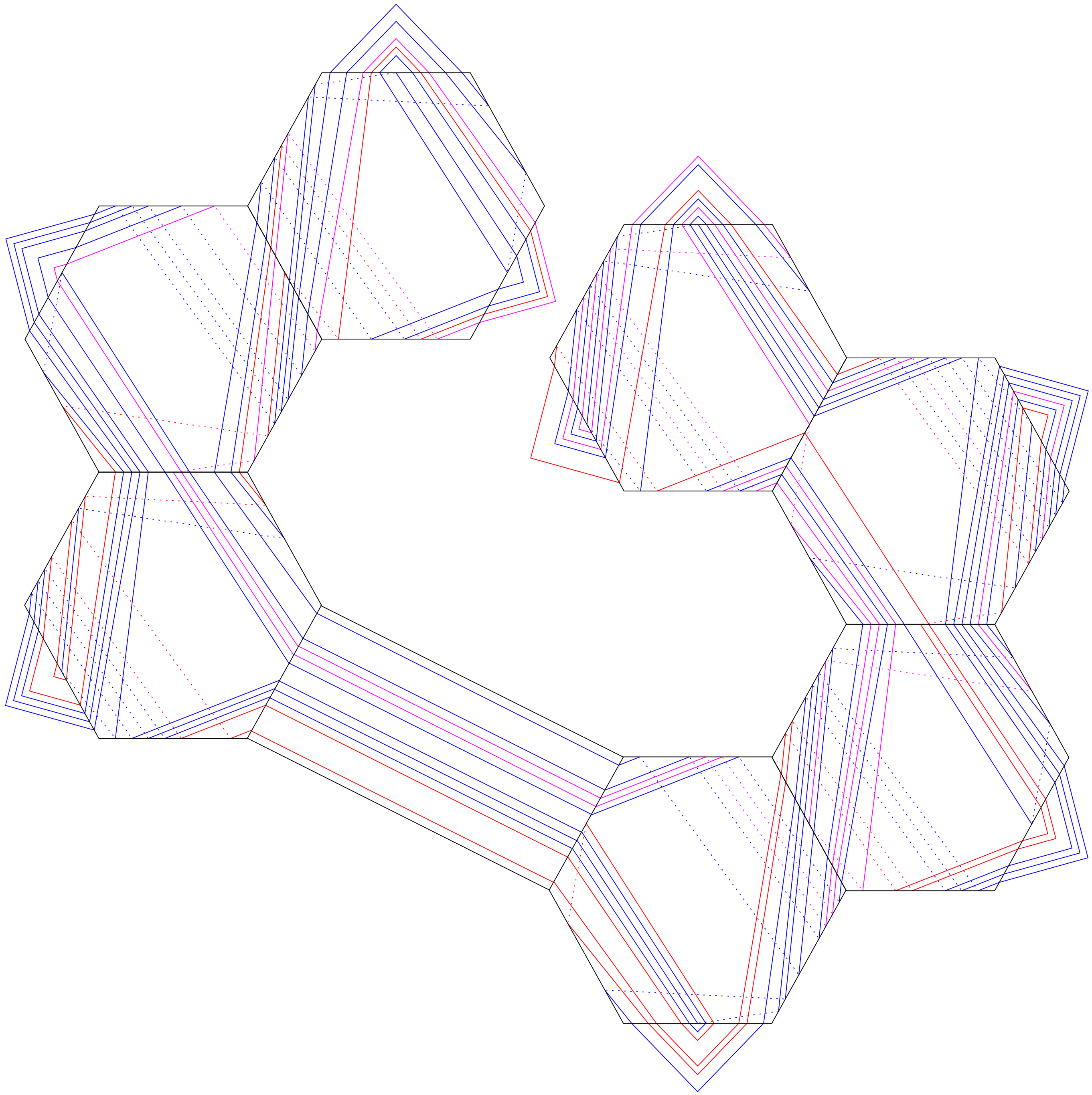
- $10_{75} = m(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}) \leftarrow 10_{145}$

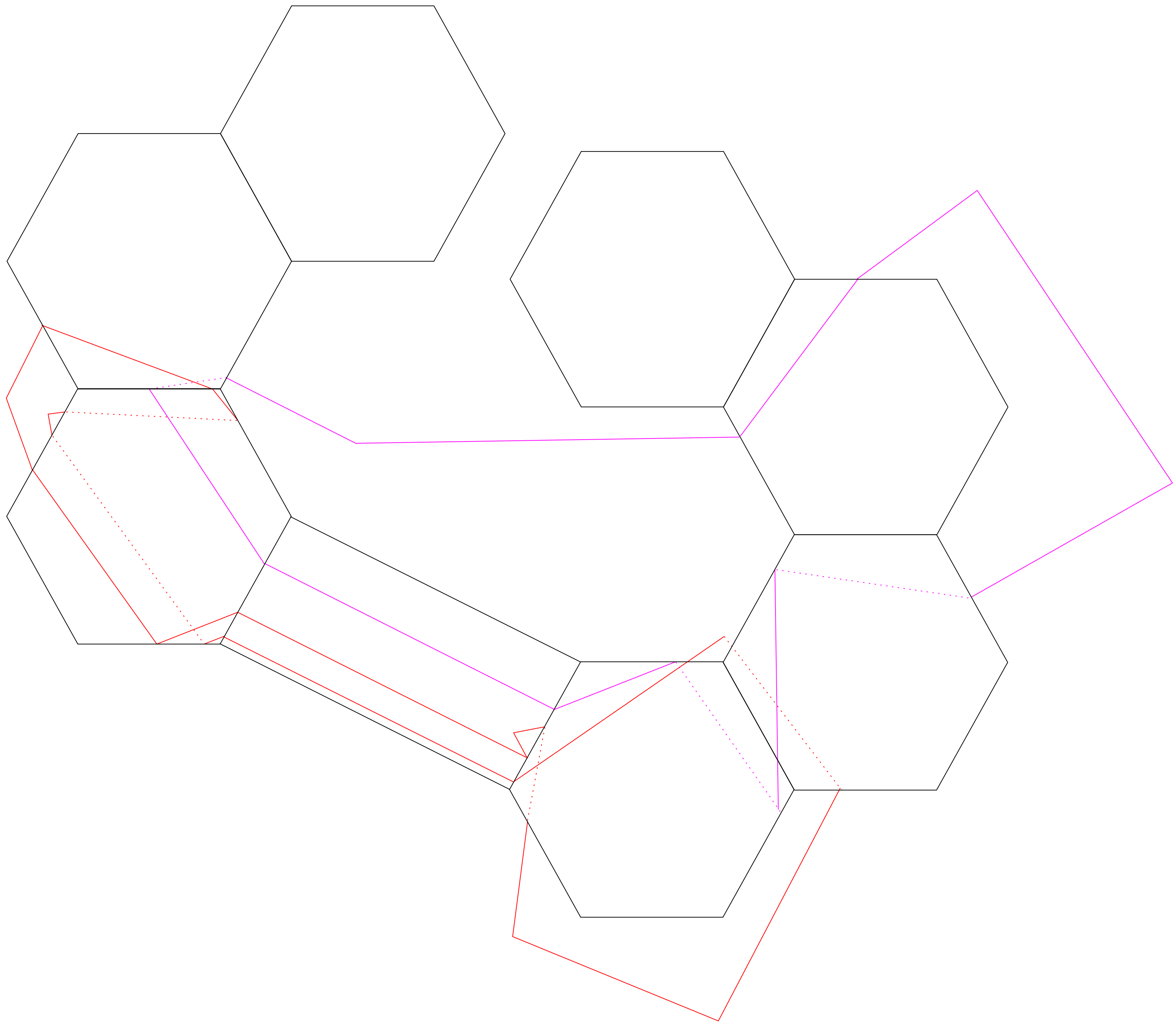
- $10_{147} = m(\frac{3}{5}, \frac{1}{3}, -\frac{1}{3}) \leftarrow 10_{145}$

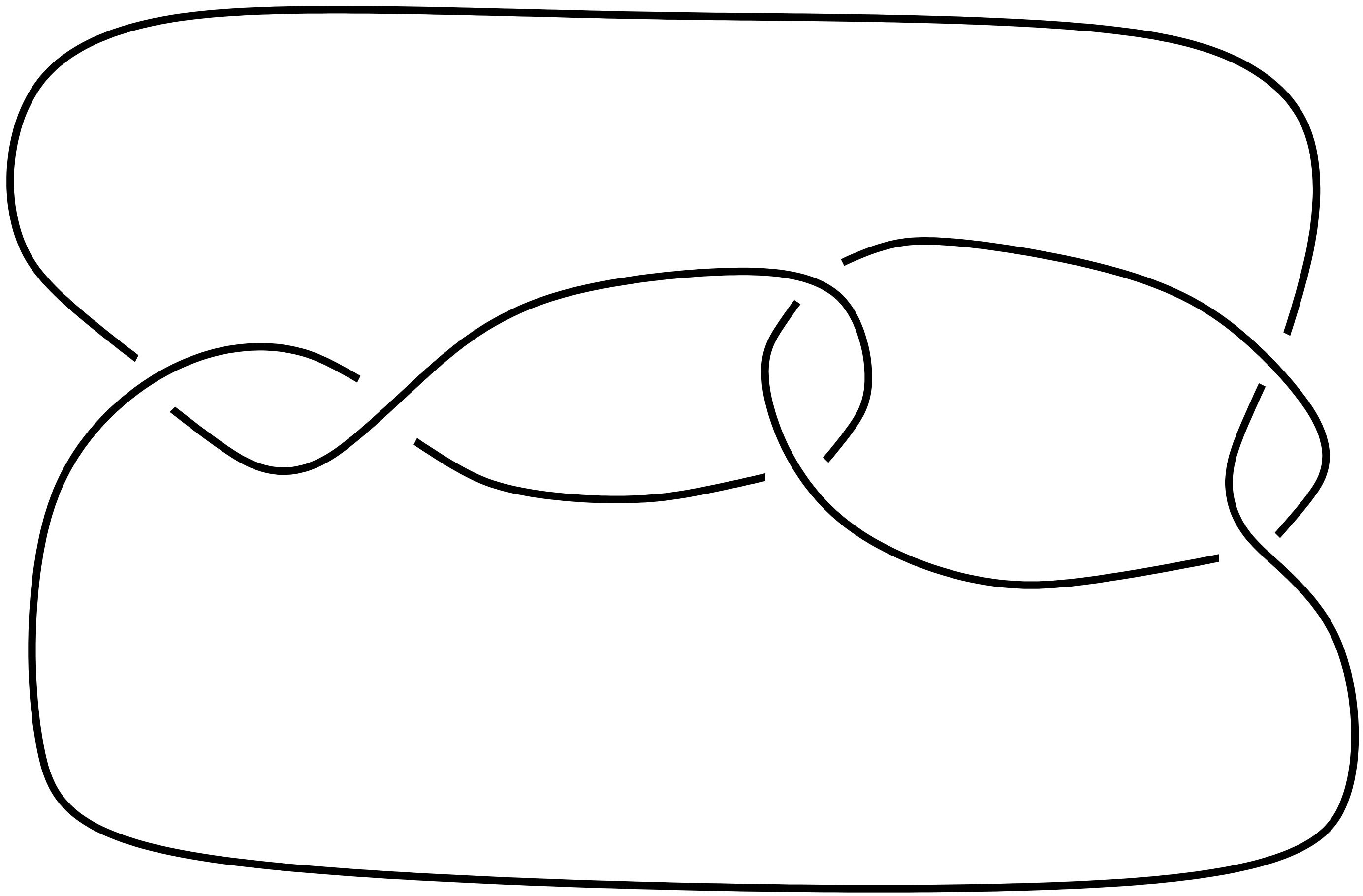
$$10_{145} = m(2/5, 2/3, -1/3)$$











$$m(2, -\frac{1}{2}, \frac{1}{2}) \sim m(-\frac{1}{2}, \frac{5}{2}) \sim m(\frac{8}{3})$$

The knot 10_{145} *is universal*

Theorem. *Let k be a Montesinos knot with less than eleven crossings.*

Then

$$k \text{ is universal} \Leftrightarrow k \text{ is hyperbolic}$$