

Quasi-stationary distributions for population processes Distribuciones cuasi-estacionarias para procesos de population

Sylvie Méléard

Ecole Polytechnique, CMAP, F- 91128 Palaiseau Cedex, France

email: sylvie.meleard@polytechnique.edu

Our aim in this course is to study the asymptotic behavior of the size $(Z_t; t \geq 0)$ of some isolated biological populations modelled by different random processes. Competition for limited resources impedes natural populations with no immigration to grow indefinitely, and they are all doomed to become extinct after some finite time T_0 . However, T_0 can be large compared to human timescale and certain species may survive for long periods before extinction actually occurs. This transient behavior is captured by the mathematical notion of quasi-stationarity. Specifically, a *quasi-stationary distribution* (in short QSD) for Z is a probability measure ν satisfying $\mathbb{P}_\nu(Z_t \in A \mid T_0 > t) = \nu(A)$ for Borel sets A .

In this course, we will study the existence and uniqueness of quasi-stationary distributions for different models of populations in continuous time, taking into account competition between individuals, multi-types populations or structured populations submitted to mutation and selection, for small or large populations. Each case uses different probabilistic and analytic tools. We will study the possible long-time convergence of the law of the process conditioned on non extinction to the QSD. Usually the existence of QSD is not obtained by a constructive method. We will explain how a particle method based on Fleming-Viot systems allows us to construct an algorithm for their simulation.