

Branching processes, Levy processes and stochastic equations

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Continuous state branching processes arise as rescaled limits of discrete branching ones. Those processes can be characterized using the Laplace transforms and as the time-change of Levy processes. They can also be characterized using martingale problems or stochastic equations. In a typical case, the stochastic equation is driven by a one-sided stable process. The strong uniqueness of solutions to those equations was proved recently. A continuous state branching process can also be decomposed into excursions away from the trap zero. A state-dependent immigration structure can be given using a stochastic equation driven by a Poisson random measure on the space excursions. This also gives an alternate approach to change the branching mechanism of the process. We shall start from the basic properties and end up with some recent results. The following topics (or some of them) will be included:

1. Discrete state branching processes: scaling limit theorems
2. Continuous state branching processes
3. Martingale problems and stochastic equations
4. Strong uniqueness of solutions
5. Entrance laws and excursion laws
6. State-dependent immigration

References

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