Normal geodesics associated to driftless control systems on step-2 simply connected nilpotent Lie groups

Felipe Monroy-Pérez
fmp@correo.azc.uam.mx
(in collaboration with Alfonso Anzaldo-Meneses)
Universidad Autónoma Metropolitana-Azapotzalco
Departamento de Ciencias Básicas

Abstract

We present in this talk some recent results on the geodesic problem defined by means of a left invariant distribution of rank $n$ on a step-2 simply connected nilpotent Lie group $G$ of dimension $n(n + 1)/2$. From the control theory point of view, the distribution defines a driftless control system which together with the energy functional for admissible curves, determines a well defined optimal control problem on the group $G$. This class of optimal control problems, has been extensively utilized in the study of certain non-holonomic problems such as chained systems, micro-swimming, etc.

We describe a transitive group action that leaves the structure of the problem invariant. By using Pontryagin Maximum Principle of optimal control, we derive necessary conditions for the length-minimality of the geodesics. We perform an integration process using the Sylvester-Lagrange representation of the exponential of a skew-symmetric matrix. The integration yields explicit expressions for the extremal curves and the corresponding geodesics, the obtained formulæ allow the complete parametrization of the exponential mapping in terms of algebraic invariants of the problem. We shall present in some detail some low dimensional cases.