CHAPTER 1

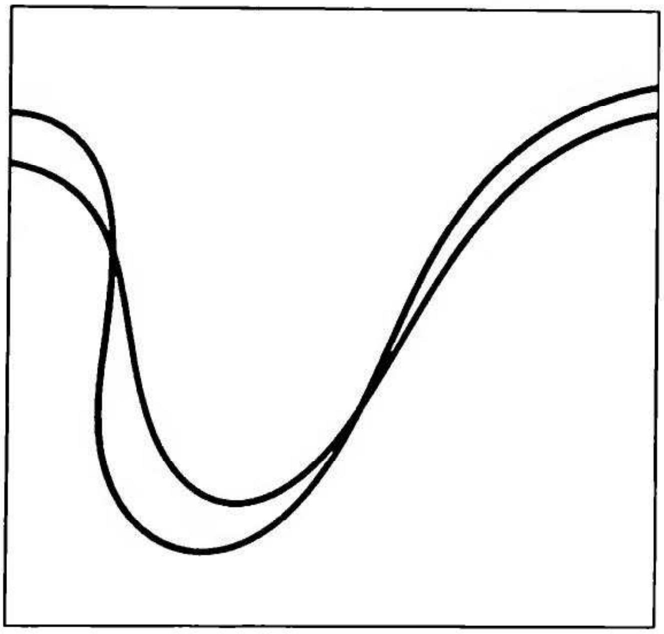
Plane Geometry

1.1 Locus

1. Which Way Did the Bicycle Go?

> "This track, as you perceive, was made by a rider who was going from the direction of the school."
>
> "Or towards it?"
>
> "No, no, my dear Watson . . . It was undoubtedly heading away from the school."
>
> —Sherlock Holmes, during his visit to the Priory School

Here's a mystery that is truly worthy of Sherlock Holmes! Imagine a 20-foot wide mud patch through which a bicycle has just passed, with its front and rear tires leaving tracks as illustrated. In which direction was the bicyclist travelling?



2. Where Can the Third Vertex Live?

Given $\triangle ABC$, describe the set of points X for which there exists a point D on BC such that $\triangle ADX$ is equilateral.

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CHAPTER 7

Plane Geometry

7.1 Locus

1. Which Way Did the Bicycle Go?

The bicyclist was going from right to left. Let $F(t)$ and $B(t)$ be the points of contact of the front and back wheels, respectively, at time t. Then, because the rear wheel does not steer, the line connecting $F(t)$ and $B(t)$ is tangent to the path of the back wheel and has *fixed length,* namely the distance between the bottoms of the bicycle wheels. It follows that the thick curve in Figure 1 cannot be the rear wheel's path, for some of its tangents fail to strike the other curve. Therefore the thin curve is the rear wheel's path.

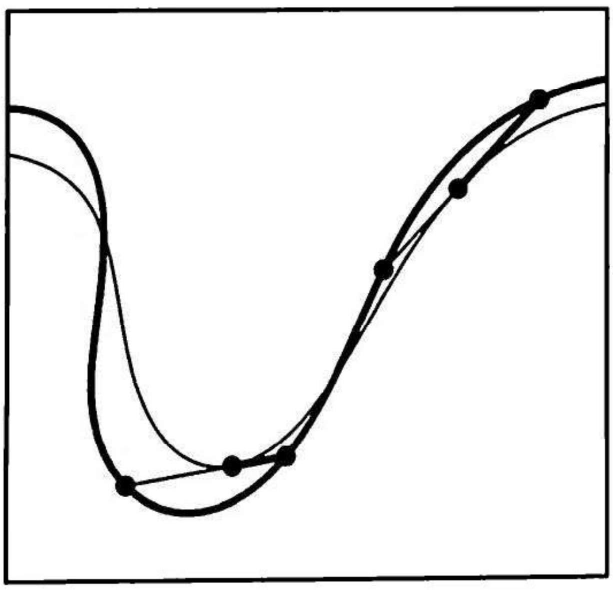

FIGURE 1
The fact that tangent segments have constant length in only one direction proves that the bicycle was traveling right to left.

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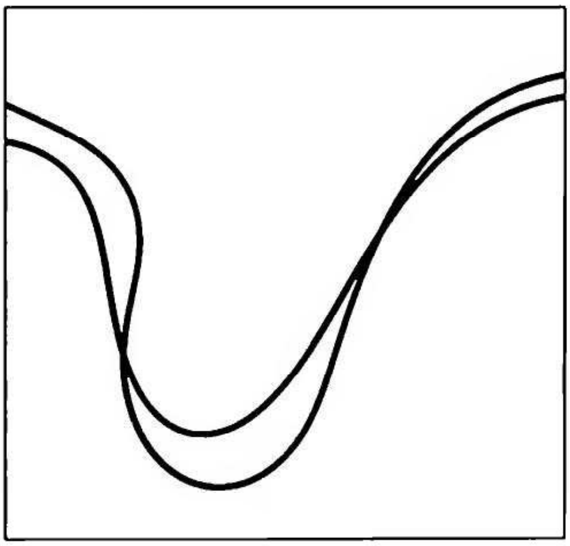

FIGURE 2
These paths show what the tracks would be if the back path were the same but the direction of travel were reversed.

We now examine some tangents to determine the direction. The lengths of the thick tangent segments in Figure 1 would be constant if the direction were from left to right. But they are not! The thin segments do have constant length, thus confirming that the direction of travel is from right to left. Figure 2 shows what the curves would look like if the back path were the same but the rider were going from left to right.

Note. Here is how Sherlock "solved" the problem:

> "No, no, my dear Watson. The more deeply sunk impression is, of course, the hind wheel, upon which the weight rests. You perceive several places where it has passed across and obliterated the more shallow mark of the front one. It was undoubtedly heading away from the school."

Balderdash! As observed by Dennis Thron (Dartmouth Medical School), it is true that the rear wheel would obliterate the track of the front wheel at the crossings, but this would be true *no matter which direction* the bicyclist was going. This information, by itself, does not solve the problem. We could, perhaps, give Holmes the benefit of the doubt and assume that he carried out the proper solution in his head. But we cannot believe that he would have expected Watson to grasp it from his comments alone. Conan Doyle has let us down badly on this one!

The bicycle paths in the figures were generated in *Mathematica* by using a Bézier curve to get the back-wheel path, and then symbolic differentiation to get the corresponding front-wheel paths in the two directions; for details see [Wag5]. We learned of this problem from materials for a geometry course at Princeton developed by John Conway, Peter Doyle, Jane Gilman, and Bill Thurston.

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