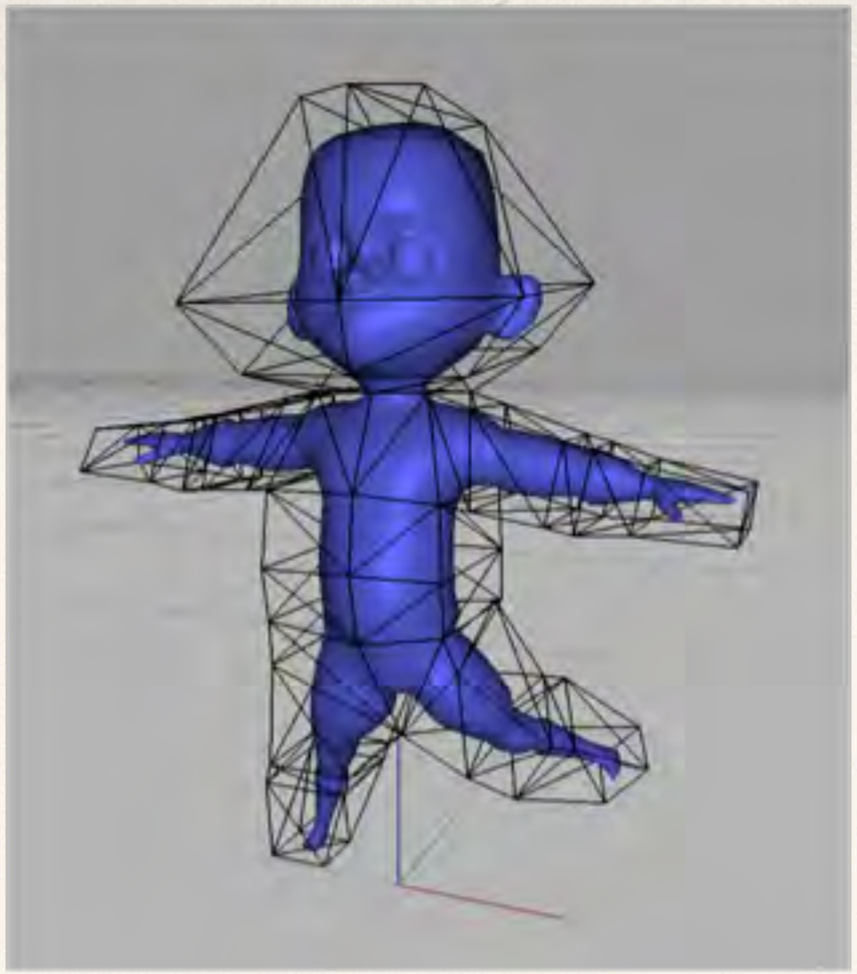

T.D. DeRose, M. Meyer, Harmonic Coordinates. Pixar Technical Memo #06-02

Free-form deformation (FFD)

Advanced Computer Animation Techniques
Aug-Dec 2014
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Free-form deformation (FFD)

2d grid-based deformation	FFD
2d grid	3d grid
bi-linear interpolation	tri-cubic interpolation

- A coordinate grid is superimposed over an objet.
- For each vertex of the object, coordinates relative to this local grid are determined that register the vertex to the grid.
- The grid is then manipulated by the user.
- Using relative coordinates, each vertex is then mapped back into the modified grid, which relocates that vertex in global space.
- Cubic interpolation is used to compute intermediate shapes.

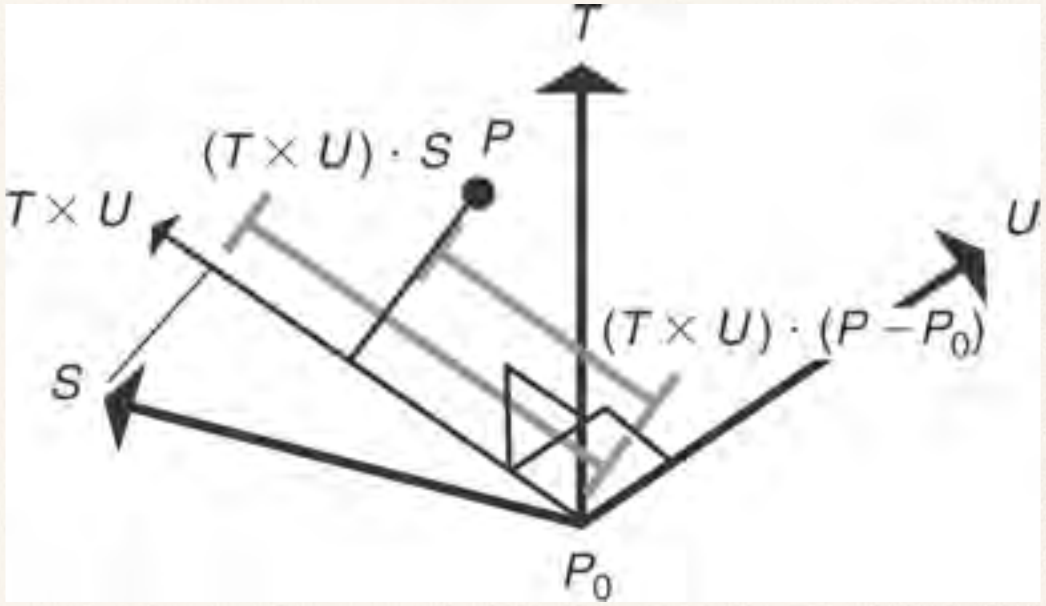
Free-form deformation (FFD)

- Define a local coordinate system *(S,T,U)*.
- Register every vertex P of the object in the local coordinate: determine its trilinear interpolants.

$$s = \frac{(T \times U) \cdot (P - P_0)}{((T \times U) \cdot S)}$$

$$t = \frac{(U \times S) \cdot (P - P_0)}{((U \times S) \cdot T)}$$

$$u = \frac{(S \times T) \cdot (P - P_0)}{((S \times T) \cdot U)}$$



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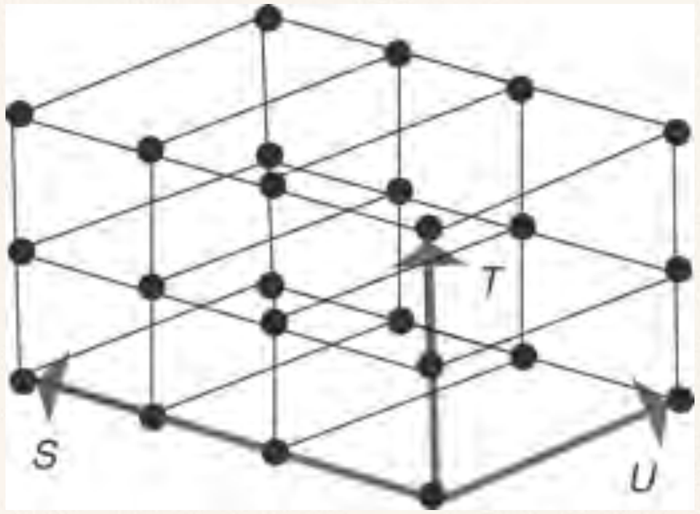
- Given the local coordinates *(s,t,u)* of a point and the unmodified local coordinate grid a point's position is reconstructed in global space:

$$P = P_0 + sS + tT + uU$$

Free-form deformation (FFD)

- Embed object in the rectilinear grid.
- Grid can be non-uniform,
  - e.g. *3(l)* points in S direction, *2(j)* in T direction, *1(n)* in U direction.



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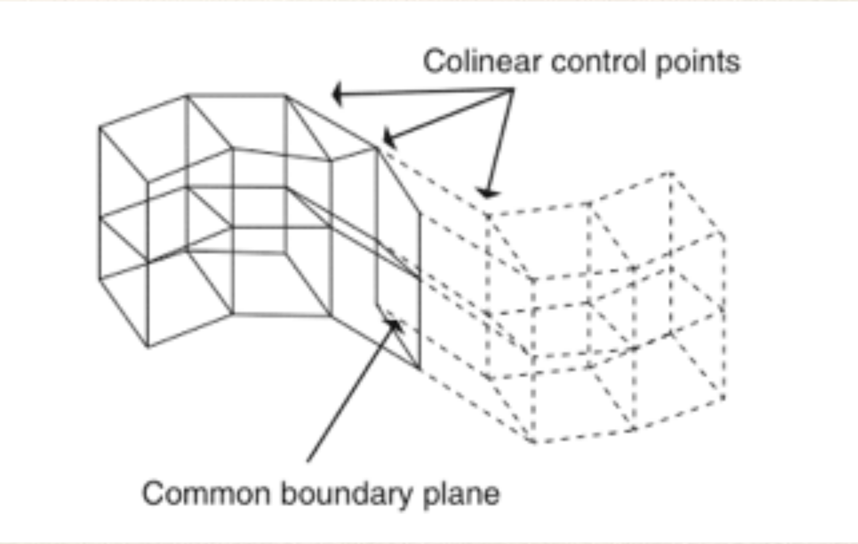
$$P_{ijk} = P_0 + \frac{i}{l}S + \frac{j}{m}T + \frac{k}{n}U \quad \text{for} \quad \begin{cases} 0 \leq i \leq l \\ 0 \leq j \leq m \\ 0 \leq k \leq n \end{cases}$$

Free-form deformation (FFD)

- The deformations are specified by moving the control points from their initial positions.
- The function that effects the deformation is a trivariate Bézier interpolating function.
- The deformed position of a point P_{stu} is determined by using its (s,t,u) local coordinates in the following Bézier interpolating function:

$$P(s,t,u) = \sum_{i=0}^{l} \binom{l}{i} (1-s)^{l-i} s^i \left(\sum_{j=0}^{m} \binom{m}{j} \left((1-t)^{m-j} t^j \left(\sum_{k=0}^{n} \binom{n}{k} (1-u)^{n-k} u^k P_{ijk} \right) \right) \right)$$

$P(s,t,u)$: global coordinates of the deformed point.
P_{ijk} : global coordinates of the control points.
C^1 continuity.



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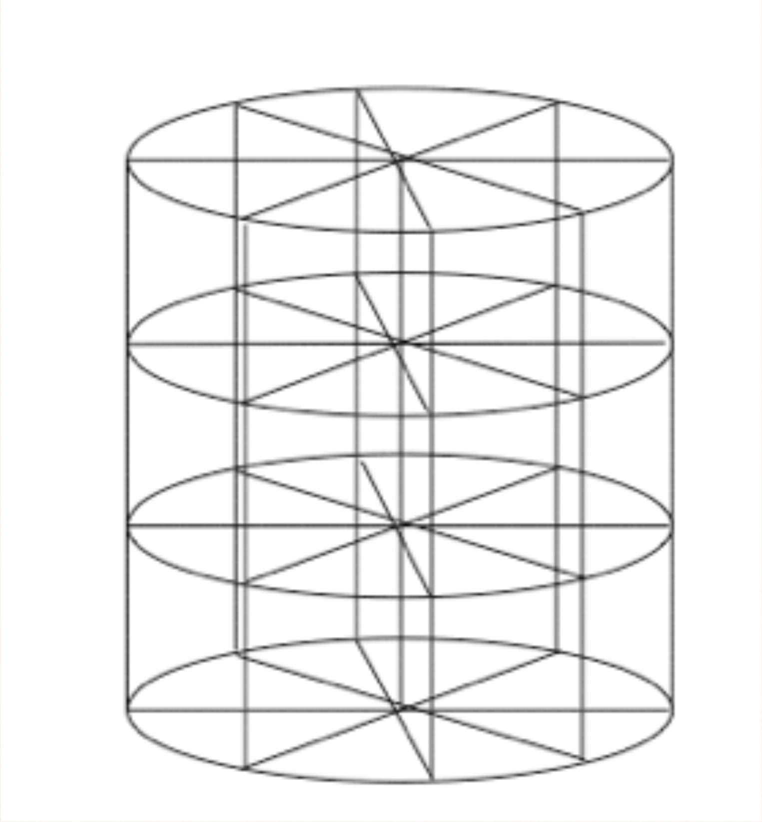
FFDs: alternate grid organizations



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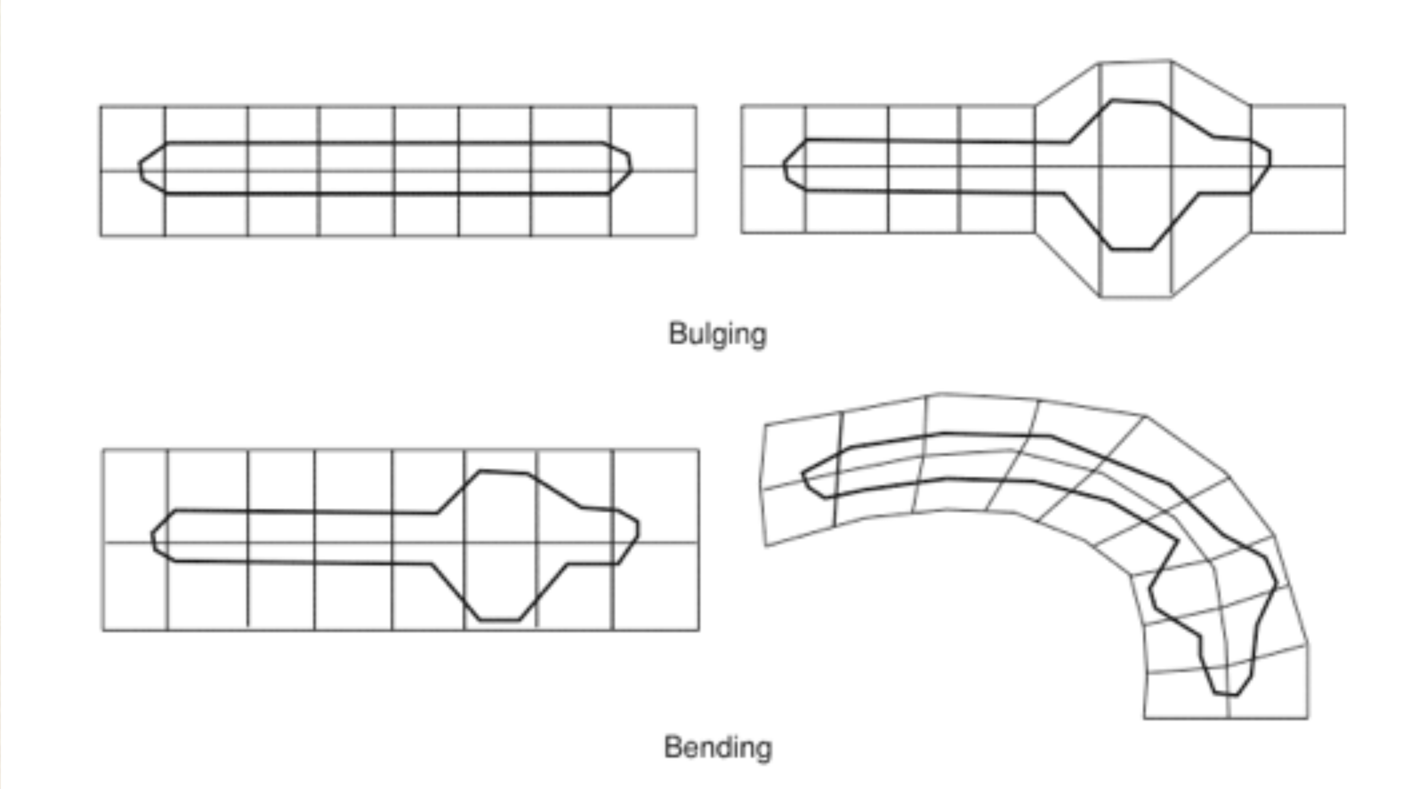
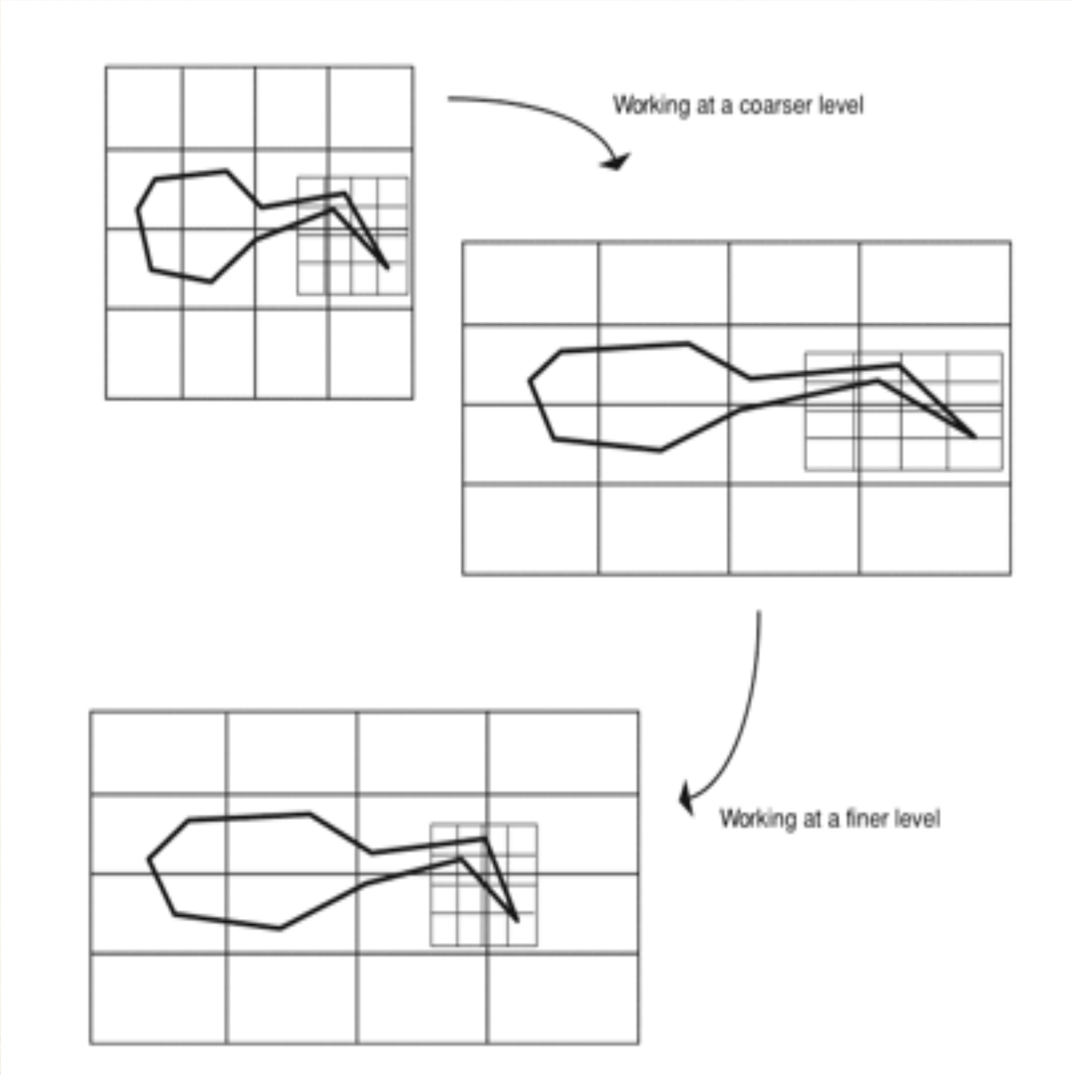
Composite FFDs: sequential and hierarchical

- Sequential:
  - an object is modeled by progressing through a sequence of FFDs, each of which imparts a particular feature to the object.
  - various detail elements can be added to an object in stages as instead of building a complex FFD.
- Hierarchical:
  - allows the user to work at various levels of detail.
  - finer-resolution FFDs are embedded inside FFDs higher in hierarchy.
  - as a coarser-level FFD is used to modify object vertices, it also modifies the control points of any children FFDs that are within space affected by the deformation.

Composite FFDs: sequential and hierarchical





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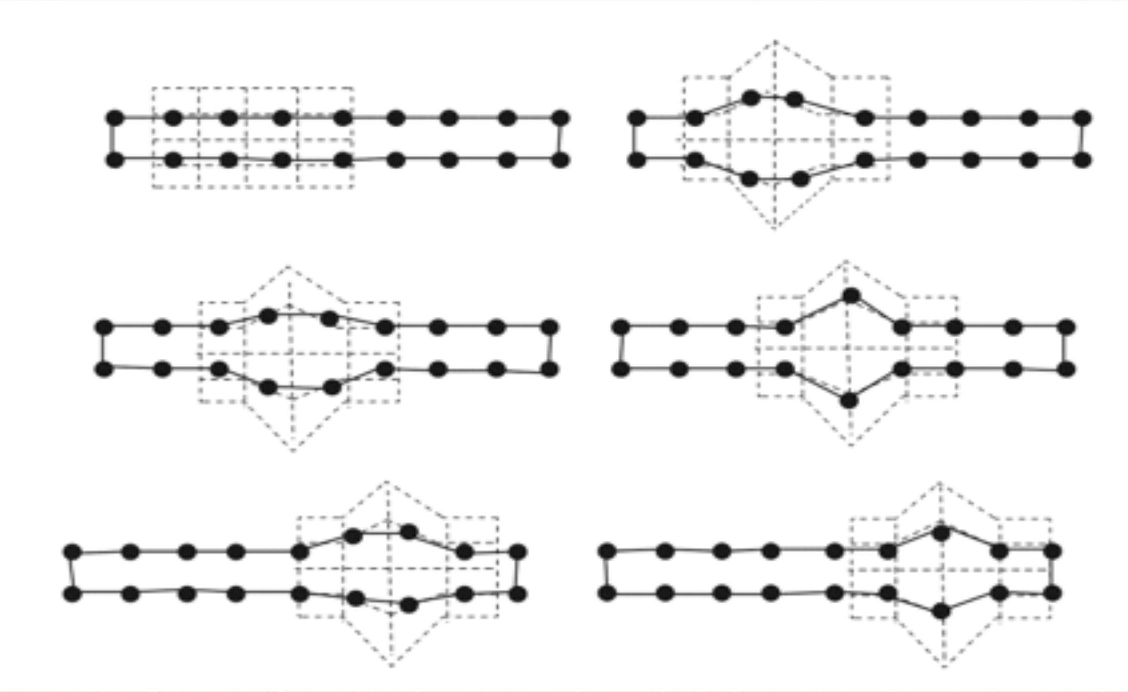
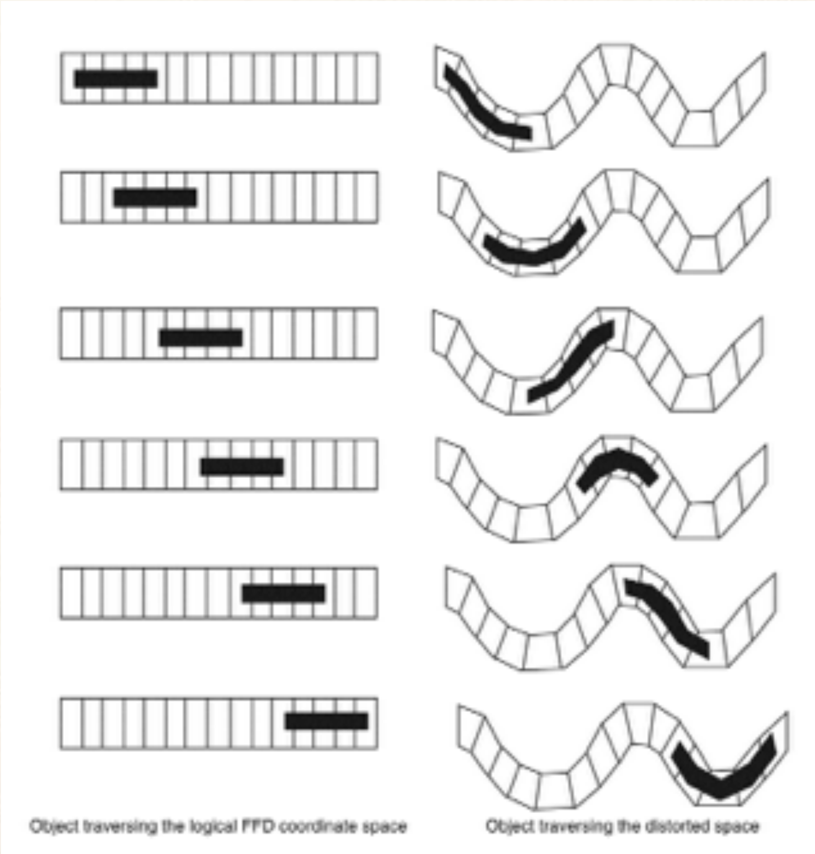
Animated FFDs

- Linear interpolation of the object's vertices on a vertex-by-vertex basis.
- Construct the FFD so that the object can traverse throughout the FFD space resulting in a continuous transformation of its shape.





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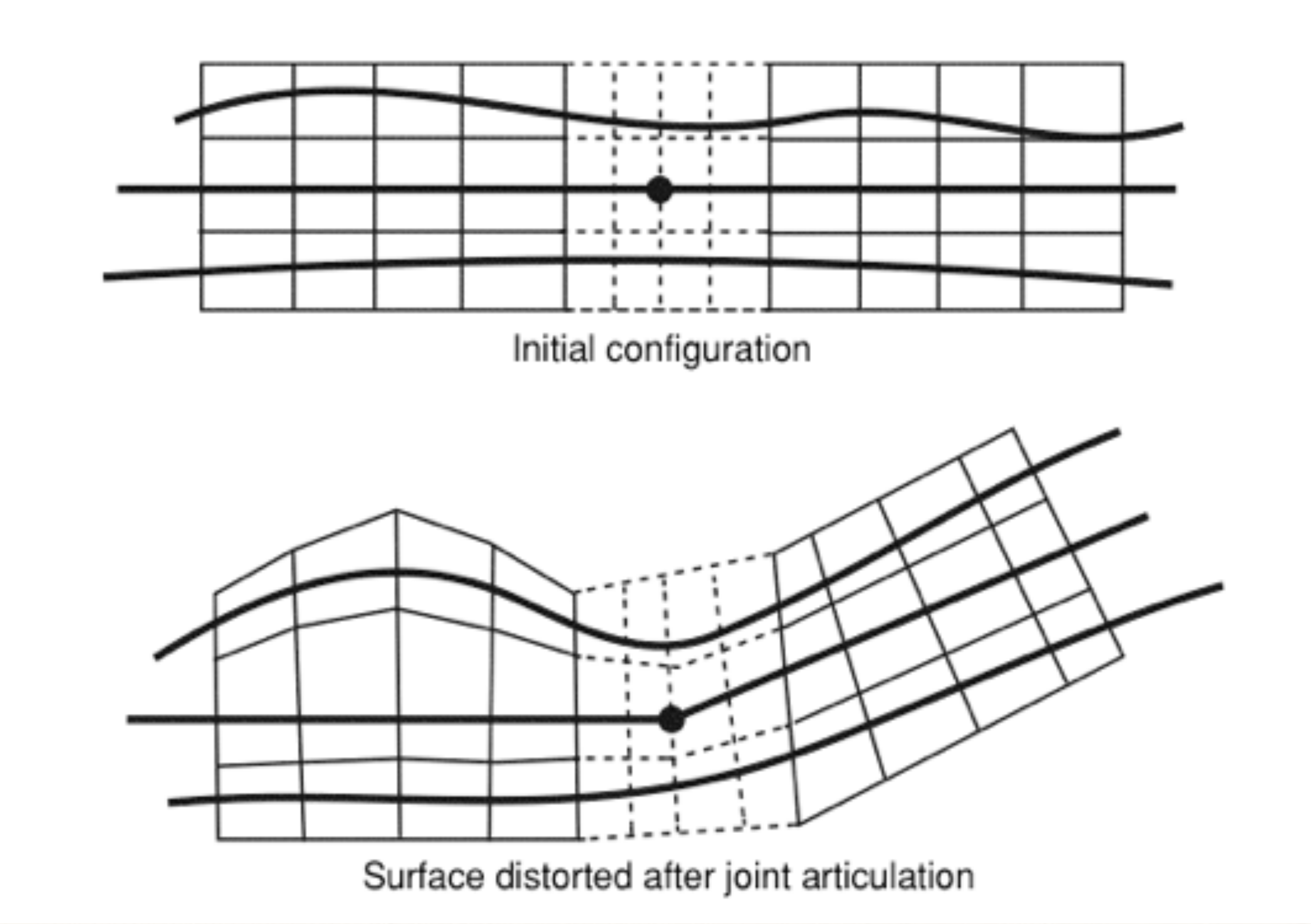
Animated FFDs

- Animate the control points of the FFD explicitely using key-frame animation or physical simulation.
- J.Chadwick, D.Haumann and R.Parent. "**Layered construction for deformable animated characters**". *SIGGRAPH 1989*. 23(3) p.p.243-252.
- Two techniques are proposed:
- **Strictly kinematic method:**
    - positions of the FFD grid vertices are located relative to a wire skeleton the animator uses to move the figure.
    - as the skeleton is manipulated, the grid vertices are repositioned relative to the skeleton automatically.
    - the skin of the figure is located relative to this local FFD coordinate grid.
    - **Joint articulation modifies the FFD grid, which modifies the surface of the figure**: exo-muscular system.

Animated FFDs

- Skeleton changes FFD, which changes skin.



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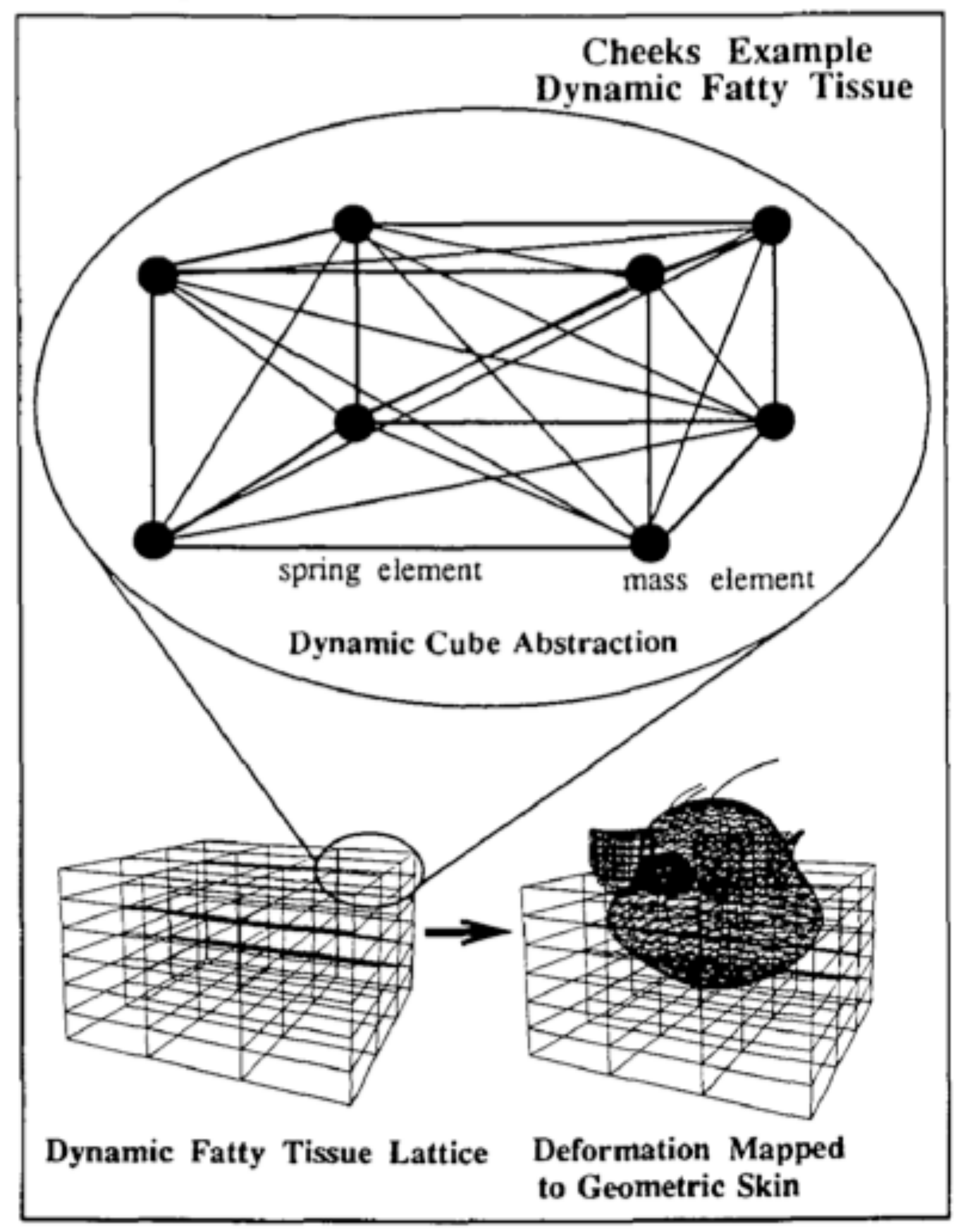
Animated FFDs

- Physically-based method.
  - animation of the FFD control points is produced by modeling the lattice with springs, dampers and mass points.
  - control points respond to gravity as well as kinematic motion of the figure.
  - the center of the FFD is fixed relative to the figure's skeleton.
  - the user kinematically controls the skeleton, and the motion of the skeleton moves the center point of the FFD lattice.
  - the rest of the FFD lattice points react to the movement of this center point via the spring-mass model.
  - This approach can also animate clothes and facial tissue.

Animated FFDs



3d shape interpolation

- Surface-based or volume-based techniques.
- Surface-based:
  - use the boundary representation of the objects and modify one or both of them so that the vertex-edge topologies of the two objects match.
  - the vertices of the object can be interpolated on a vertex-by-vertex basis.
  - handle limited types of objects (no holes).
- Volume-based:
  - consider the volume contained within the objects and blend one volume into the other.
  - less sensitive to objects with different topologies.
  - require volume-based representation of objects: computationally intensive.

3d shape interpolation

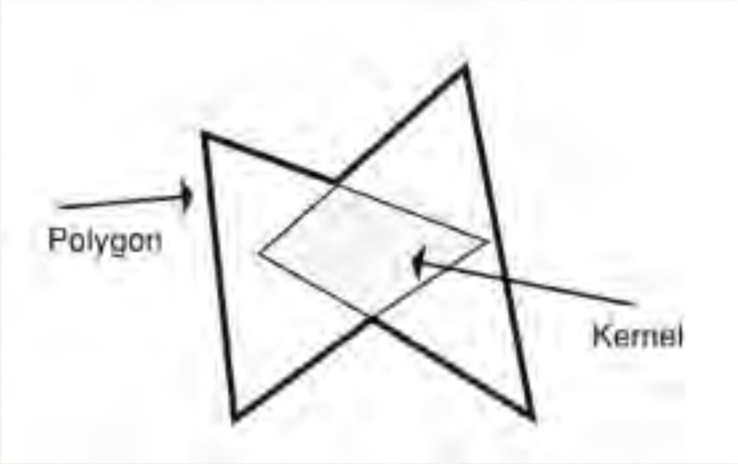
- Correspondence problem:
  - establish the mapping from a vertex (or other geometric element) on one object to a vertex on the other object.
- Interpolation problem:
  - create a sequence of intermediate objects that visually represent the transformation of one object into the other.
- Use of topological information versus geometric information.
  - Topological:
    - logical construction of the objects.
    - tends to minimize the number of new vertices and edges generated in the process.
  - Geometric:
    - considers the spacial extent of the object
    - useful for relating the position in space of one object to the position of the other.

3d shape interpolation

- Topology of surface or object
  - Surface topology
    - connectivity
    - manifold vs. non-manifold
  - Object topology
    - genus (holes)
- Topologically equivalent
  - a doughnut and a teacup are topologically equivalent.

3d shape interpolation: star-shaped polyhedra

- Star shaped polyhedra:
  - there is at least one point from which a line can be drawn to any point on the boundary of the polygon without intersecting a boundary.
  - the set of points from which the entire boundary can be seen: **kernel**.
  - Use of polar coordinates.



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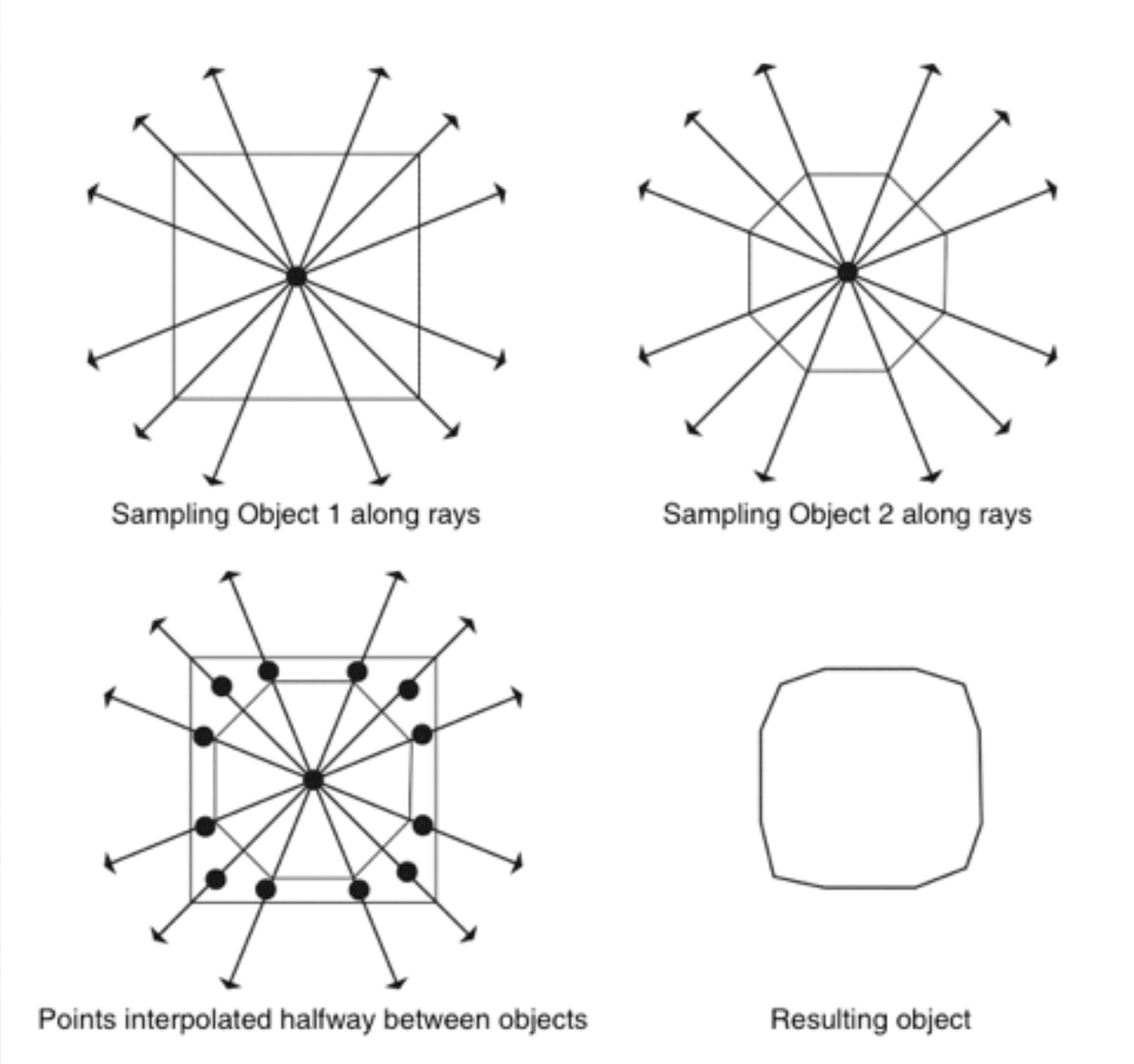
3d shape interpolation: star-shaped polyhedra



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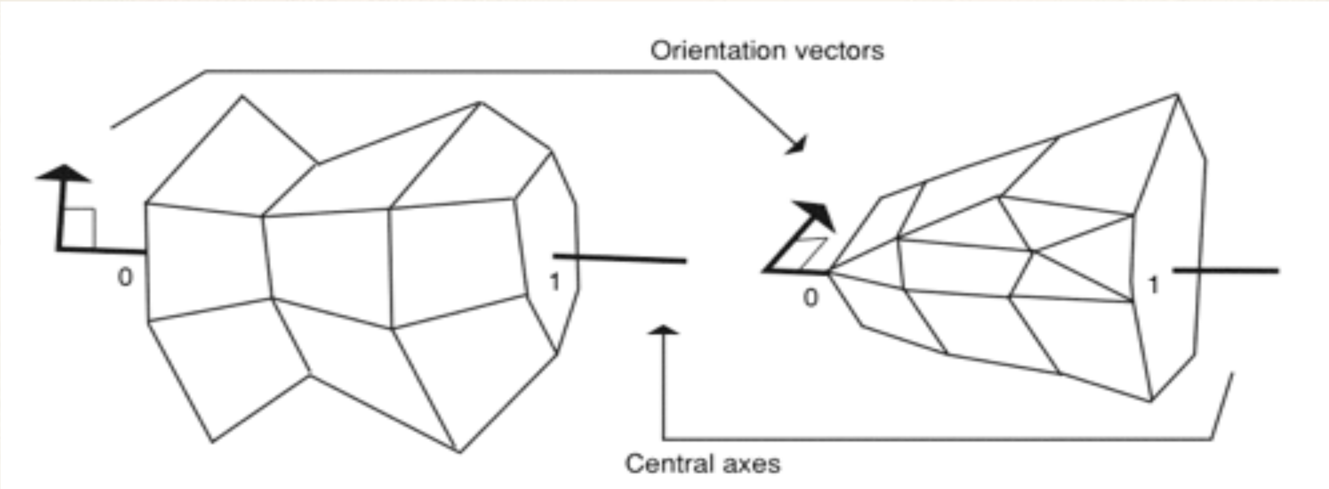
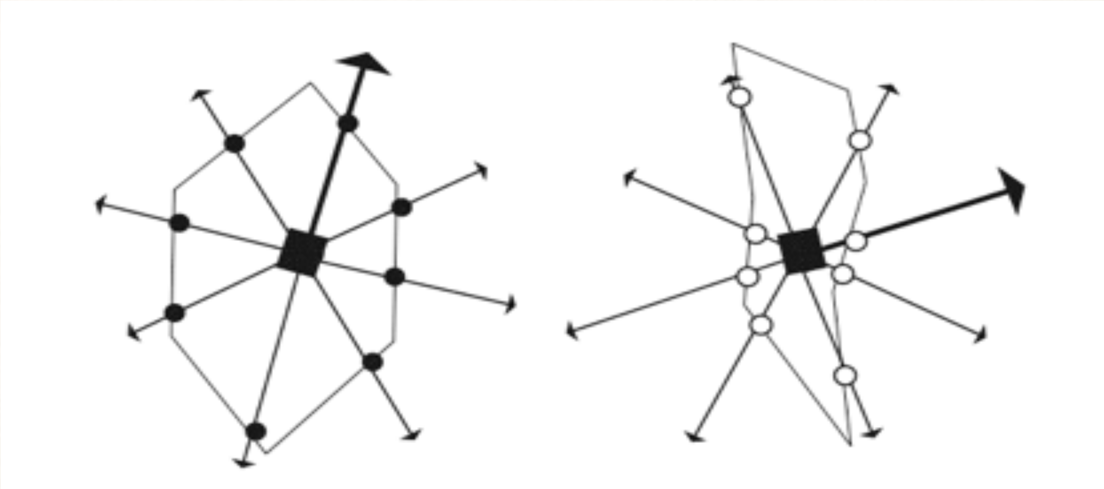
3d shape interpolation: axial slices

- Interpolate objects with respect to a central axis.
- Axis is generally user-defined
- At regular intervals along this axis, perpendicular slices are taken of the object.
- Slices must be star-shaped with respect to the point of intersection between axis and slices.
- Axes defined for both objects.
- The denser the sampling, the more accurate the approximation to the original object.





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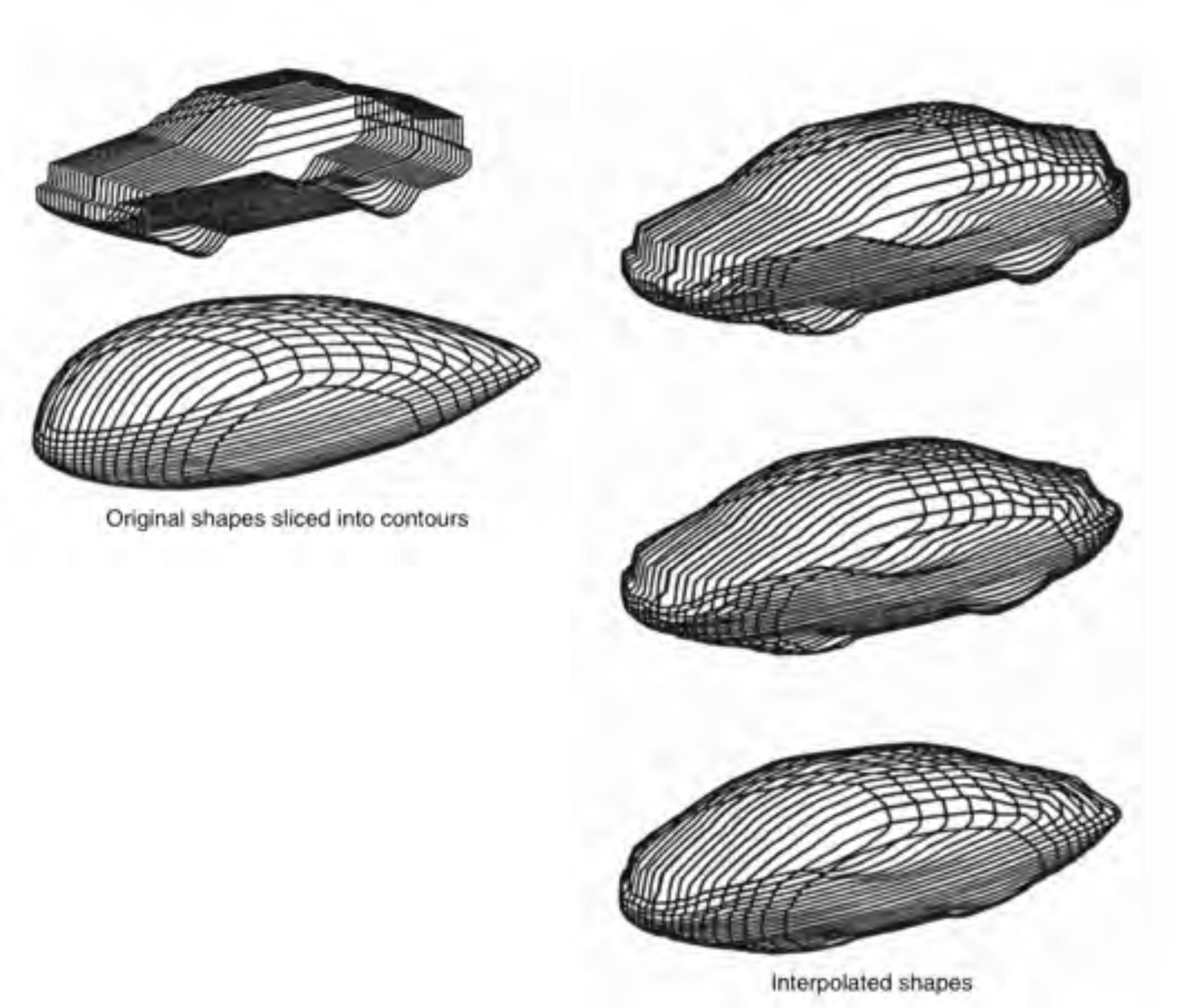
3d shape interpolation: axial slices

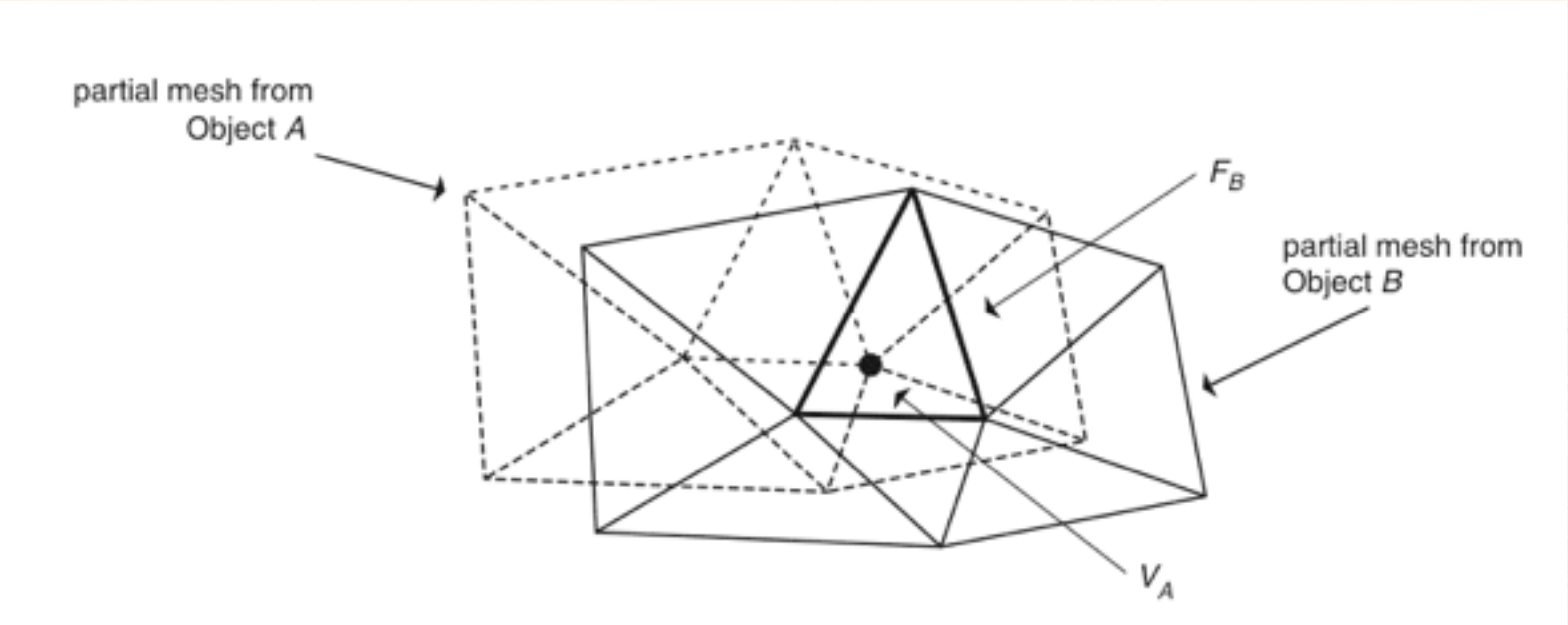


E. Chen and R. Parent. "Shape Averaging and Its Applications to Industrial Design". IEEE Computer Graphics and Applications. 9(1), January 1989. p.p.47-54

3d shape interpolation: map to sphere

- Map both objects onto a common surface such as a unit sphere.
- Spherical mapping to establish matching edge-vertex topology.
  1. Map to sphere.
  2. Intersect arc-edges.
  3. Re-triangulate.
  4. Re-map to object shapes.
  5. Vertex-to-vertex interpolation.

3d shape interpolation: map to sphere



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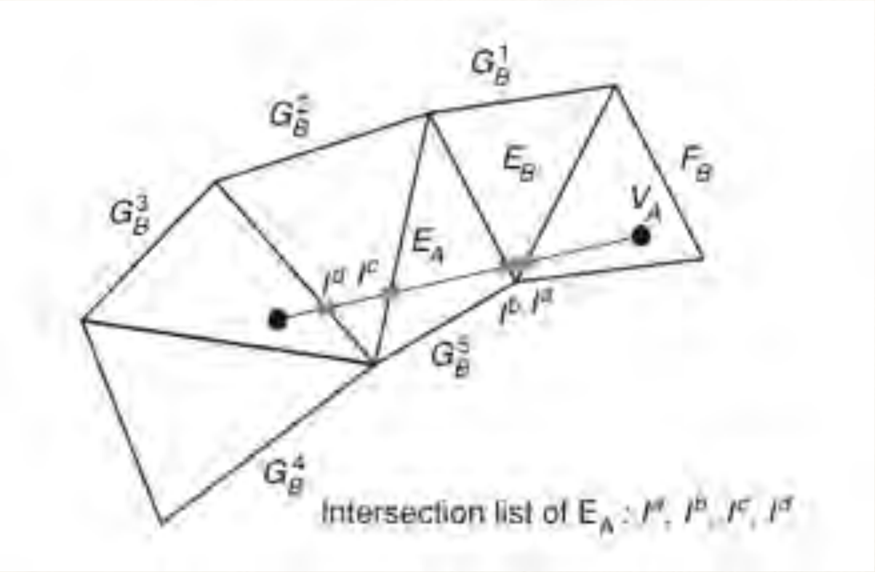
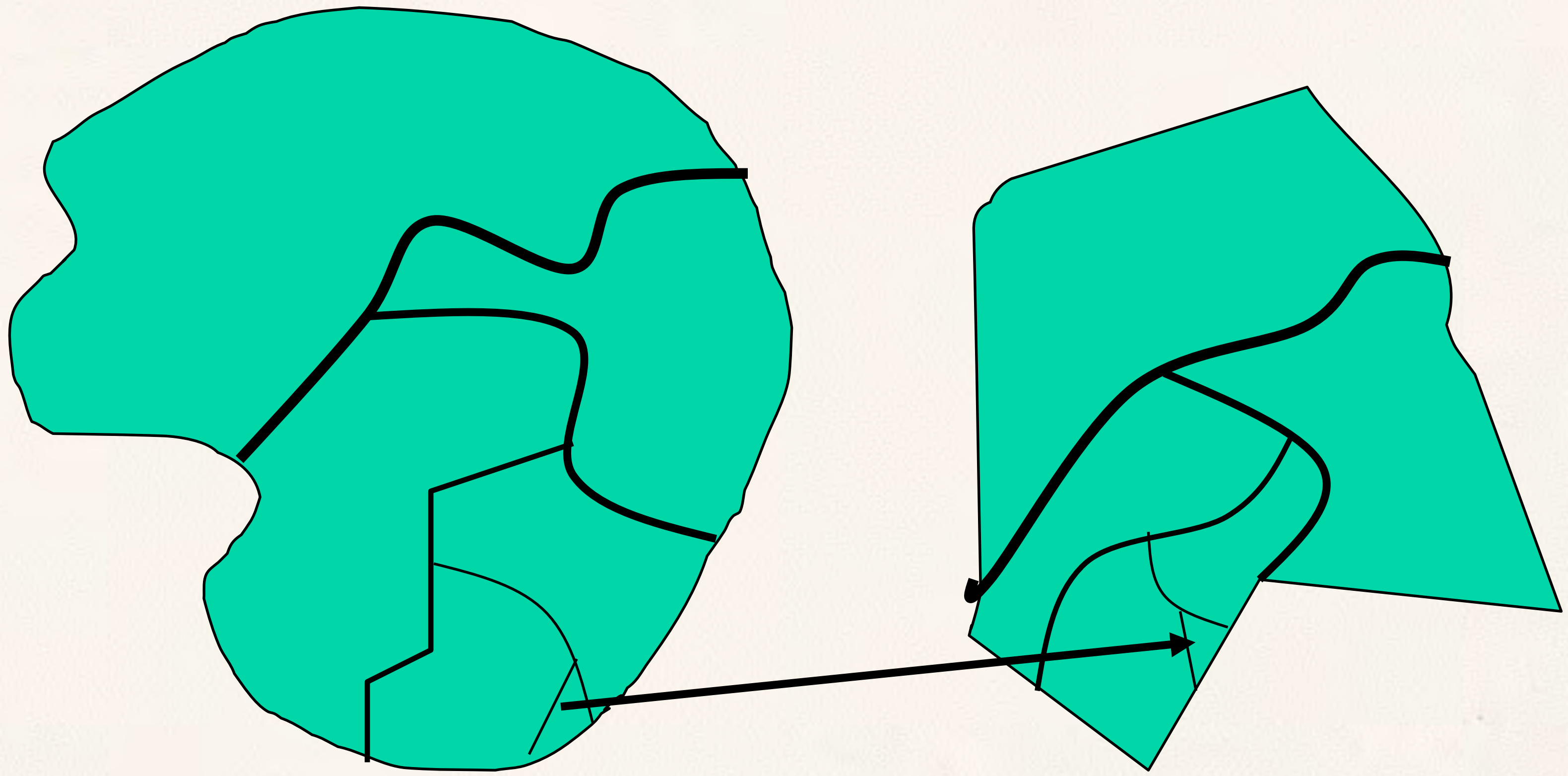


3d shape interpolation: recursive sheets



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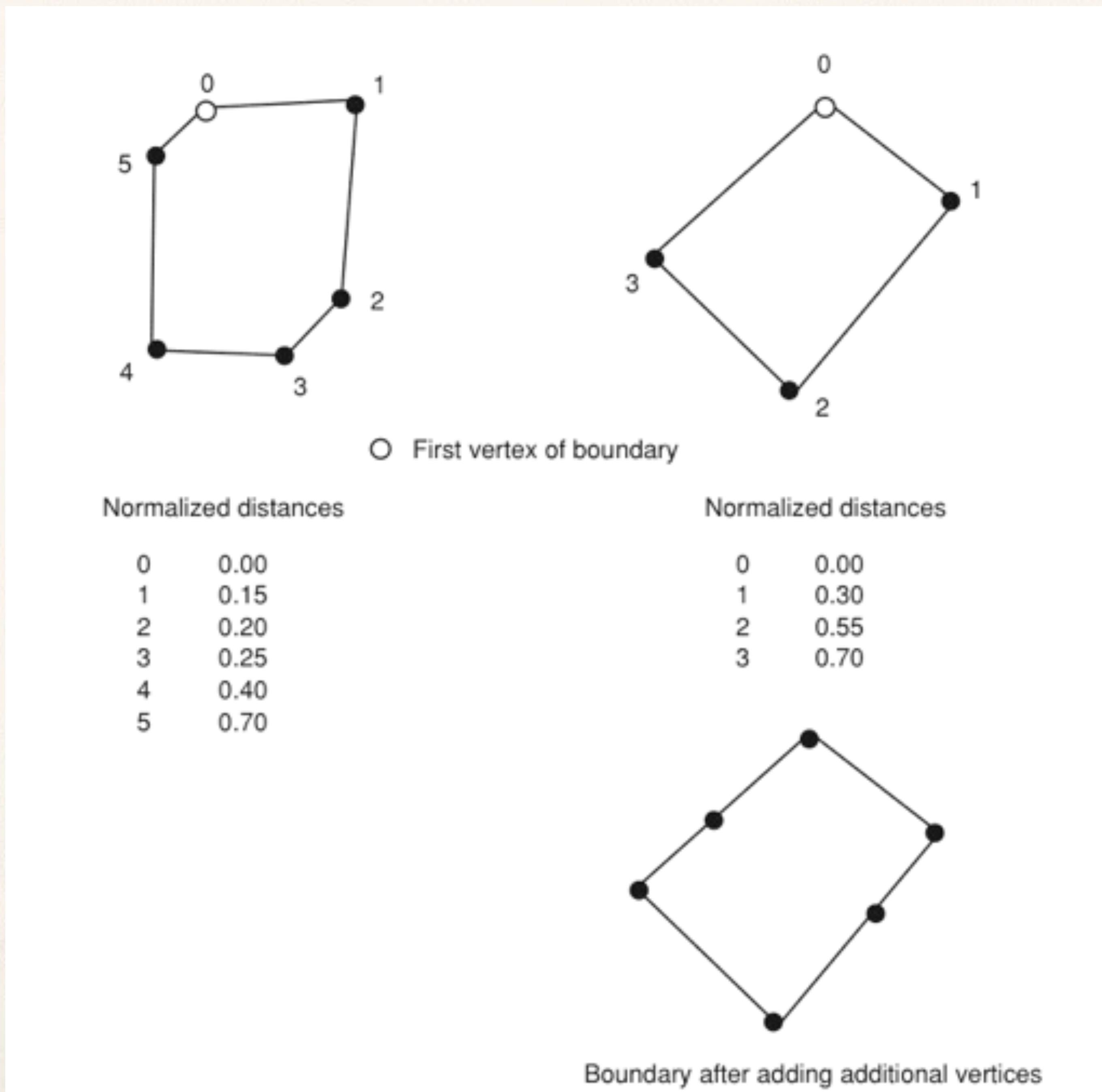
- Continually add vertices to make corresponding boundaries have an equal number.

3d shape interpolation: recursive sheets



Morphing

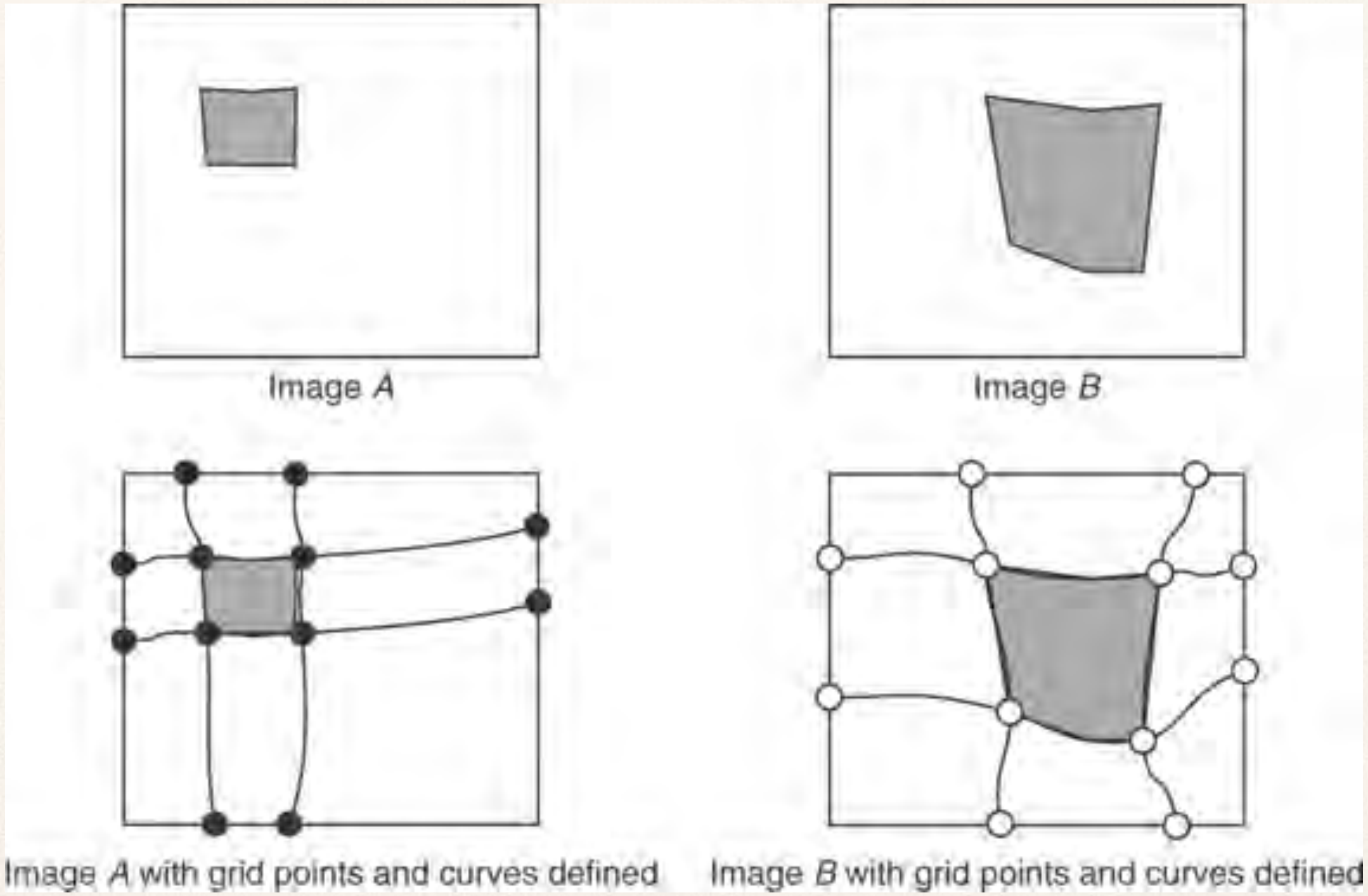
- Image blending.
- Move pixels to corresponding pixels.
- Blend colors.

- Morph from source image to destination image.
- Specify corresponding elements in the two images.
  - Coordinate grid approach.
  - Feature-based approach.

Morphing: coordinate grid approach

- User-defined curvilinear grid over each image
  - make sure corresponding elements in the images are in the corresponding cells.
  - locate the same number of grid intersection points on both images.
  - connecting curves are generated using intersection points as control points for a spline curve, e.g. Catmull-Rom spline

Morphing: coordinate grid approach



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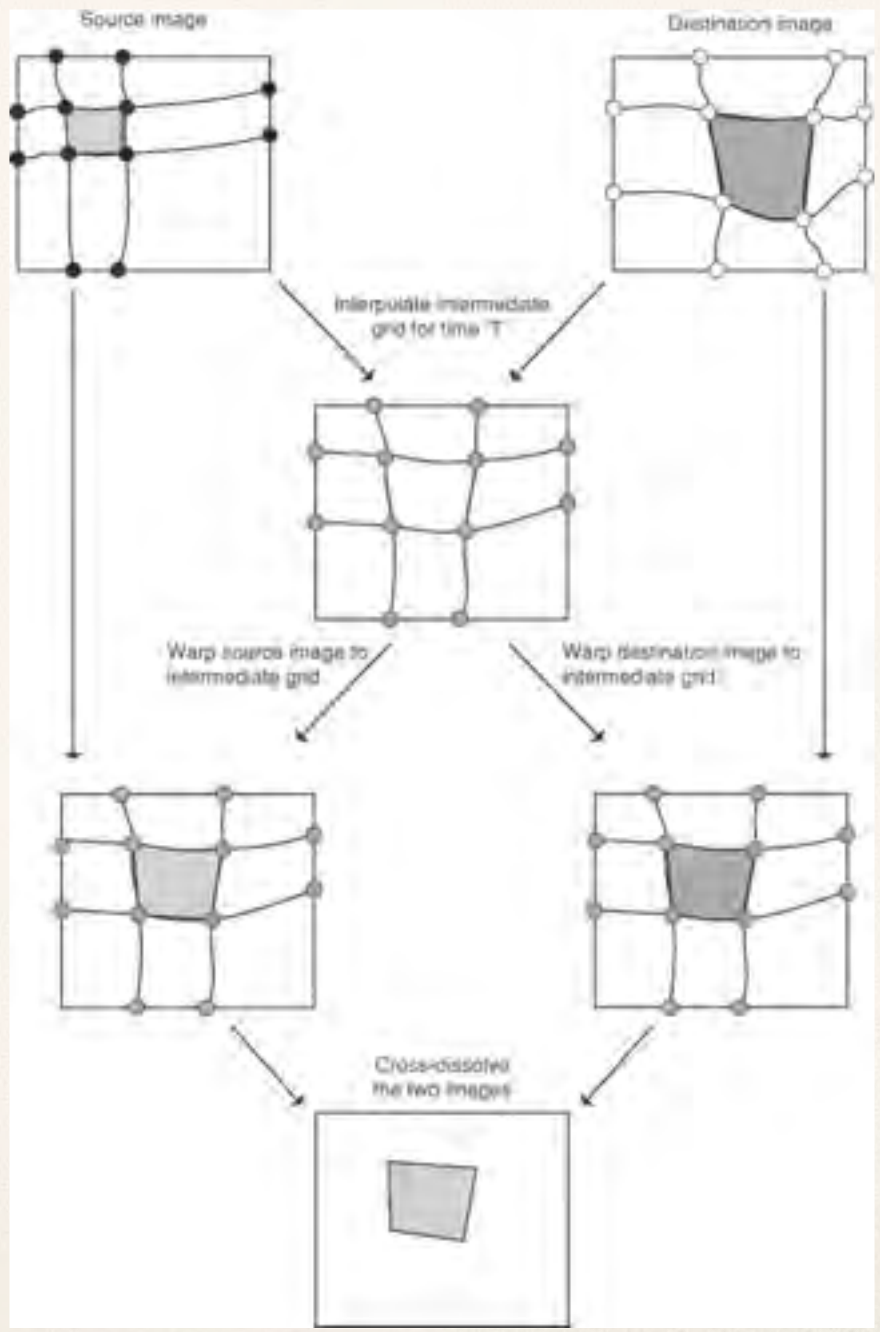
Morphing: coordinate grid approach



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