Algoritmo De Boor

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B-Spline

Un B-Spline es una curva paramétrica compuesta de una combinación lineal de B-splines base $B_{i,n}$:

$$p(t) = \sum_{i=0}^{m} P_i B_{i,k}(t)$$

P_i $(i = 0, \ldots m)$ son los puntos de control
$t_0 \leq t_i \leq \cdots \leq t_{k+m}$ subdividen el dominio en un conjunto de intervalos $[t_i, t_{i+1})$
En los splines "sujetos", la multiplicidad del primer y último nodo es 4 (repetido 4 veces)

B-Spline

Los B-splines se definen de la siguiente manera:

$$B_{i,1}(t) = \begin{cases} 1 & \text{si } t_i \leq t < t_{i+1} \\ 0 & \text{en otro caso} \end{cases}$$

$$B_{i,k}(t) = \frac{t - t_i}{t_{i+k-1} - t_i} B_{i,k-1}(t) + \frac{t_{i+k} - t}{t_{i+k-1} - t_i} B_{i+1,k-1}(t)$$

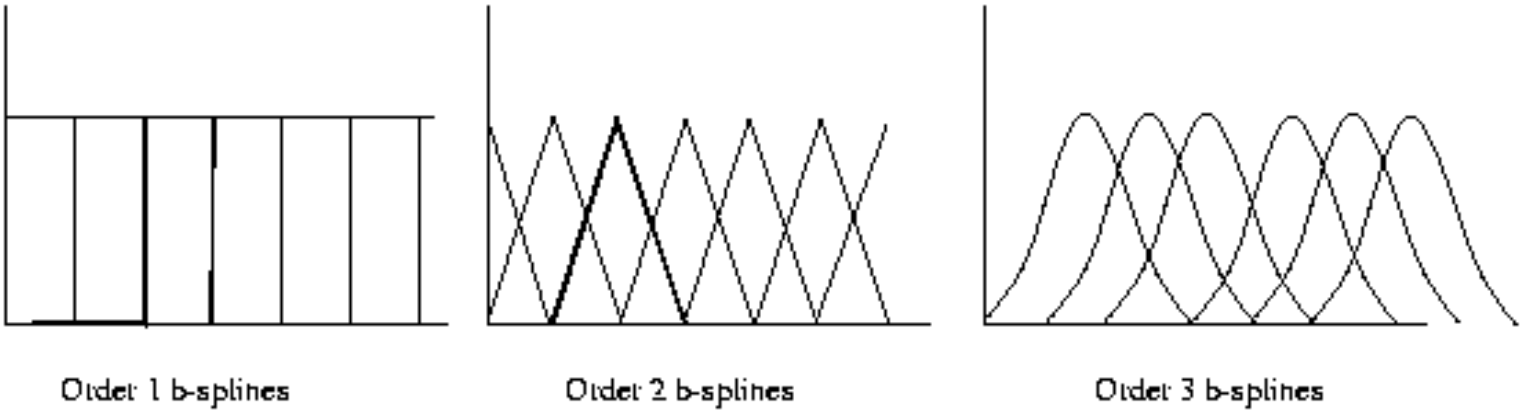


Figure: B-splines

B-Spline

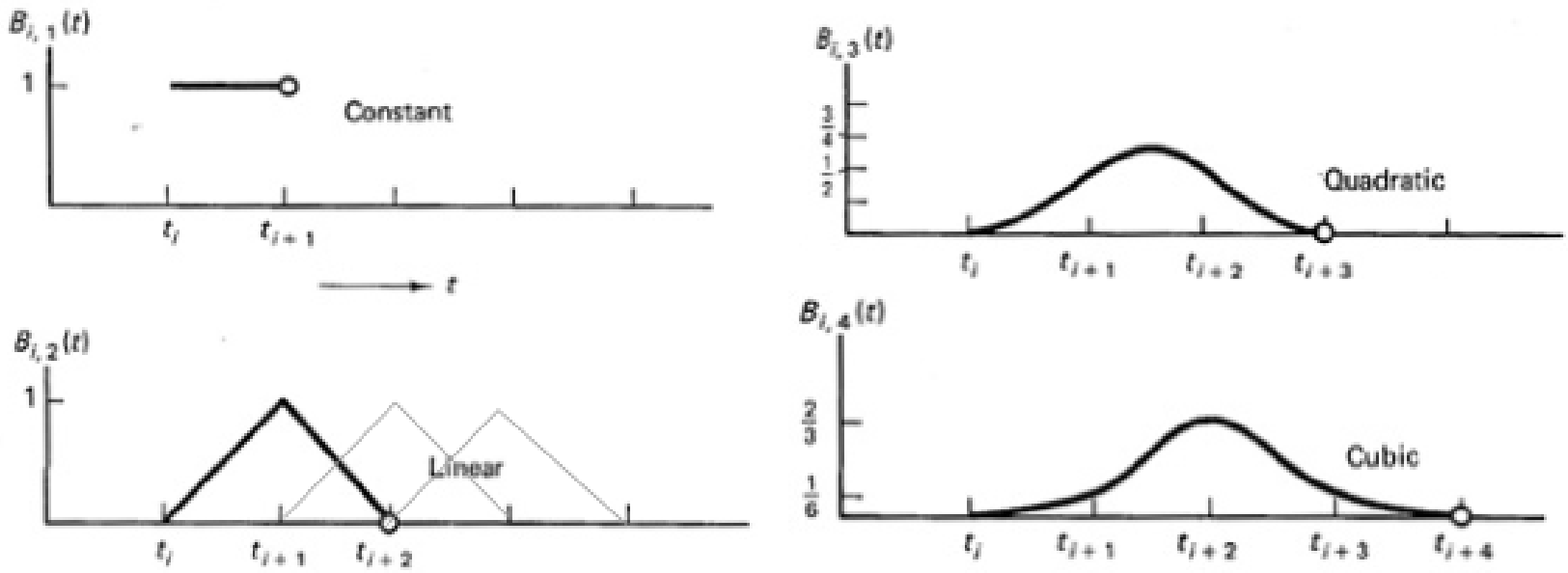


Figure: B-splines

Algoritmo De Boor

Entrada: Un valor t en $[t_k, t_{k+1})$
Salida: El punto en la curva $P(t)$

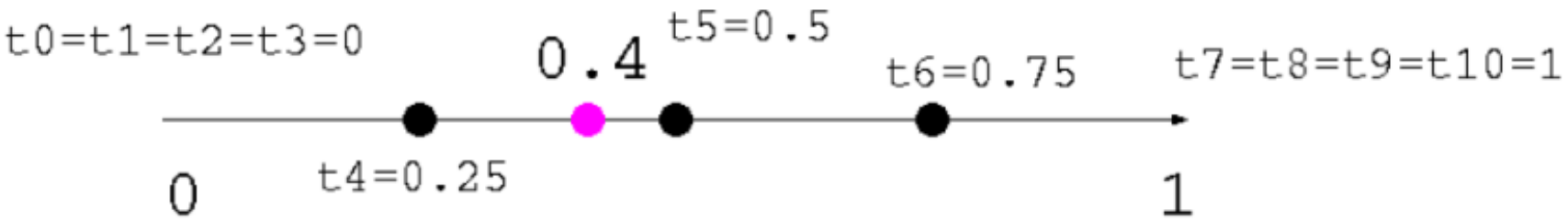
1: $P_i^0 = P_i$
2: **Para** $r = 1$ **hasta** 3 **:**
3: **Para** $i = k - 3 + r$ **hasta** k **:**
4: $a_{i,r} = \frac{t - t_i}{t_{i+k+1-r} - t_i}$
5: $P_{i,r} = (1 - a_{i,r})P_{i-1,r-1} + a_{i,r}P_{i,r-1}$
6: **Regresar** $P_{k,3}$

Ejemplo

Supongamos que tenemos los siguientes valores para t:

t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
0	0	0	0	0.25	0.50	0.75	1	1	1	1

y supongamos que queremos obtener la curva en $t = 0.4$:



Como $t_4 < t < t_5$, los puntos de control que afectan la posición final son $P1$, $P2$, $P3$, $P4$

Ejemplo

$$a_{i,r} = \frac{t - t_i}{t_{i+k+1-r} - t_i}$$

$$a_{4,1} = (t - t_4)/(t_{4+3} - t_4) = 0.2$$
$$a_{3,1} = (t - t_3)/(t_{3+3} - t_3) = 0.53$$
$$a_{2,1} = (t - t_2)/(t_{2+3} - t_2) = 0.8$$

$$P_{i,r} = (1 - a_{i,r})P_{i-1,r-1} + a_{i,r}P_{i,r-1}$$

$$P_{4,1} = (1 - a_{4,1})P_{3,0} + a_{4,1}P_{4,0} = 0.8P_{3,0} + 0.2P_{4,0}$$
$$P_{3,1} = (1 - a_{3,1})P_{2,0} + a_{3,1}P_{3,0} = 0.47P_{2,0} + 0.53P_{3,0}$$
$$P_{2,1} = (1 - a_{2,1})P_{1,0} + a_{2,1}P_{2,0} = 0.2P_{1,0} + 0.8P_{2,0}$$

Ejemplo

$$a_{i,r} = \frac{t - t_i}{t_{i+k+1-r} - t_i}$$

$$a_{4,2} = (t - t_4)/(t_{4+3-1} - t_4) = 0.3$$
$$a_{3,2} = (t - t_3)/(t_{3+3-1} - t_3) = 0.8$$

$$P_{i,r} = (1 - a_{i,r})P_{i-1,r-1} + a_{i,r}P_{i,r-1}$$

$$P_{4,2} = (1 - a_{4,2})P_{3,1} + a_{4,2}P_{4,1} = 0.7P_{3,1} + 0.3P_{4,1}$$
$$P_{3,2} = (1 - a_{3,2})P_{2,1} + a_{3,2}P_{3,1} = 0.2P_{2,1} + 0.8P_{3,1}$$

Ejemplo

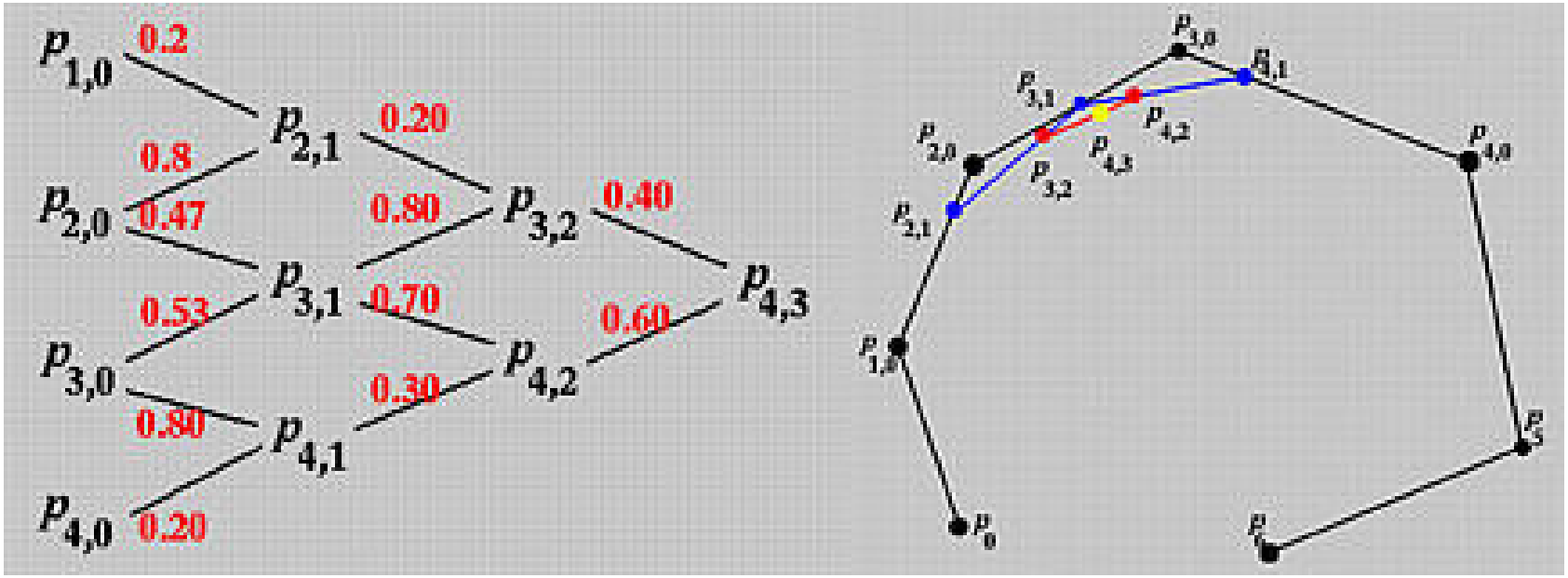
$$a_{i,r} = \frac{t - t_i}{t_{i+k+1-r} - t_i}$$

$$a_{4,3} = (t - t_4)/(t_{4+3-2} - t_4) = 0.6$$

$$P_{i,r} = (1 - a_{i,r})P_{i-1,r-1} + a_{i,r}P_{i,r-1}$$

$$P_{4,3} = (1 - a_{4,3})P_{3,2} + a_{4,3}P_{4,2} = 0.4P_{3,2} + 0.6P_{4,2}$$

Ejemplo



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