THREE GENERALIZATIONS REGARDING LIMIT SETS FOR COMPLEX KLEINIAN GROUPS

SUMMARY

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In this work, we study three problems related to the limit sets for the action of discrete subgroups of $\mathrm{PSL}(n+1,\mathbb{C})$ on the complex projective space $\mathbb{CP}^n$:

- In Chapter 2, we study the dynamics of solvable discrete subgroups of $\mathrm{PSL}(3,\mathbb{C})$. This groups present *simple* dynamics, contrary to the dynamics of irreducible groups, which have been thoroughly studied in the last few years (see [BCN11] and [CNS13]).

  We prove that solvable groups are virtually triangularizable, and therefore, we restrict our attention to triangular discrete subgroups of $\mathrm{PSL}(3,\mathbb{C})$. We do this because any finite index subgroup has the same Kulkarni limit set as the group it belongs to. With this simplification, we provide a description of the all the possible Kulkarni limit sets of solvable subgroups of $\mathrm{PSL}(3,\mathbb{C})$. Finally, we give the representations of solvable discrete subgroups of $\mathrm{PSL}(3,\mathbb{C})$.

  With this description, the full description of the Kulkarni limit set for general discrete subgroups of $\mathrm{PSL}(3,\mathbb{C})$ and their dynamics will be almost complete.

- In Chapter 3, we propose a new definition for the concept of limit set for the action of a discrete subgroup of $\mathrm{PSL}(n+1,\mathbb{C})$. In complex dimension $n=2$, the Kulkarni limit set seems to be the *right* notion of limit set. However, in complex dimension $n>2$, the Kulkarni limit set is very difficult to compute and, in general, it is bigger than it needs to be.

  The limit set we propose is based on the Cartan decomposition of a matrix and the work of C. Frances [Fra03]. This new limit set is, in general, smaller than the Kulkarni limit set. It also has the advantage of being made up of projective subspaces of the same dimension.

We prove that the action of a discrete subgroup of $\mathrm{PSL}(n+1, \mathbb{C})$ on the complement of its Frances limit set is proper and discontinuous, we also prove that this limit set is purely dimensional and unstable under deformations. We finally give several relations between this new limit set and the other definitions of limit set.

- In Chapter 4, we propose a way to generalize Patterson-Sullivan measures to the complex setting. Patterson-Sullivan measures are a family of probability measures, associated to discrete groups of $\mathrm{PSL}(2, \mathbb{C})$, supported on the limit set of the group. They give the proportion of elements of any orbit that accumulates in a given region of the limit set of the group (see [Pat76] and [Sul79]).

  We consider the Kobayashi metric on the complement of the Kulkarni limit set of an irreducible discrete subgroup of $\mathrm{PSL}(3, \mathbb{C})$. These domains are the complement of arrays of complex projective lines in general position in $\mathbb{CP}^2$. We parametrize the space of such arrays of lines and we prove that, if for some subgroup of $\mathrm{PSL}(3, \mathbb{C})$ the entropy volume of the Kobayashi metric is finite, then we can construct similar measures for that subgroup.

  We also give some concrete ideas on how to guarantee that the entropy volume of the Kobayashi metric is finite for certain groups. However, the work in this chapter is a work in progress and we show the advances made.

References

[BCN11] W. Barrera, A. Cano, and J. P. Navarrete. *The limit set of discrete subgroups of PSL$(3, \mathbb{C})$*. Math. Proc. Camb. Phil. Soc., 150:129–146, 2011.

[CNS13] A. Cano, J. P. Navarrete, and J. Seade. *Complex Kleinian Groups.* Progress in Mathematics 303. Springer Basel, 2013.

[Fra03] C. Frances. *Lorentzian kleinian groups.* Comment. Math. Helv., 80(4):883–910, 2003

[Pat76] S. J. Patterson. *The limit set of a Fuchsian group.* Acta Mathematica, 1976, Volume 136, Issue 1, pp 241-273.

[Sul79] D. Sullivan. *The density at infinity of a discrete group of hyperbolic motions.* Publ. Math. I.H.E.S. 1979, Volume 50, pp 171-202.