Chaos and period forcing for a family of piecewise monotonic real-valued functions

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In this work we consider a family of one-real parameter of piecewise monotone real-valued functions given by $f_b(x) = \wp(x) + b$, where b is a real parameter and $\wp$ denotes the Weierstrass P-function defined over a real square lattice and restricted over the real line.

Each element in the family f_b defines a periodic function over the real line with singularities at the integer multiples of its real period. When restricted to a fundamental interval, the family f_b exhibits some dynamical similarities to the quadratic family $x^2 + c$. One of the main problems addressed in this thesis is to show that under certain conditions on the parameter b and the lattice, f_b acts over the real line as a chaotic dynamical system.

The second problem considered in this work is related to Sharkovskii's Theorem, one of the most celebrated theorems in real dynamics. This theorem states a period forcing result: if f is a continuous function over the real line that has a periodic point of period n, then it must also have a periodic point of period k, with k smaller than n in the Sharkovskii ordering. Taking into account that each f_b is no longer continuous in the whole real line, we provide a partial period forcing result for the family f_b following Sharkovskii's ordering.