RRT* with Visibility Constraints for Robotic Dynamical Systems and Their Local Planners Properties

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Outline

1. **Introduction**
   - Motivation
2. **Asymptotic Convergence**
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4. **Experimental Analysis and Theoretical Insight**
   - Simulations
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5. **Mixed Cost System**
   - Problem Statement
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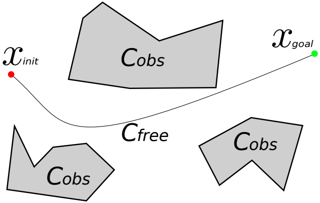
   - Definitions and Propositions
   - Simulation, Experiments and Results
6. **Conclusions**

Motion Planning

Dynamic Model of the Robot + Environment
+ +
Initial Configuration + Goal Configuration

Motion Planning Algorithms

Sequency of Commands to Run on the Robot

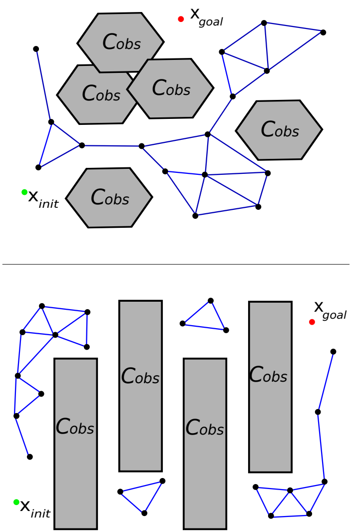


Robot Dynamic Model

Initial Configuration → **Goal Configuration**

Environment

Sampling-Based Motion Planning



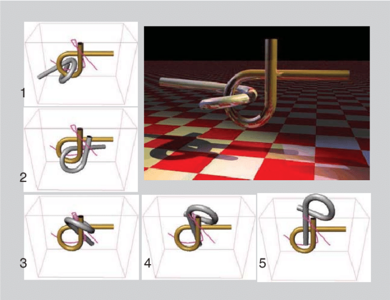
- Sampling-based algorithms represent the configuration space with a roadmap of sampled configurations.

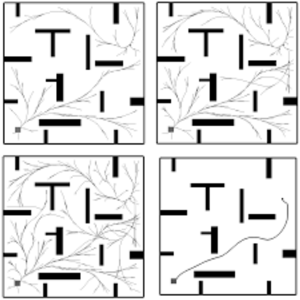
- A basic algorithm samples n configurations in C_{space}, and retains those in C_{free} to use as milestones.

- These algorithms work well for high-dimensional configuration spaces, their running time is not (explicitly) exponentially dependent on the dimension of C_{free}.

RRT-based Algorithms

Rapidly-Exploring Random Tree (RRT)

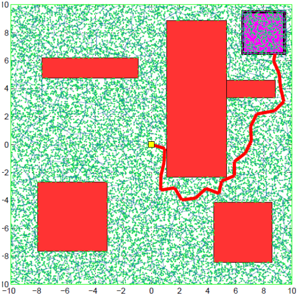




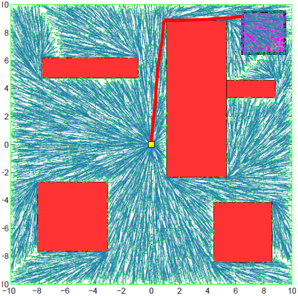
The RRT proposes the sampling of the control space U, which allows us naturally address the Kinodynamic Planning Problem.

RRT*-based Algorithms

The RRT* algorithm essentially *rewire* the tree as it discovers new lower-cost paths reaching the nodes already present in the tree.



(a)



(b)

- RRT* allows the addition of asymptotic optimality.

- Procedures such as *rewire* requires to solve a two-point boundary value problem (BVP).

Related Works

- Perez, A., Platt, R., Konidaris, G., Kaelbling, L., Lozano-Perez, T.: Lqr-rrt*: Optimal sampling-based motion planning with automatically derived extension heuristics. ICRA 2012.

- Webb, D.J., van den Berg, J.: Kinodynamic rrt*: Asymptotically optimal motion planning for robots with linear dynamics. ICRA 2013.

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Problem Definition

Let $X_{free} \subset X$ be the set of collision free states, $X_{goal} \subset X$ be the goal set, and $c : X \to \mathbb{R}_{\geq 0}$ be the cost function. The *Optimal Kinodynamic Motion Planning Problem* is defined as finding a dynamically-feasible trajectory $x : [0, T] \to X$, with $x(0) = x_0$, so the trajectory

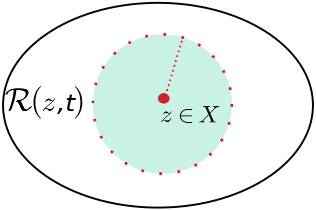
I) is collision-free, i.e. $x(t) \in X_{free}, \forall t$.

II) reaches the goal region, i.e. $x(T) \in X_{goal}$.

III) minimizes the cost functional $J(x) = \int_0^T c(x(t))dx$.

Local Controllability and Local Planner

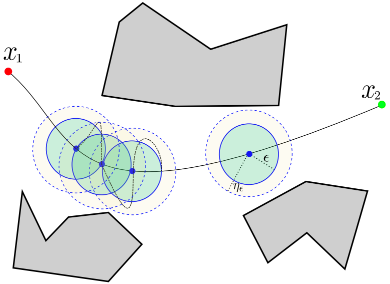
I. The considered system is Small Time Locally Controllable (STLC).

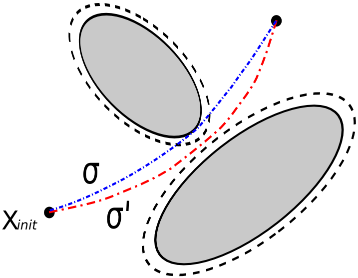


II. The local planner used in the RRT* is an optimal local planner.

Topological Property and Obstacle-Free Space

III. The local planner used in the RRT* respects the topological property.





IV. There exist an optimal path which has enough obstacle-free space around it to allow almost-sure convergence.

Research Questions

What are the necessary and sufficient conditions for the RRT* to converge in the context of kinodynamic planning?

Since the optimal local planners are hard to obtain, is this condition necessary?

Is the topological property sufficient or necessary?

Can we design an experimental setup that can offer insight regarding these questions?

Differential Drive Robot

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} v + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} w$$

Case of Study: Time-Optimal Planners for Differential Drive Robot without Obstacles

Structure of Optimal Words

Tangent	$\curvearrowright\Uparrow\curvearrowright$	$\curvearrowright\Downarrow\curvearrowright$	$\curvearrowleft\Uparrow\curvearrowleft$	$\curvearrowleft\Downarrow\curvearrowleft$
Tangent$_\pi$	$\Uparrow\curvearrowright_\pi\Downarrow$	$\Downarrow\curvearrowright_\pi\Uparrow$	$\Uparrow\curvearrowleft_\pi\Downarrow$	$\Downarrow\curvearrowleft_\pi\Uparrow$
ZigZag	$\Uparrow\curvearrowleft\Downarrow\curvearrowright\Uparrow$	$\Downarrow\curvearrowleft\Uparrow\curvearrowright\Downarrow$	$\Uparrow\curvearrowright\Downarrow\curvearrowleft\Uparrow$	$\Downarrow\curvearrowright\Uparrow\curvearrowleft\Downarrow$

	Optimal Letters	Optimal Words	Topological Property
$\mathbf{L}^+\mathbf{W}^+\mathbf{T}^+$	YES	YES	YES
$\mathbf{L}^+\mathbf{W}^*\mathbf{T}^+$	YES	Not all	YES
$\mathbf{L}^+\mathbf{W}^-\mathbf{T}^+$	YES	NO	YES
$\mathbf{L}^+\mathbf{W}^-\mathbf{T}^-$	YES	NO	NO
$\mathbf{L}^-\mathbf{W}^-\mathbf{T}^-$	NO	NO	NO

Time Optimal Planner for Differential Drive Robot

Local planner $L^+W^+T^+$ (Balkcom and Mason - 2002)

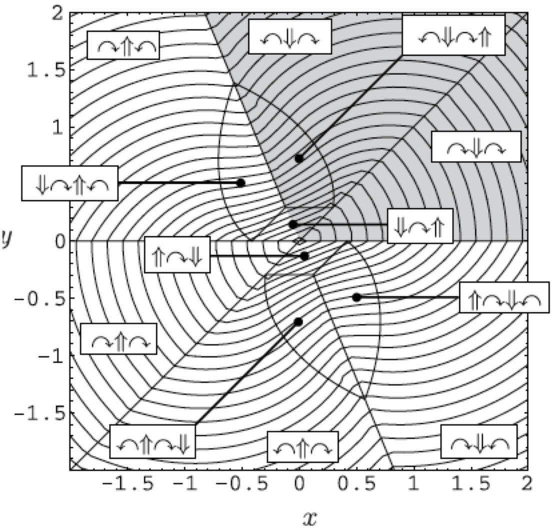
Structure of Optimal Words

Tangent	$\curvearrowright\Uparrow\curvearrowright$	$\curvearrowright\Downarrow\curvearrowright$	$\curvearrowleft\Uparrow\curvearrowleft$	$\curvearrowleft\Downarrow\curvearrowleft$
Tangent$_\pi$	$\Uparrow\curvearrowright_\pi\Downarrow$	$\Downarrow\curvearrowright_\pi\Uparrow$	$\Uparrow\curvearrowleft_\pi\Downarrow$	$\Downarrow\curvearrowleft_\pi\Uparrow$
ZigZag	$\Uparrow\curvearrowleft\Downarrow\curvearrowright\Uparrow$	$\Downarrow\curvearrowleft\Uparrow\curvearrowright\Downarrow$	$\Uparrow\curvearrowright\Downarrow\curvearrowleft\Uparrow$	$\Downarrow\curvearrowright\Uparrow\curvearrowleft\Downarrow$

	Optimal Letters	Optimal Words	Topological Property
$\mathbf{L^+W^+T^+}$	YES	YES	YES
$\mathbf{L^+W^*T^+}$	YES	Not all	YES
$\mathbf{L^+W^-T^+}$	YES	NO	YES
$\mathbf{L^+W^-T^-}$	YES	NO	NO
$\mathbf{L^-W^-T^-}$	NO	NO	NO

Sintesis

Balkcom and Mason - 2002



Time Optimal Planner for Differential Drive Robot

Local planner $L^{+}W^{*}T^{+}$ (Subset of Balkcom and Mason)

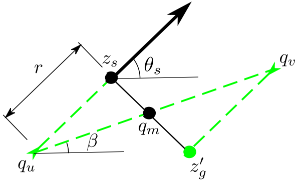
Subset of Optimal Words

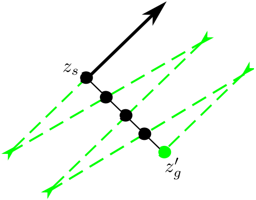
Tangent	$\curvearrowright\Uparrow\curvearrowright$	$\curvearrowright\Downarrow\curvearrowright$	$\curvearrowleft\Uparrow\curvearrowleft$	$\curvearrowleft\Downarrow\curvearrowleft$
Tangent$_{\pi}$	$\Uparrow\curvearrowright_{\pi}\Downarrow$	$\Downarrow\curvearrowright_{\pi}\Uparrow$	$\Uparrow\curvearrowleft_{\pi}\Downarrow$	$\Downarrow\curvearrowleft_{\pi}\Uparrow$

	Optimal Letters	Optimal Words	Topological Property
$L^{+}W^{+}T^{+}$	YES	YES	YES
$L^{+}W^{*}T^{+}$	YES	Not all	YES
$L^{+}W^{-}T^{+}$	YES	NO	YES
$L^{+}W^{-}T^{-}$	YES	NO	NO
$L^{-}W^{-}T^{-}$	NO	NO	NO

Time Optimal Planner for Differential Drive Robot

Local planner $\mathbf{L}^{+}\mathbf{W}^{-}\mathbf{T}^{+}$





	Optimal Letters	Optimal Words	Topological Property
$\mathbf{L}^{+}\mathbf{W}^{+}\mathbf{T}^{+}$	YES	YES	YES
$\mathbf{L}^{+}\mathbf{W}*\mathbf{T}^{+}$	YES	Not all	YES
$\mathbf{L}^{+}\mathbf{W}^{-}\mathbf{T}^{+}$	YES	NO	YES
$\mathbf{L}^{+}\mathbf{W}^{-}\mathbf{T}^{-}$	YES	NO	NO
$\mathbf{L}^{-}\mathbf{W}^{-}\mathbf{T}^{-}$	NO	NO	NO

Time Optimal Planner for Differential Drive Robot

Local planner $L^{+}W^{-}T^{-}$

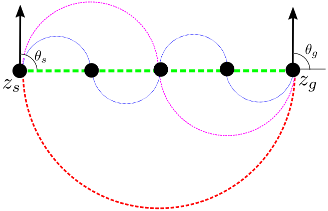
Rotation in site + Straight line + Rotation in site

$(\curvearrowleft / \curvearrowright)$ $(\Uparrow / \Downarrow)$ $(\curvearrowleft / \curvearrowright)$

	Optimal Letters	Optimal Words	Topological Property
$L^{+}W^{+}T^{+}$	YES	YES	YES
$L^{+}W^{*}T^{+}$	YES	Not all	YES
$L^{+}W^{-}T^{+}$	YES	NO	YES
$L^{+}W^{-}T^{-}$	YES	NO	NO
$L^{-}W^{-}T^{-}$	NO	NO	NO

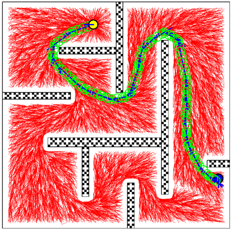
Time Optimal Planner for Differential Drive Robot

Local planner $L^{-}W^{-}T^{-}$

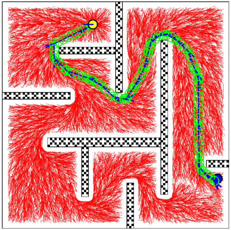


	Optimal **L**etters	Optimal **W**ords	**T**opological Property
$L^{+}W^{+}T^{+}$	YES	YES	YES
$L^{+}W^{*}T^{+}$	YES	Not all	YES
$L^{+}W^{-}T^{+}$	YES	NO	YES
$L^{+}W^{-}T^{-}$	YES	NO	NO
$L^{-}W^{-}T^{-}$	NO	NO	NO

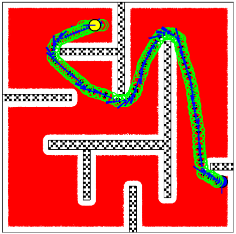
Experiment Set (Environment 1)



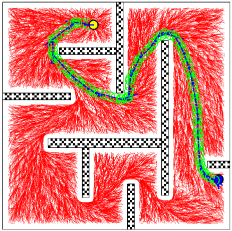
$L^+W^+T^+$



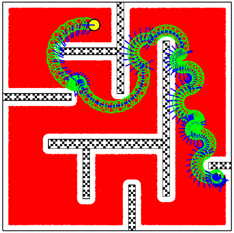
$L^+W^*T^+$



$L^+W^-T^+$

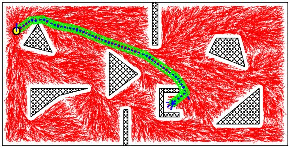


$L^+W^-T^-$

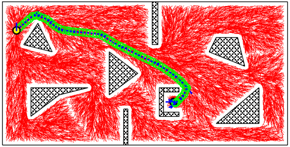


$L^-W^-T^-$

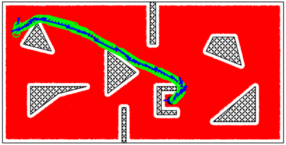
Experiment Set (Environment 2)



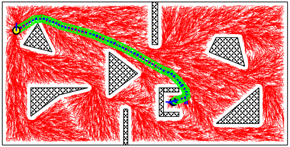
$\mathbf{L}^+\mathbf{W}^+\mathbf{T}^+$



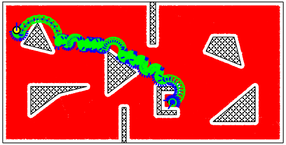
$\mathbf{L}^+\mathbf{W}^*\mathbf{T}^+$



$\mathbf{L}^+\mathbf{W}^-\mathbf{T}^+$

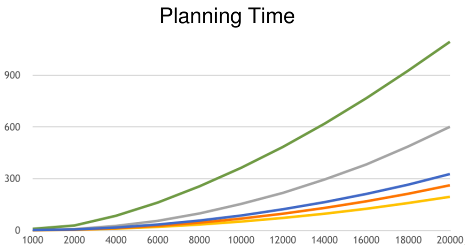


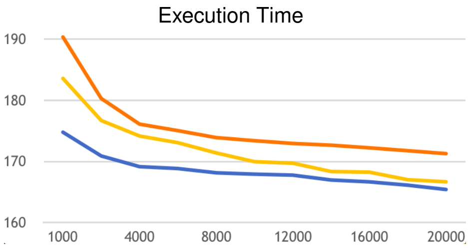
$\mathbf{L}^+\mathbf{W}^-\mathbf{T}^-$



$\mathbf{L}^-\mathbf{W}^-\mathbf{T}^-$

Results





$\mathbf{L^+W^+T^+}$ —

$\mathbf{L^+W{*}T^+}$ —

$\mathbf{L^+W^-T^+}$ —

$\mathbf{L^+W^-T^-}$ —

$\mathbf{L^-W^-T^-}$ —

Theoretical Insight

	Optimal **L**etters	Optimal **W**ords	Local Optimal **W**ords	Subset Optimal **W**ords	**T**opological Property
Necessary	YES*	NO	NO	YES*	NO
Sufficient	YES*	YES	YES	YES*	YES

Theoretical Insight

Lemma - The RRT* algorithm with local planner $\mathbf{L}^{+}\mathbf{W}^{-}\mathbf{T}^{-}$ is asymptotically optimal in the context of time-optimal trajectories for a DDR in the presence of obstacles.

Theorem - The *Optimal Words Property* on a local planner is not a necessary condition.

Theorem - The *Local Optimal Words Property* on a local planner is not a necessary condition

Theorem - The *Topological Property* on a local planner is not a necessary condition, but it is part of a set of sufficient conditions

Keeping an Object in View with a Manipulator Arm with a Camera in Hand on the Top of a DDR

Keeping an Object in View with a Manipulator Arm with a Camera in Hand on the Top of a DDR

Problem Definition

Solve the *Optimal Kinodynamic Motion Planning Problem* with a trajectory x, s.t. $\mathrm{x}(t) \in X_{FV}$ $\forall t \in [0, T]$, considering the dynamical system presented in the next equation

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \{\dot{\alpha}_i\} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\cos\theta & \frac{1}{2}\cos\theta & 0_{1\times N} \\ \frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta & 0_{1\times N} \\ \frac{-1}{2b} & \frac{1}{2b} & 0_{1\times N} \\ 0_{N\times 1} & 0_{N\times 1} & \mathrm{I}_{N\times N} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \{v_i\} \end{pmatrix}$$

and J as the cost functional given in the next equation

$$J(\mathrm{x}) = \int_0^T c_{\mathcal{T}}(\mathrm{x}(t))dt + \sum_{i=1}^{M} c_\gamma(\mathrm{x}_i) + \sum_{i=1}^{M} c_c(\mathrm{x}_i).$$

Cost Functional for a Manipulator Arm with a Camera in Hand on the Top of a DDR

$$J(\mathrm{x}) = \int_0^T c_{\mathcal{T}}(\mathrm{x}(t))dt + \sum_{i=1}^M c_\gamma(\mathrm{x}_i) + \sum_{i=1}^M c_c(\mathrm{x}_i)$$

- $c_{\mathcal{T}}(\mathrm{x}(t)) \in \mathbb{R}_{\geq 0}$: Time related to the DDR base to traverse a trajectory x.

The DDR sets the pace and the manipulator can keep up with it. For a given goal state $[x' \quad y' \quad \theta' \quad \{\alpha'_i\}]^{\mathbf{T}}$, the manipulator will reach its respective state at least as fast as the DDR:

$[x(t) \quad y(t) \quad \theta(t) \quad \{\alpha_i(\tau)\}]^{\mathbf{T}} = [x' \quad y' \quad \theta' \quad \{\alpha'_i\}]^{\mathbf{T}}$, with $\tau \leq t$.

This will be referred to as *dominance in time* of the DDR base over the manipulator.

Cost Functional for a Manipulator Arm with a Camera in Hand on the Top of a DDR

$$J(\mathrm{x}) = \int_0^T c_{\mathcal{T}}(\mathrm{x}(t))dt + \sum_{i=1}^{M} c_{\gamma}(\mathrm{x}_i) + \sum_{i=1}^{M} c_c(\mathrm{x}_i)$$

- $c_{\mathcal{T}}(\mathrm{x}(t)) \in \mathbb{R}_{\geq 0}$: Time related to the DDR base.

- $c_{\gamma}(\mathrm{x}(t)) \in [0, \pi]$: Deviation of the Roll of the camera considering the scene vertical.

- $c_c(\mathrm{x}(t)) \in \mathbb{R}_{\geq 0}$: Distance from the projection of the center of mass of a reference object into the image plane to the center of the image.

Detachability

Definition Detachability Property:

Consider a cost functional $J(\mathrm{x}_{i,i+1}) = \sum_{k=1}^{N} C_k(\mathrm{x}_{i,i+1})$. Summands $C_k(\mathrm{x}_{i,i+1})$ and $C_{k'}(\mathrm{x}_{i,i+1})$ are said to be *detachable*, if $C_k(\mathrm{x}_{i,i+1}) = \int_{t_i}^{t_{i+1}} c_k(\mathrm{x}_{i,i+1}(t))dt$ and $C_{k'}(\mathrm{x}_{i,i+1}) = c_{k'}(\mathrm{x}_{\mathrm{i+1}})$.

J is said to **respect** the *detachability property*, if it contains at least a pair of terms $C_k(\mathrm{x}_{i,i+1})$ and $C_{k'}(\mathrm{x}_{i,i+1})$.

Propositions

Proposition -

Under the consideration of time dominance of the DDR base over the manipulator arm, the cost functional J_r from the equation $J_r(\mathrm{x}) = \int_0^T [c_{\mathcal{T}}(\mathrm{x}(t)) + c_\gamma(\mathrm{x}(t)) + c_c(\mathrm{x}(t))]dt$, is approximated up to a resolution by J in the equation $J(\mathrm{x}) = \int_0^T c_{\mathcal{T}}(\mathrm{x}(t))dt + \sum_{i=1}^{M} c_\gamma(\mathrm{x}_i) + \sum_{i=1}^{M} c_c(\mathrm{x}_i)$

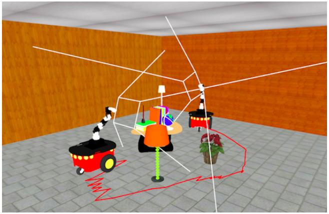
Remark -

The quality of this approximation depends on step size considered in the RRT* construction, the smaller the step size the better the approximation.

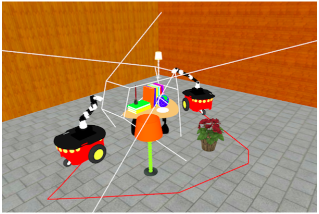
Proposition -

Consider cost functional J given by by the equation
$J(\mathrm{x}) = \int_0^T c_{\mathcal{T}}(\mathrm{x}(t))dt + \sum_{i=1}^{M} c_\gamma(\mathrm{x}_i) + \sum_{i=1}^{M} c_c(\mathrm{x}_i)$. Such cost functional does respect the detachability property.

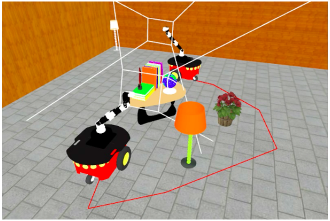
Simulations: Comparison of different implementations



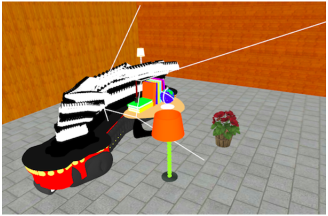
Imp-A.



Imp-C. Homotopy Class 1



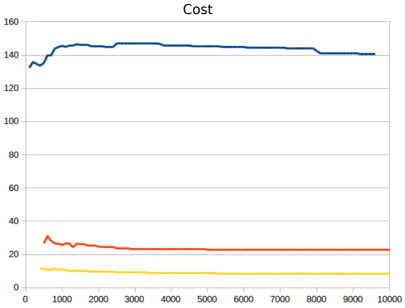
Imp-B.

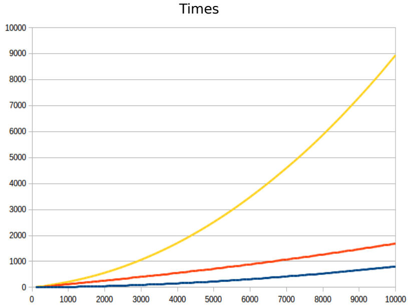


Imp-C. Homotopy Class 2

Simulations

Performance Review





Imp-A —

Imp-B —

Imp-C —

Experiments in a real robot

Experiments

Analysis and Conclusions

The methodology of reusing simpler local planners within general complex systems, may be used for other dynamical systems. The steps to follow are the next ones:

1. Start from a single simple system for which an optimal local planner is available for a running cost term.
2. Add new subsystems to the original simpler system, yielding a more complex system, and design a cost functional that has the form of a sum of terms.
3. For the terms related to the newly added subsystems, design cost terms that are detachable to the running cost for the original subsystem.
4. Reuse the optimal local planner for the original simple system in the context of the more complex system.

Analysis and Conclusions

- We present an experimental setup that allowed us to obtain theoretical insight on what are the sufficient and necessary conditions for the RRT* to be asymptotically optimal in a kinodynamic planning context.

- A local planner with **optimal words** is not necessary.

- We show that planner with **optimal letters** can obtain global asymptotic optimality, which alleviates the lack of the synthesis in optimal control problems, in the context of time-optimal trajectories.

Analysis and Conclusions

- The topological property is not necessary but it is part of a set of sufficient conditions.

- The problem of achieving asymptotical global optimality is different than approximating a geometric path by paths computed by a local planner, respecting the nonholonomic constraints.

- Formal results about the local planners were presented.

- A complex robotic system was contemplated, namely, a mobile manipulator robot including visibility constraints in the planning requirements.

Analysis and Conclusions

- Relevant concepts for complex robotic systems such as detachability and time dominance were introduced.

- Experiments in a physical robot that validate the theoretical modeling were presented.

Future Work

- Second order dynamics
- Other dynamical systems
  - Drones
  - Applications to self-driving cars

Publications

- I. Becerra, H. Yervilla-Herrera and R. Murrieta-Cid, An Experimental Analysis on the Necessary and Sufficient Conditions for the RRT* Applied to Dynamical Systems, 13th International Workshop on the Algorithmic Foundations of Robotics, WAFR 2018, Mérida México, 2018
- I. Becerra, H. Yervilla-Herrera, E. Cuevas and R. Murrieta-Cid, RRT* with Visibility Constraints for Robotic Dynamical Systems and their Local Planners Properties, Submitted to IEEE Transactions on Robotics. In second review, 2019.

Thanks

Questions?

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- Planning algorithms, Steven M. LaValle, Cambridge, 2006.
- Robot motion planning and control, Jean-Paul Laumond, Springer, 1998.