# Search and Pursuit-Evasion with Robots

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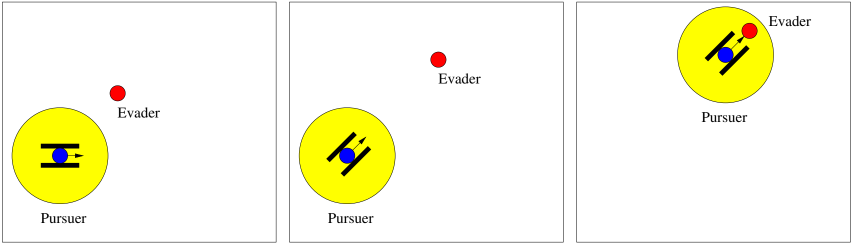
# Outline


1. Introduction
2. Time-Optimal Motion Strategies for Capturing an Omnidirectional Evader using a Differential Drive Robot
3. Maintaining Visibility of an Evader in an Environment with Obstacles
4. Object Detection
5. Conclusions and Future Work

# Previous work

**Target capturing in an environment without obstacles**

- R. Isaacs. Differential Games: A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization. John Wiley and Sons. Inc., 1965.
- Y.C. Ho et. al., Differential Games and Optimal Pursuit-Evasion Strategies, IEEE Transactions on Automatic Control, 1965.




(a) (b) (c)

**Figure:** Target capturing.

# Previous work

**Target tracking in an environment with obstacles**

- S. M. LaValle et. al., "Motion strategies for maintaining visibility of a moving target", in Proc. IEEE Int. Conf. on Robotics and Automation, 1997.
- H.H. González-Baños et. al., Motion Strategies for Maintaining Visibility of a Moving Target In Proc IEEE Int. Conf. on Robotics and Automation, 2002.
- R. Murrieta-Cid et. al., Surveillance Strategies for a Pursuer with Finite Sensor Range, International Journal on Robotics Research, Vol. 26, No 3, pages 233-253, March 2007.
- S. Bhattacharya and S. Hutchinson , On the Existence of Nash Equilibrium for a Two Player Pursuit-Evasion Game with Visibility Constraints, The International Journal of Robotics Research, December, 2009.

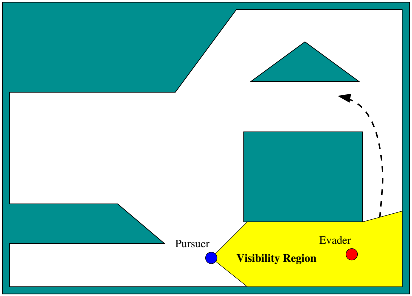
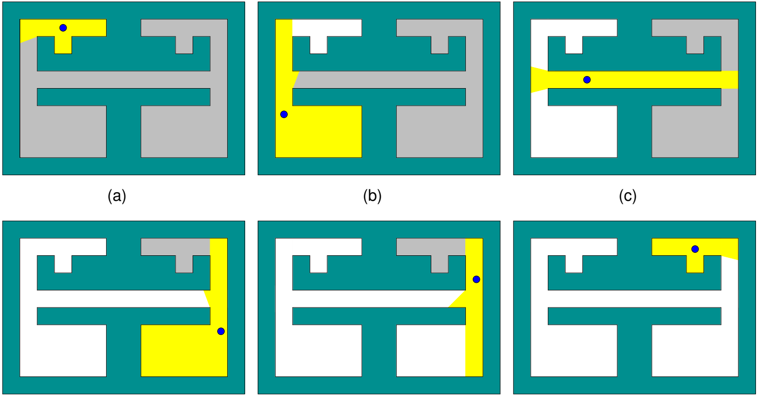



**Figure:** Target tracking.

# Previous work

**Target finding in an environment with obstacles**

- V. Isler et. al., “Randomized pursuit-evasion in a polygonal enviroment”, IEEE Transactions on Robotics, vol. 5, no. 21, pp. 864-875, 2005.
- R. Vidal et. al., “Probabilistic pursuit-evasion games: Theory, implementation, and experimental evaluation”, IEEE Transactions on Robotics and Automation, vol. 18, no. 5, pp. 662-669, 2002.

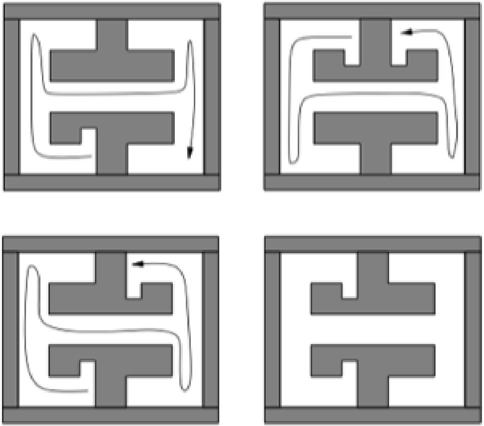


(a) (b) (c) (d) (e) (f)

**Figure:** Target finding.

# Target finding in an environment with obstacles

- L. Guibas, J.-C. Latombe, S. LaValle, D. Lin and R. Motwani, "Visibility-based pursuit-evasion in a polygonal environment", International Journal of Computational Geometry and Applications, vol. 9, no. 4/5, pp. 471-494, 1999.



# Possible Applications

- Transportation of items in airports or supermarkets.

# Possible Applications

- Monitoring and surveillance.

# Possible Applications

- Convoys of vehicles and assisted driving.

**Time-Optimal Motion Strategies for Capturing an Omnidirectional Evader using a Differential Drive Robot**

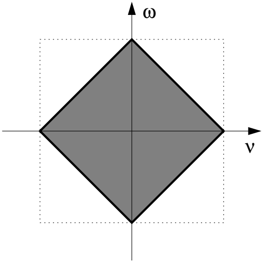
# Time-Optimal Motion Strategies for Capturing an Omnidirectional Evader using a Differential Drive Robot

## Problem formulation

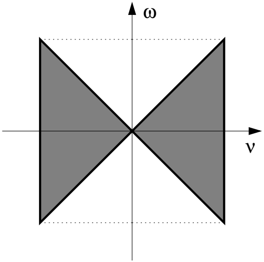
- A Differential Drive Robot (DDR) and an omnidirectional evader move on a plane without obstacles.
- The game is over when the distance between the DDR and the evader is smaller than a critical value $l$.
- Both players have maximum bounded speeds $V_p^{\max}$ and $V_e^{\max}$, respectively. The DDR is faster than the evader, $V_p^{\max} > V_e^{\max}$.
- The DDR wants to minimize the capture time $t_f$ while the evader wants to maximize it.
- We want to know the time-optimal motion strategies of the players that are in Nash Equilibrium.

# The Homicidal Chauffeur Problem

- A driver wants to run over a pedestrian in a parking lot without obstacles.
- The pursuer is a vehicle with a minimal turning radius (car-like).
- The question to be solved is: under what circumstances, and with what strategy, can the driver of the car guarantee that he can always catch the pedestrian, or the pedestrian guarantee that he can indefinitely elude the car?




(a) DDR



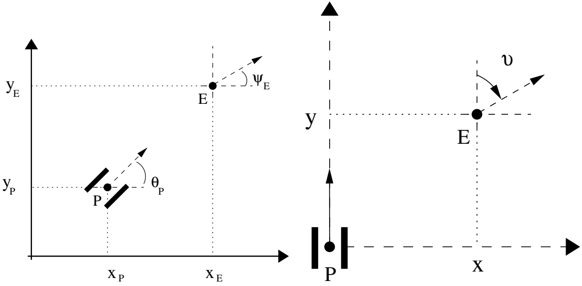

(b) Car-like

**Figure:** Control domains

# Model

## Reduced space

The problem can be stated in a coordinate system that is fixed to the body of the DDR. The state of the system is expressed as $\mathbf{x}(t) = (x(t), y(t)) \in \mathbb{R}^2$.




(a) Realistic space (b) Reduced space

# Model

The evolution of the system in the DDR-fixed coordinate system is described by the following equations of motion

$$\begin{aligned} \dot{x}(t) &= \left(\frac{u_2(t) - u_1(t)}{2b}\right) y(t) + v_1(t) \sin v_2(t) \\ \dot{y}(t) &= -\left(\frac{u_2(t) - u_1(t)}{2b}\right) x(t) - \left(\frac{u_1(t) + u_2(t)}{2}\right) + v_1(t) \cos v_2(t) \end{aligned} \tag{1}$$

This set of equations can be expressed in the form $\dot{\mathbf{x}} = f(t, \mathbf{x}(t), u(t), v(t))$, where $u(t) = (u_1(t), u_2(t)) \in \widehat{U} = [-V_p^{\max}, V_p^{\max}] \times [-V_p^{\max}, V_p^{\max}]$ and $v(t) = (v_1(t), v_2(t)) \in \widehat{V} = [0, V_e^{\max}] \times [0, 2\pi)$.

# Preliminaries

**Payoff**

A standard representation [Isaacs65, Basar95] of the payoff is

$$J(\mathbf{x}(t_s), u, v) \quad = \quad \int_{t_s}^{t_f} \underbrace{L(\mathbf{x}(\bar{t}), u(\bar{t}), v(\bar{t}))}_{\text{running cost}} \, d\bar{t} \quad + \quad \underbrace{G(\mathbf{x}(t_f))}_{\text{terminal cost}}$$

For problems of *minimum time* [Isaacs65, Basar95], as in this game, $L(\mathbf{x}(t), u(t), v(t)) = 1$ and $G(t_f, \mathbf{x}(t_f)) = 0$. Therefore in our game, the payoff is represented as

$$J(\mathbf{x}(t_s), u, v) = \int_{t_s}^{t_f(\mathbf{x}(t_s), u, v)} d\bar{t} = t_f(\mathbf{x}(t_s), u, v) - t_s \tag{2}$$

Note that $t_f(\mathbf{x}(t_s), u, v)$ depends on the sequence of controls $u$ and $v$ applied to reach the point $\mathbf{x}(t_f)$ from the point $\mathbf{x}(t_s)$.

# Preliminaries

**Value of the game**

For a given state of the system $\mathbf{x}(t_s)$, $V(\mathbf{x}(t_s))$ represents the outcome if the players implement their optimal strategies starting at the point $\mathbf{x}(t_s)$, and it is called the *value of the game* or the *value function* at $\mathbf{x}(t_s)$ [Isaacs65, Basar95]

$$V(\mathbf{x}(t_s)) = \min_{u(t)\in\widehat{U}} \max_{v(t)\in\widehat{V}} J(\mathbf{x}(t_s), u, v) \tag{3}$$

where $\widehat{U}$ and $\widehat{V}$ are the set of valid values for the controls at all time $t$. $V(\mathbf{x}(t))$ is defined over the entire state space.

# Preliminaries

**Open and closed-loop strategies**

Let $\gamma_p(\mathbf{x}(t))$ and $\gamma_e(\mathbf{x}(t))$ denote the closed-loop strategies of the DDR and the evader, respectively, therefore $u(t) = \gamma_p(\mathbf{x}(t))$ and $v(t) = \gamma_e(\mathbf{x}(t))$.
A strategy pair $(\gamma_p^*(\mathbf{x}(t)), \gamma_e^*(\mathbf{x}(t)))$ is in closed-loop (saddle-point) equilibrium [Basar95] if

$$\begin{aligned} J(\gamma_p^*(\mathbf{x}(t)), \gamma_e(\mathbf{x}(t))) &\leq J(\gamma_p^*(\mathbf{x}(t)), \gamma_e^*(\mathbf{x}(t))) \\ &\leq J(\gamma_p(\mathbf{x}(t)), \gamma_e^*(\mathbf{x}(t))) \forall \gamma_p(\mathbf{x}(t)), \gamma_e(\mathbf{x}(t)) \end{aligned} \tag{4}$$

where $J$ is the payoff of the game in terms of the strategies. An analogous relation exists for open-loop strategies.

# Necessary Conditions for Saddle-Point Equilibrium Strategies

**Theorem (Pontryagin's Maximum Principle - PMP)**

*Suppose that the pair* $\{\gamma_p^*, \gamma_e^*\}$ *provides a saddle-point solution in closed-loop strategies, with* $\mathbf{x}^*(t)$ *denoting the corresponding state trajectory. Furthermore, let its open-loop representation* $\{u^*(t) = \gamma_p(\mathbf{x}^*(t)), v^*(t) = \gamma_e(\mathbf{x}^*(t))\}$ *also provide a saddle-point solution (in open-loop polices). Then there exists a costate function* $p(\cdot) : [0, t_f] \rightarrow R^n$ *such that the following relations are satisfied:*

$$\dot{\mathbf{x}}^*(t) = f(\mathbf{x}^*(t), u^*(t), v^*(t)),\ \mathbf{x}^*(0) = \mathbf{x}(t_s) \tag{5}$$

$$H(p(t), \mathbf{x}^*(t), u^*(t), v(t)) \leq H(p(t), \mathbf{x}^*(t), u^*(t), v^*(t)) \leq H(p(t), \mathbf{x}^*(t), u(t), v^*(t)) \tag{6}$$

$$p(t) = \nabla V(\mathbf{x}(t)) \tag{7}$$

$$\dot{p}^T(t) = -\frac{\partial}{\partial x} H(p(t), \mathbf{x}^*(t), u^*(t), v^*(t)) \ \textbf{(Adjoint Equation)} \tag{8}$$

$$p^T(t_f) = \frac{\partial}{\partial x} G(t_f, \mathbf{x}^*(t_f)) \ along\ \zeta(\mathbf{x}^*(t)) = 0 \tag{9}$$

*where*

$$H(p(t), \mathbf{x}(t), u(t), v(t)) = p^T(t) \cdot f(\mathbf{x}(t), u(t), v(t)) + L(\mathbf{x}(t), u(t), v(t)) \ \textbf{(Hamiltonian)} \tag{10}$$

*and* $T$ *denotes the transpose operator.*

# Time-Optimal Motion Primitives

## Optimal controls

### Lemma

*The time-optimal controls for the DDR that satisfy the Isaacs' equation in the reduced space are given by*

$$u_1^* = -sgn\left(\frac{-yV_x}{b} + \frac{xV_y}{b} - V_y\right) V_p^{\max}$$
$$u_2^* = -sgn\left(\frac{yV_x}{b} - \frac{xV_y}{b} - V_y\right) V_p^{\max} \tag{11}$$

*We have that both controls are always saturated. The controls of the evader in the reduced space are given by*

$$v_1^* = V_e^{\max}, \ \sin v_2^* = \frac{V_x}{\rho}, \ \cos v_2^* = \frac{V_y}{\rho} \tag{12}$$

*where* $\rho = \sqrt{V_x^2 + V_y^2}$. *The evader will move at maximal speed.*

# Decision problem

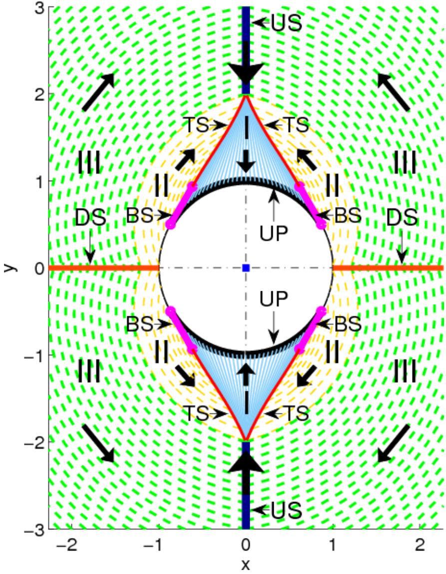
**Theorem**

*If $V_e^{max}/V_p^{max} < l|\tan S|/b$ the DDR can capture the evader from any initial configuration in the reduced space. Otherwise the barrier separates the reduced space into two regions:*

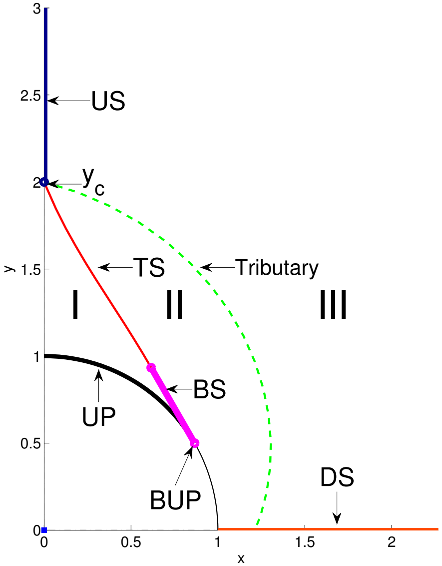
1. *One between the UP and the barrier.*
2. *Another above the barrier.*

*The DDR can only force the capture in the configurations between the UP and the barrier, in which case, the DDR follows a straight line in the realistic space when it captures the evader.*

# Partition of the reduced space



# Partition of the first quadrant



# Global optimality

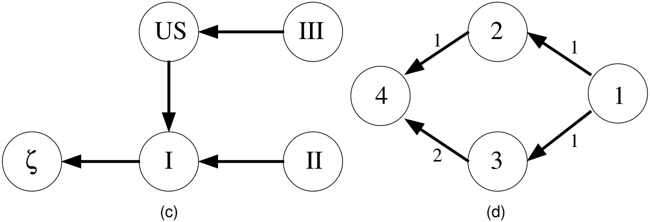



**Figure:** Graphs

## Simulations - Optimal Strategies

The parameters were $V_p^{max} = 1$ , $V_e^{max} = 0.5$, $b = 1$ and $l = 1$. Capture time $t_c = 1.2s$.

# Simulations - Evader avoids capture

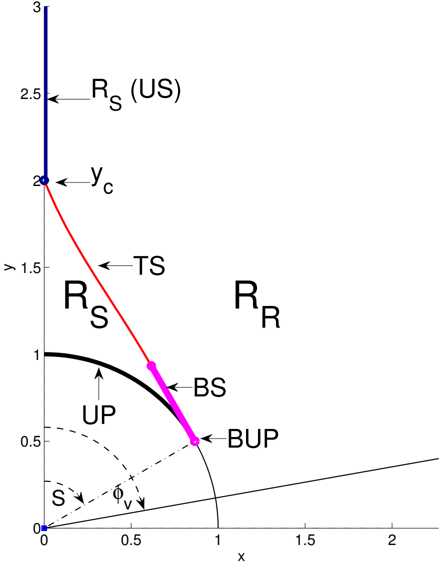
The parameters were $V_p^{\max} = 1$ , $V_e^{\max} = 0.787$, $b = 1$ and $l = 1$.

# A bound for the angle delimiting the field of view of the pursuer

**Theorem**

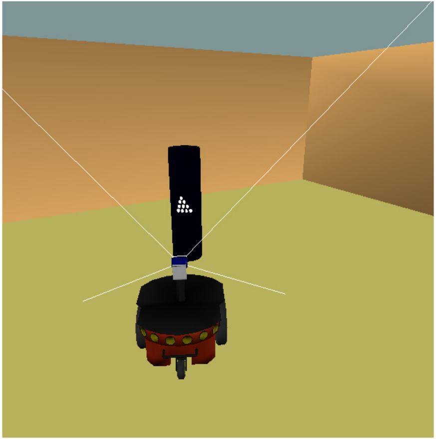
*If the evader is in position $(r_0, \phi_0)$ in the reduced space at the beginning of the game with $\phi_0 < \phi_v$ and $S < \phi_v$ then, if the pursuer applies its time-optimal feedback policy the evader's position $(r, \phi)$ will satisfy $\phi < \phi_v$ at all times until the capture is achieved regardless of the evader's motion strategy.*

# Feedback-based motion strategies for the DDR



# State Estimation

3D View



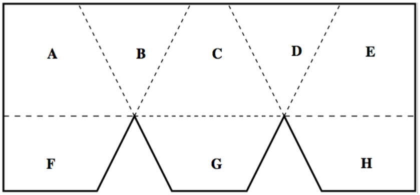
**Simulations**

## Maintaining Visibility of an Evader in an Environment with Obstacles

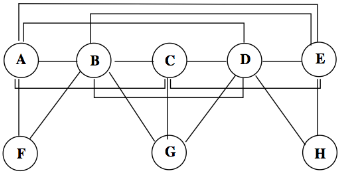
# The Objective

We address our target tracking problem as a game of kind consisting in the next decision problem: is the pursuer able to maintain surveillance of an evader at all time?

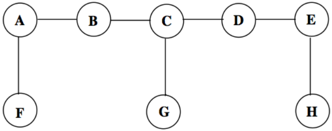
# Environment Partition and Graphs




(a) Environment partition




(b) Mutual visibility graph



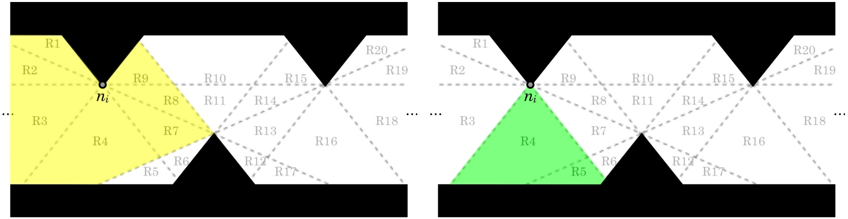

(c) Accessibility graph

**Figure:** Strong Mutual Visibility

### Definition

A *guard polygon* for a given point $q$ is the set of all regions in which each of them is mutually visible to all the regions that own the point $q$. Let $Q(q) = \{R : q \in R\}$, a guard polygon $gP(q)$ for a given point $q$ is defined by:

$$gP(q) = \{R : (R, R_k) \in MVG, \forall R_k \in Q(q)\} \tag{13}$$



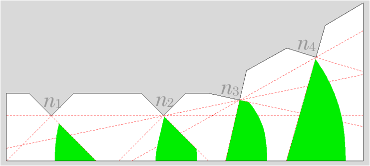

(a) If the evader stands on $n_i$, it is simultaneously over regions $\{R1, R2, R3, R4, R7, R8, R9\}$

(b) The guard polygon for point $n_i$ is $gP(n_i) = \{R4, R5\}$

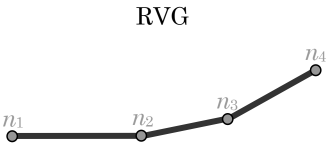
**Figure:** Guard polygon

$$\mathbf{R}_{i,i+1} = \{(w, z) : t_p(w, z) \leq t_e(q_i, q_{i+1}) \text{ where } w \in gP(q_i) \text{ and } z \in gP(q_{i+1})\} \tag{14}$$

# Safe Areas and RGV with Tree Topology



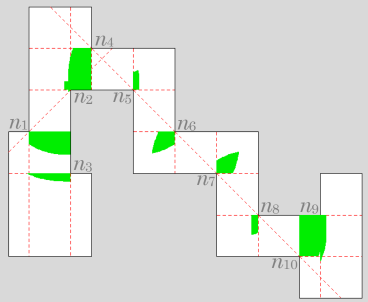

(a)



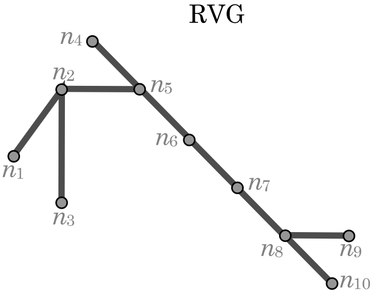

(b)

**Figure:** Example 1 with a tree topology *RVG* and its calculated safe areas, $\frac{V_p}{V_e} = 0.9$

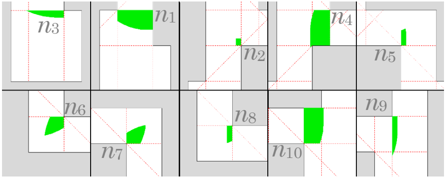
# Safe Areas and RGV with Tree Topology




(a)



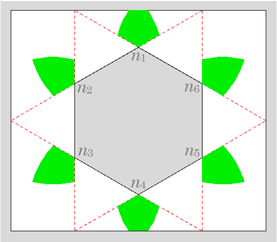

(b)



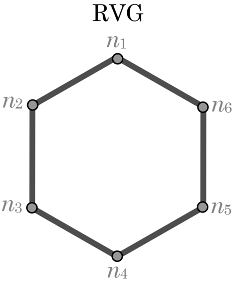

(c)

**Figure:** Example 2 with a tree topology *RVG* and its calculated safe areas, $\frac{V_p}{V_e} = 1.1$

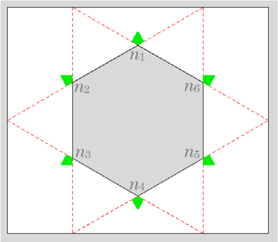
# Safe Areas and *RVG* with Cycles



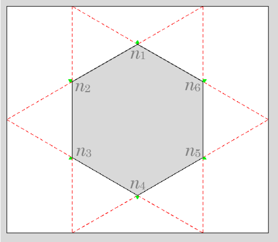

(a) $\frac{V_p}{V_e} = 1.2$



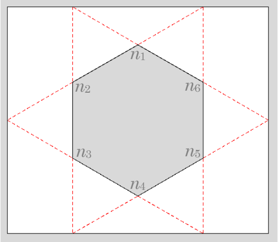

(b)




(c) $\frac{V_p}{V_e} = 1.05$

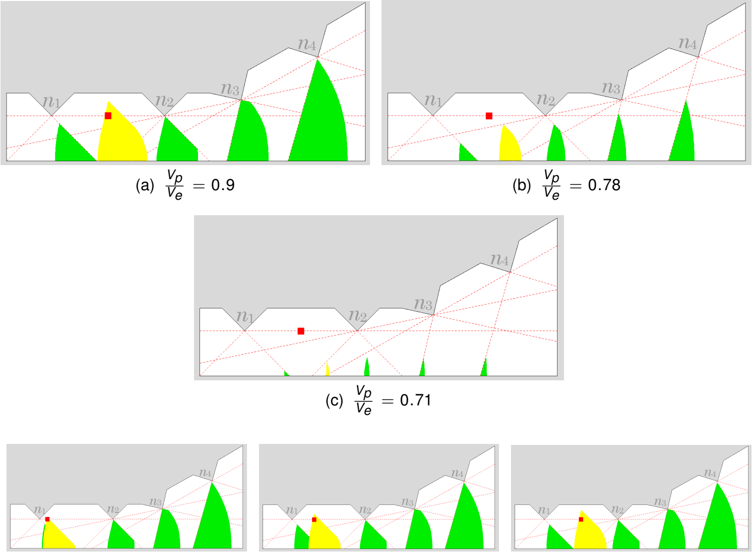



(d) $\frac{V_p}{V_e} = 1.015$




(e) $\frac{V_p}{V_e} = 0.999$

# The $S$ Set



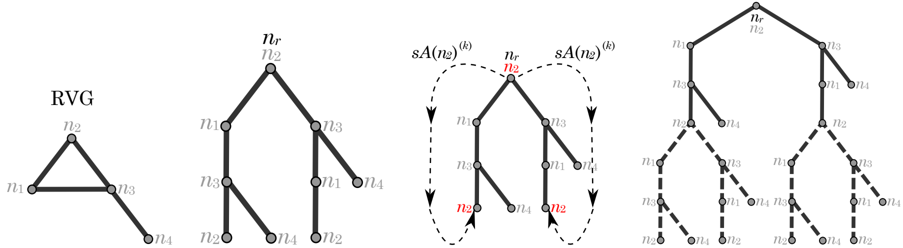

(a) $\frac{V_p}{V_e} = 0.9$ (b) $\frac{V_p}{V_e} = 0.78$

(c) $\frac{V_p}{V_e} = 0.71$

(d) (e) (f)

**Figure:** Example 4 with its calculated safe areas and sample $S$ sets

# Cycles algorithm




(a) Original RVG

(b) Unfolded RVG

(c) Feedback procedure

(d) Tree that considers 2 laps

**Figure:** Cycles algorithm

# Decidability and Complexity

**Theorem**

*The proposed algorithm always converges in a finite number of iterations, hence, the problem of deciding whether or not a pursuer is able to maintain SMV of an evader that travels over the RVG, both players moving at bounded speed, is decidable.*

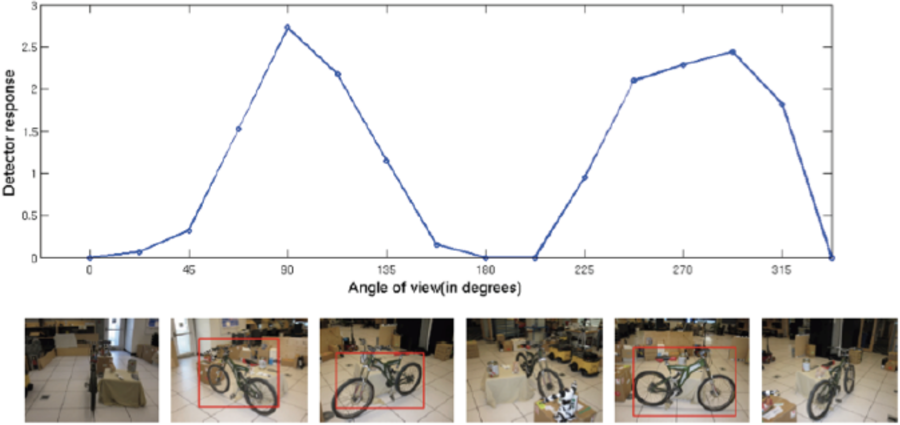
**Theorem**

*The problem of deciding whether or not the pursuer is able to maintain SMV of an evader that travels over the RVG, both players moving at bounded speed, is NP-complete.*

**Object Detection**

# Related Work

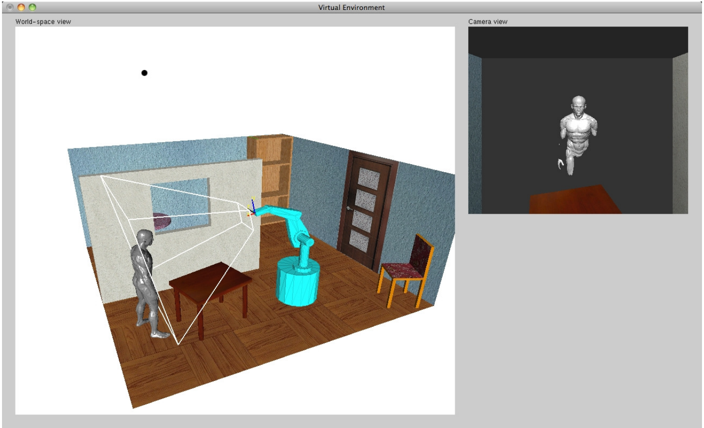
- D. Meger, A. Gupta and J. Little, “Viewpoint Detection Models for Sequential Embodied Object Category Recognition”, ICRA, 2010.



# Object Finding

**Previous work.**

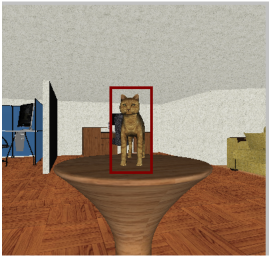
- Judith Espinoza, Alejandro Sarmiento, Rafael Murrieta-Cid and Seth Hutchinson, Motion Planning Strategy for Finding an Object with a Mobile Manipulator in Three-Dimensional Environments, *Journal Advanced Robotics, 25(13-14):1627-1650*, August 2011.




**Simulations**

# Observation Model

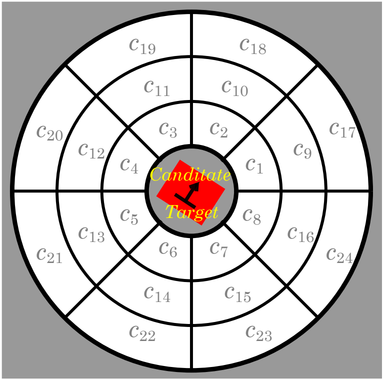
- The robot is equipped with a software module DT (detector), capable of identifying **T**



- DT returns a discrete detection score $o_1 < o_2 < \cdots < o_3$ where $y \in \{o_1, o_2, \ldots, o_n\}$, measuring how well the image matches the appearance of **T**, hence the confidence of the identification

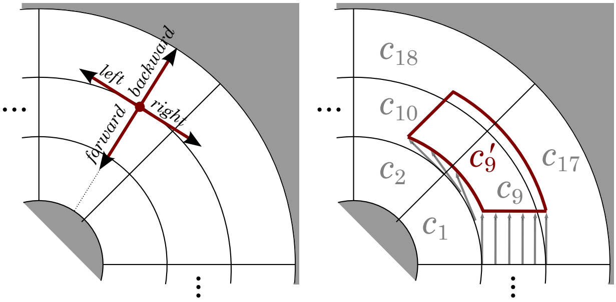
# Observation Model

- The observation model of T is then created in the form of a probability distribution $P(o_j|c_i)$



# Motion Model

- The motion model is given by the probability distribution $P(x_t|x_{t-1}, u_{t-1})$.
- We have 4 motion commands.




(a) Motion commands

(b) $c_9' = R(x_{t-1} = c_9, u_{t-1} = \mathit{left})$

**Figure:** Motion model

# Confirmation of Detection

- The target is declared as detected if the detector returns a confidence score greater than $\hat{o}$ at time $t+1$ and If the robot reaches at time $t$ a position where the condition $P(y_{t+1} \geq \hat{o} | I_t, u_t)$ is satisfied.
- This gives us a twofold goal that mixes robot localisation relatively to the candidate object and target identification using its appearance.

# Computation of motion strategy

- We use SDP to calculate the motion policy $\Pi(I_t, t)$.

$$J_{N-1}(I_{N-1}) = \max_{u_{N-1}\in U_{N-1}} \left[\tilde{g}(I_{N-1}, u_{N-1}) + \underset{x_{N-1}}{E} \left\{ \underset{x_N}{E} \left\{ g_F(x_N) | x_{N-1}, u_{N-1} \right\} | I_{N-1}, u_{N-1} \right\}\right]$$

$$\pi(N-1, I_{N-1}) = \arg \max_{u_{N-1}\in U_{N-1}} \left[\tilde{g}(I_{N-1}, u_{N-1}) + \underset{x_{N-1}}{E} \left\{ \underset{x_N}{E} \left\{ g_F(x_N) | x_{N-1}, u_{N-1} \right\} | I_{N-1}, u_{N-1} \right\}\right]$$

and for $t < N - 1$

$$J_t(I_t) = \max_{u_t \in U_t} \left[\tilde{g}(I_t, u_t) + \underset{y_{t+1}}{E} \left\{ J_{t+1}(I_t, y_{t+1}, u_t) | I_t, u_t \right\}\right]$$

$$\pi(t, I_t) = \arg \max_{u_t \in U_t} \left[\tilde{g}(I_t, u_t) + \underset{y_{t+1}}{E} \left\{ J_{t+1}(I_t, y_{t+1}, u_t) | I_t, u_t \right\}\right]$$

# Gain Function

- Since we want the robot to achieve a position where $P(y_{t+1} \geq \hat{o}|I_t, u_t) > \lambda$ holds, we set the gain function $\tilde{g}(I_t, u_t)$ to:

$$P(y_{t+1} \geq \hat{o}|I_t, u_t) = \sum_{x_{t+1}} P(y_{t+1} \geq \hat{o}|x_{t+1}) \sum_{x_t} P(x_{t+1}|x_t, u_t)P(x_t|I_t) \tag{15}$$

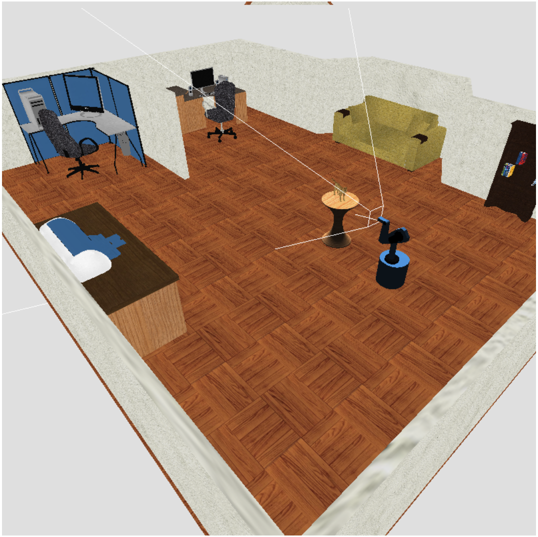
# Simulations and Experimental Results

- We use a 24-cell decomposition.
- For each target T, the detector DT uses a deformable part model algorithm [1] trained on a set of images taken from a single cell cg of the decomposition.

[1] P. F. Felzenszwalb, R. B. Girshick, D. McAllester, and D. Ramanan, “Object Detection with Discriminatively Trained Part Based Models”, Trans. on Pattern Analysis and Machine Intelligence, 2010.

- 6 score values as observation

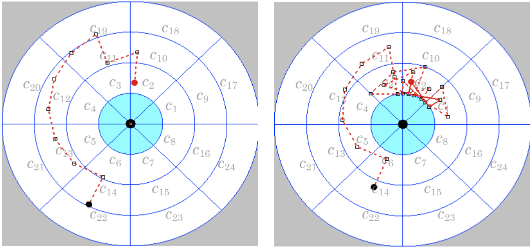
# Simulation



**<u>Simulations</u>**

# Simulation


(a) (b)




(c) Path generated with $\lambda = 0.8$ (true bottle)

(d) Path generated with $\lambda = 0.8$ (false bottle)

# Similar Bottles

| Scene object | $\lambda$ | # of sensing locations | Path length | Planning time (ms) | % of confirmation |
|---|---|---|---|---|---|
| True Bottle | 0.80 | 10.820 | 9.346 | 367.723 | 100 |
| | 0.85 | 10.825 | 9.122 | 361.993 | 100 |
| | 0.90 | 12.030 | 9.244 | 415.965 | 99.5 |
| False Bottle | 0.80 | 21.333 | 18.002 | 721.861 | 1.5 |
| | 0.85 | 17 | 14.561 | 621.074 | 0.5 |
| | 0.90 | - | - | - | 0 |

**Table:** Statistics for similar bottles

# Object Detection

**Experiments with the Robot**

# Conclusions

In this work, we made the following contributions:

1. Pursuit/Evasion: DDR vs Omnidirectional Agent
   - We presented time-optimal motion strategies and the conditions defining the winner for the game of capturing an omnidirectional evader with a differential drive robot.
2. Surveillance with Obstacles
   - We proved decidability of this problem for any arbitrary polygonal environment.
   - We provided a complexity measure to our evader surveillance game.
3. Object Detection
   - We proposed a motion policy mixing robot localisation and target confirmation using the target's appearance.
   - We presented experimental results.

# Future Work

1. Pursuit/Evasion: DDR vs Omnidirectional Agent
   - The results will be extended for capturing an omnidirectional agent using two o more differential drive robots when one is not able to do it.
   - We will include acceleration bounds in the solution of the problem.
2. Surveillance with Obstacles
   - A moving evader that is free to travel any path within the workspace.

# Future Work

3 Object Detection

- Propose a motion policy for a robot with many degrees of freedom.

**Many Degrees of Freedom**

# Publications

- Published papers
  - Ubaldo Ruiz, Rafael Murrieta-Cid and Jose Luis Marroquin, Time-Optimal Motion Strategies for Capturing an Omnidirectional Evader using a Differential Drive Robot, *IEEE Transaction on Robotics 29(5):1180-1196*, 2013.
  - Israel Becerra, Luis M. Valentin, Rafael Murrieta-Cid and Jean-Claude Latombe, Appearance-based Motion Strategies for Object Detection, *Proc IEEE International Conference on Robotics and Automation*, pages 6455-6461, 2014.
- Submited papers
  - Israel Becerra, Rafael Murrieta-Cid, Raul Monroy, Seth Hutchinson and Jean-Paul Laumond, Maintaining Strong Mutual Visibility of an Evader Moving over the Reduced Visibility Graph, *Submitted to Journal Autonomous Robots, 2013*. **In second review**.
  - David Jacobo, Ubaldo Ruiz, Rafael Murrieta-Cid, Hector Becerra and Jose Luis Marroquin, A Visual Feedback-based Time-Optimal Motion Policy for Capturing an Unpredictable Evader, *Submitted to International Journal of Control, 2014*.

# Thanks

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R. Isaacs. *Differential Games: A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization*. Wiley, New York, 1965.

L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mishchenko. *The Mathematical Theory of Optimal Processes.* JohnWiley, 1962.