

1. ALGEBRAIC TOPOLOGY

Marcelo Aguilar *Instituto de Matemáticas, UNAM* A classification of cohomology transfers for ramified covering maps

abstract We construct a cohomology transfer for n -fold ramified covering maps. Then, we define a general concept of transfer for ramified covering maps and prove a classification theorem for these transfers. This generalizes Roush's classification of transfers for n -fold ordinary covering maps. We characterize those representable cofunctors which admit a family of transfers that have two naturalness properties, as well as normalization and stability. This is analogous to Roush's characterization theorem for the case of ordinary covering maps. Finally, we classify these families and give some examples.

Boris N. Apanasov *University of Oklahoma, USA* Fibre bundle structure of almost nilpotent manifolds with pinched negative curvature

abstract The aim of our talk is to describe geometry and topology of manifolds with pinched negative sectional curvature whose fundamental groups are almost nilpotent, especially such noncompact locally symmetric rank one manifolds. Such manifolds classify thin ends of geometrically finite manifolds with pinched negative sectional curvature and play an important role in their geometry. We describe topology of such almost nilpotent manifolds as non-trivial fibre bundles—see B.Apanasov, Almost Nilpotent Manifolds With Pinched Negative Curvature, Preprint No 601, Centre de Reserca Matematica, Bellaterra, 2004. As a tool for this study we use our structural theorem for dynamics of discrete groups acting by isometries on nilpotent groups, see B.Apanasov and X.Xie, Int. J. Math. **8**(1997), 703-757; Diff. Geom. Appl. **20**(2004), 11-29.

Jose Luis Cisneros Molina *Instituto de Matemáticas, UNAM, Unidad Cuernavaca* Stiefel-Whitney classes and singularities

abstract In this talk we give a geometric construction of the Stiefel-Whitney classes of a vector bundle ξ using the singular set of a bundle morphism $h: \epsilon^{n-i+1} \rightarrow \xi$, with ϵ^{n-i+1} the trivial bundle of rank $n-i+1$. In general the singular set is not a manifold, but for generic h is a stratified manifold and it has a fundamental class. The i -th Stiefel-Whitney class of ξ is the Poincaré dual of such a fundamental class.

T.M. Gendron *Instituto de Matemáticas, UNAM* Geometric Galois Theory, Nonlinear Number Fields and a Galois Group Interpretation of the Idele Class Group

abstract This talk is concerned with the description of geometric parallels of the Galois theory of algebraic number fields. There has been a longstanding meta-principle that given an algebraic number field K over Q , there should exist a Riemann surface-like object whose field of meromorphic functions may be identified with (a certain complexification of) K . We give the following concrete expression to this meta-principle. After expanding the notion of adèle class group to number fields of infinite degree over Q , a hyperbolized adèle class group \mathfrak{S}_K is assigned to every number field K/Q . The projectivization of the Hardy space $PH_\bullet[K]$ of graded holomorphic functions on \mathfrak{S}_K possesses two operations \oplus and \otimes giving it the structure of a *nonlinear number field extension* of K . We show that the Galois theory of these nonlinear number fields coincides with their discrete counterparts in that $\text{Gal}(PH_\bullet[K]/K) = 1$ and $\text{Gal}(PH_\bullet[L]/PH_\bullet[K]) \cong \text{Gal}(L/K)$ if L/K is Galois.

If K^{ab} denotes the maximal abelian extension of K and C_K is the idele class group, we show that there is an embedding $C_K \hookrightarrow \text{Gal}_{\otimes}(PH_{\bullet}[K^{\text{ab}}]/K) =$ the group of \otimes -automorphisms of $PH_{\bullet}[K^{\text{ab}}]$ that fix K . This work is joint with A. Verjovsky.

Samuel Gitler Manifolds of the type of smooth projective algebraic manifolds
abstract S. Gitler with L. Astey, E. Micha and G. Pastor. In this talk I will describe current research on the topology of manifolds of dimension $4n$ plus 2 which have CP^n as $2n$ skeleton. These manifold contain twisted projective spaces as cores. We define analogues of complete intersections and try to give a homotopy classification of these, similar to the one we have obtained for algebraic complete intersections.

J. González *Cinvestav* Robotics in lens spaces: an approach to the immersion problem for real projective spaces

abstract Using primary operations in Brown-Peterson cohomology (and in fact just BP-Euler classes) Donald M. Davis obtained in the mid 80's a strong non-immersion result for real projective spaces. Such a result gave new insights on the kind of results one should expect for this extremely difficult problem. In this talk I will explain how to expand this philosophy by considering the immersion problem for (2-torsion) lens spaces. I will discuss possible generalizations of Davis' result and indicate how one can give an alternative (and possibly more manageable) approach to the immersion problem for real projective spaces in terms of the topological complexity of lens spaces, a concept that arises naturally in robotics.

Hiroaki Hamanaka *Department of Natural Science, Hyogo University of Teacher Education* On unstable \tilde{K}^1 -theory

Abstract Let $U(n)$ be the unitary group. For a CW-complex X , we define $U_n(X)$ as the homotopy set $[X, U(n)]$ with the group structure defined by the point-wise multiplication. We may call $U_n(X)$ the unstable \tilde{K}^1 -theory of X , because, when n is sufficiently large ($2n > \dim X$), $U_n(X)$ merely equals to $\tilde{K}^1(X)$.

When $\dim X = 2n$, $U_n(X)$ is a central extension of $\tilde{K}^1(X)$ as follows. Denote the infinite Stiefel manifold $U(\infty)/U(n)$ by W_n and consider the fibration sequence:

$$(1.1) \quad \Omega U(\infty) \xrightarrow{\Omega p} \Omega W_n \xrightarrow{\delta} U(n) \xrightarrow{i} U(\infty) \xrightarrow{p} W_n.$$

Since W_n is $2n$ -connected and $\pi_{2n+1}(W_n) = \mathbb{Z}$, the above sequence induces the exact sequence

$$(1.2) \quad \tilde{K}^0(X) \xrightarrow{\Theta(X)} H^{2n}(X) \rightarrow U_n(X) \rightarrow \tilde{K}^1(X) \rightarrow 0.$$

We omit the coefficient ring of cohomology, when it is the integer ring. Concerning with this sequence, the following is known.

Theorem 1.1. [1]

(i) *The sequence (1.2) induces the central extension*

$$0 \rightarrow \text{Coker} \Theta(X) \rightarrow U_n(X) \rightarrow \tilde{K}^1(X) \rightarrow 0.$$

(ii) $\Theta(X)$ is equal to $n!ch_n$, where ch_n is the n -th Chern character.

Also, the commutator of $U_n(X)$ can be described by means of ordinary cohomology and some characteristic classes ([1]).

Next, when $\dim X$ is $2n+1$ or $2n+2$, the situation of $U_n(X)$ depends on whether n is even or odd. Recall the exact sequence obtained by (1.1):

$$(1.3) \quad \tilde{K}^0(X) \xrightarrow{\Theta(X)} [X, \Omega W_n] \rightarrow U_n(X) \rightarrow \tilde{K}^1(X) \xrightarrow{T(X)} [X, W_n].$$

Here one can see that $[X, \Omega W_n]$ is abelian when $\dim X < 4n$, but the following extension induced by (1.3) is not central in general when n is even and $\dim X = 2n+1$.

$$(1.4) \quad 0 \rightarrow \text{Coker} \Theta(X) \rightarrow U_n(X) \rightarrow \text{Ker} T(X) \rightarrow 0.$$

On the other hand the above is central extension when n is odd and $\dim X \leq 2n+2$ (See [4]).

When $\dim X$ is $2n+1$ and n is odd, one have the result of $U_n(X)$ almost similar to Theorem 1.1([3]).

When $\dim X$ is $2n+1$ and even, we should introduce a intermediate group $\tilde{K}^1(X)$ in general, which is a central extension of a subgroup of $\tilde{K}^1(X)$, and $U_n(X)$ is a central extension of $\tilde{K}^1(X)$ ([3]).

Then, we consider the case $\dim X = 2n+2$ and n is odd. We set $W(X) = (1 \oplus \rho^{-1} \text{Sq}^2 \rho)(\mathbb{H}^{2n}(X))$ where ρ is the mod 2 reduction.

Theorem 1.2. *Let n be odd and $\dim X \leq 2n+2$. If $\text{Tor}(\mathbb{H}^{2n+2}(X), \mathbb{Z}/2) = 0$, we have the exact sequence:*

$$\tilde{K}^0(X) \xrightarrow{\Theta'(X)} W(X) \rightarrow U_n(X) \rightarrow \tilde{K}^1(X) \xrightarrow{T(X)} \mathbb{H}^{2n+1}(X).$$

Here $T(X) = \Sigma^{-1} c_{n+1}$, c_{n+1} is the $n+1$ -th Chern class and $\Theta'(X) = (n)! ch_n \oplus (n+1)! ch_{n+1}$.

Also the commutator of $U_n(X)$ can be described by means of ordinary cohomology and some characteristic classes. Using these result, $U_n(X)$ can be determined for several spaces .

[1]H.Hamanaka & A.Kono, *On $[X, U(n)]$, when $\dim X$ is $2n$* , J. Math. Kyoto Univ. **43** (2003), no. 2, 333–348. [2]H.Hamanaka, *Adams e -invariant, Toda bracket and $[X, U(n)]$* , J. Math. Kyoto Univ. **43** (2003), no. 4, 815–828. [3]H.Hamanaka, *On $[X, U(n)]$, when $\dim X$ is $2n+1$* , J. Math. Kyoto Univ., To appear. [4]H.Hamanaka, *Nilpotency of unstable K -theory*, to appear.[5]M.J.Hopkins, *Nilpotence and finite H -spaces*, Israel J. Math. **66**(1989),238–246. [6]M.Mimura & H.Ōshima, *Self homotopy groups of Hopf spaces with at most three cells*, J. Math. Soc. Japan **51** (1999), no. 1, 71–92

Norio Iwase *Kyushu University, Japan* L-S category of principal fibre bundles
abstract We give new lower and upper estimates for L-S category of principal fibre bundles, which implies some results on L-S category of spinor groups.

Yasuhiko Kamiyama *University of the Ryukyus* Configuration spaces and rational functions
abstract

Definition 1 . We set

$$P_{k,n}^l = \{f(z) = z^k + a_1 z^{k-1} + \cdots + a_k : a_i \in \mathbb{C},$$

the number of n -fold roots of $f(z)$ is at most $l\}$.

Here two n -fold roots may coincide. We write $P_{k,2}^0$ as $C_k(\mathbb{C})$.

Example 1 .

- (1) For $l \geq d$, $P_{nd,n}^l = \mathbf{C}^{nd} \simeq \{\text{a point}\}$.
- (2) $P_{nd,n}^{d-1} \cong \mathbf{C}^{nd} - \mathbf{C}^d \simeq S^{2(n-1)d-1}$.

Definition 2 . We set

$X_{k,n}^l = \{(p_1(z), \dots, p_n(z)) : \text{each } p_i(z) \text{ is a monic polynomial over } \mathbf{C}$
of degree k and such that there are at most l roots common to all $p_i(z)\}$.

Here two common roots may coincide. We write $X_{k,n}^0$ as $\text{Rat}_k(\mathbf{C}P^{n-1})$.

Example 2 .

- (1) For $l \geq d$, $X_{d,n}^l = (\mathbf{C}^d)^n \simeq \{\text{a point}\}$.
- (2) $X_{d,n}^{d-1} \cong (\mathbf{C}^d)^n - \{\text{diagonal set}\} \simeq S^{2(n-1)d-1}$.

Homotopy fiber

- (1) $J^l(2n-2)$: the l -th stage of the James construction which builds ΩS^{2n-1} .
- (2) $W^l(n)$: the homotopy theoretic fiber of the inclusion $J^l(2n-2) \hookrightarrow \Omega S^{2n-1}$.

$$W^l(n) @>>> J^l(2n-2) \hookrightarrow \Omega S^{2n-1}.$$

- (3) We can generalize Snaith's stable splitting as follows.

$$W^l(n) \simeq \bigvee_{1 \leq q} D_q \xi^l(n).$$

Theorem There is an unstable map $\alpha_{k,n}^l : X_{k,n}^l \rightarrow W^l(n)$ which is a homotopy equivalence up to dimension $\left[\frac{k}{l+1} \right] (2(l+1)(n-1) - 1)$.

Theorem

$$X_{k,n}^l \simeq \bigvee_{q=1}^k D_q \xi^l(n).$$

Theorem 2 for $l = 0$ is the theorem of Cohen et al.

Theorem Except when $(n, l) = (2, 0)$, there is a homotopy equivalence

$$P_{k,n}^l \simeq X_{\left[\frac{k}{n} \right], n}^l.$$

Theorem 3 for $l = 0$ is essentially the theorem of Vassiliev. Combining the results of Brown-Peterson and Cohen et al., Theorem 3 holds stably when $(n, l) = (2, 0)$. Note that Theorem 3 indeed holds between Examples 1 and 2.

2. TABLES

The following tables are taken from V.I. Arnold: On some topological invariants of algebraic functions. Trudy Moscov. Mat. Obshch. **21**, 27–46 (1970); English transl. in Trans. Moscow Math. Soc. **21**, 30–52 (1970). Stability Theorem: Fix q . In each column, we go downward. Then the homology stabilizes when $k \geq 2q$.

$H_*(P_{2k+i,2}^{k-1}; \mathbf{Z})$

- (1) For $1 \leq q \leq 2k-2$, $H_q(P_{2k+i,2}^{k-1}; \mathbf{Z}) = 0$.
- (2) For $2k-1 \leq q \leq 2k+3$, $H_q(P_{2k+i,2}^{k-1}; \mathbf{Z})$ are cyclic and the orders are given by the following table.

Here

TABLE 1. The groups $H_q(C_k(\mathbf{C}); \mathbf{Z})$ ($1 \leq q \leq 5$).

$k \setminus q$	1	2	3	4	5
0, 1	0	0	0	0	0
2, 3	\mathbf{Z}	0	0	0	0
4, 5	\mathbf{Z}	$\mathbf{Z}/2$	0	0	0
6, 7	\mathbf{Z}	$\mathbf{Z}/2$	$\mathbf{Z}/2$	$\mathbf{Z}/3$	0
8, 9	\mathbf{Z}	$\mathbf{Z}/2$	$\mathbf{Z}/2$	$\mathbf{Z}/6$	$\mathbf{Z}/3$
10, 11	\mathbf{Z}	$\mathbf{Z}/2$	$\mathbf{Z}/2$	$\mathbf{Z}/6$	$\mathbf{Z}/6$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
∞	\mathbf{Z}	$\mathbf{Z}/2$	$\mathbf{Z}/2$	$\mathbf{Z}/6$	$\mathbf{Z}/6$

TABLE 2. The orders of the groups $H_q(P_{2k+i,2}^{k-1}; \mathbf{Z})$ ($2k-1 \leq q \leq 2k+3$).

$i \setminus q$	$2k-1$	$2k$	$2k+1$	$2k+2$	$2k+3$
0, 1	∞	0	0	0	0
2, 3	∞	$k+1$	0	0	0
4, 5	∞	$k+1$	$2/k$	$(k+2)/2$	0
6, 7	∞	$k+1$	$2/k$	$((k+2)/2)(2/k)$	$3/k$
8, 9	∞	$k+1$	$2/k$	$((k+2)/2)(2/k)$	$6/kv$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
∞	∞	$k+1$	$2/k$	$((k+2)/2)(2/k)$	$6/kv$

(i) We introduce the notation

$$a/b = \frac{a}{\gcd(a, b)},$$

where $\gcd(a, b)$ is the greatest common divisor of the integers a and b .

(ii) Stability Theorem: Fix k and q . In each column, we go downward. Then the homology stabilizes when $i \geq 2(q - 2k + 1)$.

(iii) We have

$$v = \begin{cases} 1 & \text{if } k \not\equiv 1 \pmod{4} \\ 1 \text{ or } 2 & \text{if } k \equiv 1 \pmod{4}. \end{cases}$$

But the exact value is left unknown.

Table 1 is obtained from Table 2 by setting $k = 1$ and rewriting $2k + i$ as k .

Reconstruction of Table 2

(1) By Theorem 3,

$$P_{2k+i,2}^{k-1} \simeq X_{k+\lfloor \frac{i}{2} \rfloor, 2}^{k-1}.$$

Hence we calculate the right-hand side.

(2) By Theorem 2, as a vector space, $H_*(X_{k+\lfloor \frac{i}{2} \rfloor, 2}^{k-1}; \mathbf{Z}/p)$ (where p is a prime) is isomorphic to the subspace of $H_*(W^{k-1}(2); \mathbf{Z}/p)$ spanned by monomials of weight $\leq k + \lfloor \frac{i}{2} \rfloor$.

- (3) We can determine $H_*(W^{k-1}(2); \mathbf{Z}/p)$ from the mod p Serre spectral sequence for the fibration

$$\Omega^2 S^3 \rightarrow W^{k-1}(2) \rightarrow J^{k-1}(2).$$

- (4) If we follow the steps (1)-(3), then we can prove that the value of the indeterminacy v in Table 2 is 1 when $k \equiv 1 \pmod{4}$.

Akira Kono *Department of Mathematics Kyoto University* Homotopy type of gauge groups of $SU(3)$ -bundles over S^6

abstract Let G be a compact Lie group, $\pi : P \rightarrow B$ a principal G -bundle over a finite complex B . The group of G -equivariant self-maps covering the identity map of B is called the gauge group of P . The purpose of this talk is to show the following:

THEOREM. Denote by ϵ' a generator of $\pi_6(BSU(3)) \cong \mathbb{Z}$ and by \mathcal{G}_k , the gauge group of the principal $SU(3)$ -bundle over S^6 classified by $k\epsilon'$. Then $\mathcal{G}_k \simeq \mathcal{G}_{k'}$ if and only if $(120, k) = (120, k')$.

Ernesto Lupercio *Cinvestav* Orbifold String Topology

abstract In this talk I will present the theory of orbifold string topology (developed in collaboration with B. Uribe and M. Xicotencatl). In a seminar paper a few years ago M. Chas and D. Sullivan discovered a BV-algebra structure in the homology of the free loop space of a symplectic manifold. This was transposed to the homotopy theory realm by R. Cohen and J. Jones. Here I will introduce the original theory, its relation to quantum field theory (Cohen-Godin). Then thinking of a group action as a topological Deligne-Mumford stack I will explain how we have developed the orbifold (equivariant) version of the theory inspired by the work of Chen-Ruan and that of Dixon-Harvey-Vafa-Witten. This relates to the original theory much in the way in which equivariant K-theory relates to K-theory. Finally I will turn on the physicist flat B-field known as discrete torsion.

G. Moreno *Cinvestav* TBA

Frank Neumann *Department of Mathematics, University of Leicester, United Kingdom* On the algebraic K-theory of the category of unstable modules over the Steenrod algebra

abstract Using the Gabriel-Krull filtration, we construct a spectral sequence of homological type converging to the algebraic K-theory of the Noetherian objects in the category of unstable modules over the Steenrod algebra. This is in direct analogy with the Brown-Gersten-Quillen spectral sequence converging to the algebraic K-theory of a Noetherian scheme via the codimension of support filtration.

Ivonne J. Ortiz *Miami University* The lower algebraic K-theory of Γ_3

abstract We will present the lower algebraic K-theory of Γ_3 a discrete subgroup of the group of isometries of hyperbolic 3-space. This group forms part of a family of hyperbolic, non-cocompact, n -simplex reflection groups from which to study the problem of computing the K-theory of infinite groups with torsion. The main result is that for Γ_3 , the Whitehead group of Γ_3 is zero, $\tilde{K}_0(\mathbb{Z}\Gamma_3) = \mathbb{Z}/4 \oplus \mathbb{Z}/4$, $K_{-1}(\mathbb{Z}\Gamma_3) = \mathbb{Z} \oplus \mathbb{Z}$ and $K_n(\mathbb{Z}\Gamma_3)$ is zero for all $n < -1$.

Andres Pedroza *Universidad de Colima* A Generalization of the localization formula

abstract We will present a generalization of the Atiyah-Bott-Berline-Vergne localization theorem for the equivariant cohomology of a torus action: replacing the torus action by a compact connected Lie group action. This provides a systematic method for calculating the Gysin homomorphism in ordinary cohomology of an equivariant map.

Mikhail Shchukin *Belarusian state university* About the K -theory of n -homogeneous C^* -algebras

abstract The classical result by Gelfand and Naimark describes the K -theory of commutative C^* -algebras as the K -theory of the space of maximal ideals of the algebra. We extend the result for the class n -homogeneous C^* -algebras. We say that the algebra A is n -homogeneous if all its irreducible representations are of dimension n for some positive integer n . We prove that the K -theory of a n -homogeneous C^* -algebra is canonically isomorphic to the K -theory of the space of primitive ideals of the algebra A in the appropriate topology.

Dai Tamaki *Department of Mathematical Sciences, Shinshu University* On the E^1 -term of the gravity spectral sequence

abstract The author constructed a spectral sequence strongly converging to $h_*(\Omega^n \Sigma^n X)$ for any homology theory in 1994. In this talk, we prove that the E^1 -term of the spectral sequence is isomorphic to the cobar construction, and hence the spectral sequence is isomorphic to the classical cobar-type Eilenberg-Moore spectral sequence based on the geometric cobar construction from the E^1 -term. Similar arguments can be also applied to its variants constructed in 2002 by the author.

Shuichi Tsukuda *University of the Ryukyus* On the mod 2 cohomology of $\text{Map}(X, \text{BSp}(n))$

abstract In this talk, we describe the ring structure of the mod 2 cohomology of the mapping space $\text{Map}(S^4, \text{BSp}(1))$ from 4-dimensional sphere to the classifying space of $\text{Sp}(1)$. We also study the action of the Steenrod algebra. As an application, we show the non triviality of certain evaluation fibrations.

Miguel A. Xicotencatl *CINVESTAV, Instituto Politécnico Nacional* Homology calculations and operadic structure in orbifold string topology.

abstract In recent work (with B. Uribe and E. Lupercio) we have developed an “orbifold analogue” of the Chas-Sullivan product in the homology of the free loop space of a manifold. In this talk I will present explicit calculations in the string homology ring of the free loop space of classifying space of an orbifold. Also, I will show that the homology of the orbifold loop space can be given a natural action of the cactus operad, and thus it inherits a BV-algebra structure.