ON THE FRACTAL BURGERS EQUATION WITH A STOCHASTIC NOISY TERM

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1 Introduction

The classical Burgers equation

$$\frac{\partial}{\partial t}u(t,x) = \nu\Delta u(t,x) - \lambda\nabla u^2(t,x)$$

was proposed by Burgers [3] as a particular case of the Navier-Stokes equation, and has been used extensively to study turbulence and other physical phenomena (see e.g. [9],[6],[12], [16]). Burgers equation involving fractional powers $\Delta_\alpha := -(-\Delta)^{\alpha/2}$, $\alpha \in (0,2]$, of the Laplacian in its linear part has also been studied in connection with models of several hydrodynamical phenomena (see e.g. [17], [5], [4] and the references therein for applications).

In [4] Biller, Funaki and Woyczynski studied existence, uniqueness, regularity and asymptotic behavior of solutions of the multidimensional fractal Burgers-type equation

$$\frac{\partial}{\partial t}u(t,x) = \nu\Delta_\alpha u(t,x) - a\nabla u^r(t,x), \tag{1.1}$$

where $x \in \mathbb{R}^d$, $d \geq 1$, $\alpha \in (0,2]$, $r \geq 1$, and $a \in \mathbb{R}^d$ is a fixed vector. For $\alpha > 3/2$ and $d = 1$ they prove existence of a unique regular weak solution of (1.1) with initial conditions in $H^1(\mathbb{R})$.

In [10] it is proved existence of a weak solution of the one-dimensional stochastic Burgers equation perturbed by a white noise term with a non-Lipschitz coefficient

$$\begin{aligned}
\frac{\partial}{\partial t}u(t,x) &= \Delta u(t,x) + \lambda\nabla u^2(t,x) + \gamma\sqrt{u(t,x)(1-u(t,x))}\,\frac{\partial^2}{\partial t\partial x}W(t,x),\\
u(t,0) &= u(t,1) = 0,\\
u(0,x) &= f(x),\ x \in [0,1],
\end{aligned} \tag{1.2}$$

where $f : [0,1] \to [0,1]$ is continuous and $\frac{\partial^2}{\partial t \partial x} W(t,x)$ is the space-time white noise. The method of proof in [10] consists in approximating (1.2) by finite systems of stochastic differential equations possessing a unique strong solution. Using bounds for the fundamental solution of the discrete Laplacian, it is shown tightness of the approximating systems, and that each weak limit is a weak solution of (1.2).

In this paper we consider the one-dimensional fractal Burgers equation given by

$$\begin{aligned} \frac{\partial}{\partial t} u(t,x) &= \Delta_\alpha u(t,x) + \lambda \nabla u^2(t,x) + \gamma \sqrt{u(t,x)(1-u(t,x))}\, \frac{\partial^2}{\partial t \partial x} W(t,x), \\ u(t,0) &= u(t,1) = 0,\ x \in [0,1], \end{aligned} \tag{1.3}$$

where the random positive initial condition $u(0,x)$ is bounded by 1.

Due to the presence of non-Lipschitz coefficients, existence and uniqueness of a weak solution of (1.3) cannot be achieved by classical results. Following the method of proof of [10], in this paper we consider a discrete version of (1.3) and obtain, similarly as in [10], existence of a strong solution of the corresponding finite system of SDEs. The principal difficulty we are dealing with in this paper, which is originated by the presence of the fractional power of the discrete Laplacian, consists in proving tightness of the approximating systems. This is solved by using Fourier analysis methods developed by D. Blount in [1] and [2], where he applies such approach to systems of SDEs related to diffusion limits of population models.

2 Notations and basic results

We recall some notations from [1]. Let $\mathbf{S} = [0,1)$ and let $\mathbf{T}$ denote the quotient space obtained from $[0,1]$ by identifying 0 and 1. We put $\varphi_0(x) = 1$ for $x \in [0,1]$, and

$$\varphi_n(x) = \sqrt{2}\cos(\pi n x), \quad \psi_n(x) = \sqrt{2}\sin(\pi n x), \quad x \in [0,1], \quad n = 2,4,\ldots.$$

This system of functions, which we also denote by e_m, $m = 0,1,2,...$, is the usual orthonormal basis in $L^2([0,1])$. Moreover, for all n, $\Delta e_n = -\pi^2 n^2 e_n$. For any $\beta \in \mathbb{R}$ we define H_β as the Hilbert space obtained from $L^2(S)$ by completion with respect to the norm

$$|f|_\beta = \left(\sum \langle f, e_m \rangle^2 (1 + \pi^2 m^2)^\beta \right)^{1/2},$$

where $\langle \cdot, \cdot \rangle$ denotes the usual inner product in $L^2(S)$.

For any integer $N \geq 1$, let $H(N)$ denote the set of functions $f : [0,1] \to \mathbb{R}$ that are constant on $[\frac{k}{N}, \frac{k+1}{N})$ for $k = 0, 1, 2, ..., N-1$. Clearly we have $H(N) \subset L^2([0,1])$.

Let $P_N : L^2(\mathbf{T}) \to H(N)$ be the orthogonal projection of $L^2(\mathbf{T})$ onto $H(N)$, which is given by

$$P_N f(r) = N \int_{\frac{k}{N}}^{\frac{k+1}{N}} f(s)\, ds, \quad r = \frac{k}{N}, \quad k = 0, 1, 2, ..., N-1.$$

For N odd and $0 \leq m \leq N-1$ we define $\hat{e_m} = \frac{P_N e_m}{|P_N e_m|_0}$. Then $\{\hat{e_m}\}$ is an orthonormal basis of $H(N)$ as a subspace of $L^2([0,1])$, and $\Delta_N \hat{e_m} = -\hat{\beta_m} \hat{e_m}$, where $\hat{\beta_m} \in [4m^2, \pi^2 m^2]$. Writing $|.|_0$ for the usual norm in $L^2([0,1])$, it follows that $\lim_{N\to\infty} |e_m - \hat{e_m}|_0 = 0$.

For $f \in H(N)$ and any β we define

$$|f|_{\beta,N} = \left(\sum \langle f, \hat{e_m} \rangle^2 (1 + \hat{\beta_m})^{\beta} \right)^{-1/2}.$$

The next result follows from [1] (Lemma 3.1): For $f \in H(N)$ and $\beta > 0$, we have $|f|_{0,N} = |f|_0$ and

$$2^{-1/2} |f|_{-\beta} \leq |f|_{-\beta,N} \leq (\pi/2)^{\beta+1} |f|_{-\beta}. \tag{2.4}$$

We define $P_n : H_\beta \to \bigcap_\gamma H_\gamma$ as the projection

$$P_n(f) = \sum_{m \leq n} \langle f, e_m \rangle e_m,$$

and put $P_n^\perp := I - P_n$, where I is the identity operator. Similarly, for $f \in H(N)$, let

$$P_{n,N}(f) = \sum_{m \leq n} \langle f, \hat{e_m} \rangle \hat{e_m},$$

and $P_{n,N}^\perp := I - P_{n,N}$. Without loss of generality we assume that $\lambda = \gamma = 1$. Let N be a fixed positive integer. Similarly as in [10], let us consider the discretized version of (1.3), namely

$$\begin{aligned} \frac{\partial}{\partial t} X^N(t,r) &= \Delta_{N,\alpha} X^N(t,r) + \nabla_N X^N(t,r)^2 + \sqrt{X^N(t,r)(1 - X^N(t,r))}\, dB_N(t,r), \\ X^N(0,r) &= X(0,r), \;\; r = 0, \frac{1}{N}, ..., \frac{N-1}{N}, \;\; t \geq 0, \end{aligned} \tag{2.5}$$

where $\Delta_{N,\alpha}$ is the fractional power of the discrete Laplacian, and $\{N^{-1/2}B_N(t,r)\}_r$ is a sequence of independent Brownian motions. Now we state our results.

Theorem 2.1. *a) For any positive initial random condition $X^N(0)$ bounded by 1, there exists a unique strong solution $X^N(t)$ of (2.5) in $C([0,\infty), L^2([0,1])$.*

b) The distributions of $\{X^N\}$ are relatively compact on $C((0,\infty) : H_\beta)$ if $\beta \leq 0, \alpha > \beta + 3/2$, and on $C([0,\infty) : H_\beta)$ for $\alpha > \beta + 3/2, \beta < -1/2$.

c) For any $\alpha > 3/2$, equation (1.3) has a weak solution in $C((0,\infty), L^2([0,1]))$.

Remark 2.1. Theorem 2.1 is consistent with results obtained in [4] for the case $\gamma = 0$. In our case, we were not able to prove uniqueness of weak solutions of (1.3); this remains to be investigated.

Theorem 2.2. *The solution $X(t)$ has a modification which is Holder continuous in time: it satisfies*

$$P\left(\sup_{0<s_0\leq s<t\leq T} \frac{|X(t)-X(s)|_\beta}{|t-s|^\delta} < \infty\right) = 1$$

for each $0 < \delta < [(2\alpha - 2\beta - 3)/(2\alpha)] \wedge 1/2$, $3/2 < \alpha \leq 2$, *and* $\beta < (2\alpha - 3)/2$.

Remark 2.2. In particular, when $\alpha = 2$ and $0 \leq \beta < 1/2$, we can take $0 < \delta < \frac{1-2\beta}{4}$, and obtain

$$P(X \in C((0,\infty) : H_\beta)) = 1,$$

thus $X(t)$ is smoother than an $L^2([0,1])$ function for $t > 0$.

3 Proofs

Let us write $x_r^N(t) = X^N(t,r)$. The system (2.5) then can be written in the more compact form

$$dx_i^N(t) = \left(\sum_{j=1}^N a_{ij}^N x_j^N(t) + b_{ij}^N x_j^N(t)^2\right) dt + \sqrt{x_i^N(t)\left(1 - x_i^N(t)\right)}\, dB_i(t) \tag{3.1}$$

where

$$b_{ij}^N = \begin{cases} N & \text{if} \quad j = i+1, \\ -N & \text{if} \quad j = i, \\ 0 & \qquad \text{otherwise,} \end{cases}$$

and a_{ij} are the coefficients of $\Delta_{N,\alpha}$.

Proof of Theorem 2.1.a The proof is similar to that of Theorem 2.1 in [10]. First we show existence of a weak solution using the Skorokhod's existence theorem [16, 8]. Pathwise uniqueness of weak solutions follows from the classical method of Ikeda and Watanabe, using the local time techniques of Le Gall (e.g. [14], Chapter V, §43). By a classical theorem of Yamada and Watanabe [18], this is sufficient for existence of a unique strong solution of (3.1). The proof is developed thoroughly for the case $\alpha = 2$ in [10]. $\square$

Since $X^N(t, \cdot)$ is defined on a discrete system of points $\{r = k/N, k = 0, 1, ..., N-1\}$, by assigning to $X^N(t, \cdot)$ the constant value $X^N(t, k/N)$ in the interval $[k/N, (k+1)/N)$, $k = 0, 1, ..., N-1$, we can view the function $X^N(t)$ as an element of the space $H(N)$. By variation of constants, we can write (2.5) in the equivalent form

$$\begin{aligned} X^N(t) &= T_{N,\alpha}(t)X^N(0) + \int_0^t T_{N,\alpha}(t-s)[\nabla_N X^N(s)^2]\, ds \\ &\quad + \int_0^t T_{N,\alpha}(t-s)\sqrt{X^N(t)(1-X^N(t))}\, dB_N(s,r) \\ &:= T_{N,\alpha}(t)X^N(0) + V_N(t) + M_N(t), \end{aligned} \tag{3.2}$$

where $T_{N,\alpha}(t)$ is the semigroup on $H(N)$ generated by $\Delta_{N,\alpha}$.

Let $Y_N(t) = \int_0^t \sqrt{X^N(s)(1-X^N(s)}\, dB_N(s)$.

Lemma 3.1. *(i) For $\beta < -1/2$, $\{Y_N\}$ is relatively compact in $C([0,\infty) : H_\beta)$.*

(ii) For any fixed n, and any β, $\{P_n X^N\}$ is relatively compact in $C([0,\infty) : H_\beta)$.

Proof. (i) For $\beta < -1/2$ and $0 \le t \le t+s \le T$, we have

$$\begin{aligned} & E[|Y_N(t+s) - Y_N(t)|_\beta^2 | \sigma(X_r), r \le t] \\ = \;& E[\sum_{m=1}^\infty \int_t^{t+s} \langle X_N(r)(1-X_N(r)), (P_N e_m)^2\rangle dr (1+\beta_m)^\beta | \sigma(X_r), r \le t], \end{aligned}$$

hence from a well-known criterion (see e.g. [7]), $\{Y_N\}$ is relatively compact in $C([0,\infty : H_\beta)$, which proves (i).

Let consider the equality

$$P_n X^N(t) = P_n X^N(0) + \int_0^t P_n \Delta_{N,\alpha} X^N(s)\, ds + \int_0^t P_n \nabla_N X^N(s)^2\, ds$$

$$+ \quad \int_0^t P_n \sqrt{X^N(s)(1-X^N(s))}\, dW_N(s).$$

For fixed n, using the fact that Δ_N is self-adjoint on H_N and $X^N(t)$ is bounded, we obtain from Ascoli's theorem and (i) that the distributions of $P_n[X^N(t) - Y_N(t)]$ are relatively compact. $\square$

Lemma 3.2. *For any $\varepsilon > 0$ and $T > 0$,*

(i) $\lim_{n\to\infty} \sup_N P(\sup_{0\le t\le T} |P^{\perp}_{n,N} M^N(t)|_{\beta,N} \ge \varepsilon) = 0$ *for any* $\beta < 1/2$.

(ii) $\lim_{n\to\infty} \sup_N P(\sup_{s\le t\le T} |P^{\perp}_{n,N} X^N(t)|_{\beta,N} \ge \varepsilon) = 0$ *for* $s > 0$ *and* $\alpha > \beta + 3/2$, *or* $s = 0$ *and* $\alpha > \beta + 3/2$, $\beta < -1/2$.

(iii) $\lim_{n\to\infty} \sup_N P(\sup_{s\le t\le T} |P^{\perp}_{n,N} X^N(t)|_{\beta} \ge \varepsilon) = 0$ *for* $s > 0$ *and* $\alpha > \beta + 3/2$, $\beta \le 0$, *or* $s = 0$ *and* $\alpha > \beta + 3/2$, $\beta < -1/2$.

Proof. From the equality

$$\langle M_N(t), \hat{e_m}\rangle = \int_0^t \exp[-\hat{\beta}_m(t-s)]\langle X^N(t)(1-X^N(t)), (\hat{e_m})^2\rangle dB(s)$$

and [1] (Lemma 1.1), we obtain

$$P\left(\sup_{t\le T}\langle M_N(t), \hat{e_m}\rangle^2 \ge a^2\right) \le \pi^2 m^2 T[\exp(Cm^2a^2) - 1]^{-1}, \tag{3.3}$$

where $C > 0$ is a constant. For $\beta < 1/2$, let δ be such that $0 < \delta < 1$, $\beta - \delta < -1/2$. Then, for given $\varepsilon > 0$, there exists $n_0 > 0$ such that for all $n \ge n_0$ there holds $\sum_{m\ge n} m^{2(\beta-\delta)} < \varepsilon$ and

$$\begin{aligned} P\left(\sup_{0\le t\le T} |P^{\perp}_{n,N} M^N(t)|_{\beta,N} \ge \varepsilon\right) &\le P\left(\sup_{t\le T}\sum_{m\ge n}\langle M_N(t), \hat{e_m}^2\rangle m^{2\beta} \ge \sum_{m\ge n} m^{2(\beta-\delta)}\right) \\ &\le \sum_{m\ge n} P\left(\sup_{t\le T}\langle M_N(t), \hat{e_m}^2\rangle \ge m^{-2\delta}\right) \\ &\le \sum_{m\ge n} \pi^2 m^2 T[\exp(Cm^{2(1-\delta)}) - 1]^{-1}, \end{aligned}$$

where we used (3.3) to obtain the last inequality. Letting $n \to \infty$ yields (i).

Let denote by $T_N(t)$ the semigroup generated by Δ_N. By definition we have

$$T_{N,\alpha}(t)(x) = \int_0^\infty f_{t,\alpha}(s) T_N(s) x\, ds,$$

where $f_{t,\alpha}(s) := \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{zs-tz^{\alpha/2}}\,dz$ for $s \geq 0$. Since for $m = 0,1,2...,N-1$, we have $T_N(s)\hat{e_m} = e^{-s\hat{\beta_m}}\hat{e_m}$, by Proposition 1, p.260 in [19],

$$\begin{aligned} T_{N,\alpha}(t)(\hat{e_m}) &= \int_0^\infty f_{t,\alpha}(s)e^{-s\hat{\beta_m}}\,ds\,\hat{e_m} \\ &= e^{-t\hat{\beta_m}^{\alpha/2}}\hat{e_m}. \end{aligned}$$

Hence $|\langle T_{N,\alpha}(t)X^N(0),\hat{e_m}\rangle| < \exp(-\hat{\beta_m}t)$ and for all natural N and $\beta < -1/2$, we have

$$\sup_{0\leq t\leq T} |T_{N,\alpha}(t)X^N(0)|^2_{\beta,N} \leq C_1 \sum_{m=0}^{N-1} m^{2\beta} < \infty, \tag{3.4}$$

and for $s > 0$ and $\alpha > \beta + 1/2$, we obtain

$$\sup_{s\leq t\leq T} |T_{N,\alpha}(t)X^N(0)|^2_{\beta,N} \leq \sum_{m=0}^{N-1} m^{2\beta-2\alpha} < \infty. \tag{3.5}$$

Using the selfadjointness of the operators $T_{N,\alpha}(t)$ and ∇_N on $H(N)$, it follows that

$$\begin{aligned} \langle V_N(t),\hat{e_m}\rangle &= \langle \int_0^t T_{N,\alpha}(t-s)[\nabla_N X^N(s)^2]\,ds,\hat{e_m}\rangle \\ &= \langle \int_0^t T_{N,\alpha}(t-s)\hat{e_m}, \nabla_N X^N(s)^2\rangle\,ds \\ &= \int_0^t -e^{-(t-s)\hat{\beta_m}^{\alpha/2}}\langle \nabla_N\hat{e_m}, X_N^2(s)\rangle\,ds. \end{aligned} \tag{3.6}$$

Since $4m^2 \leq \hat{\beta_m} \leq \pi^2 m^2$ and $\sup_x |\nabla_N \hat{e_m}(x)| \leq cm$ for some constant $c > 0$ independent of N, (see [1]), we obtain from (3.6), for all natural N, $s \geq 0$ and $\alpha > 3/2 + \beta$,

$$\sup_{s\leq t\leq T} |V_N(t)|^2_{\beta,N} = \sup_{s\leq t\leq T}\sum_{m=0}^{N-1}\langle V_N(t),\hat{e_m}\rangle^2(1+\pi^2m^2)^\beta \leq C_1\sum_{m=0}^{N-1} m^{2(1-\alpha)}m^{2\beta} < \infty, \tag{3.7}$$

where $C_1 = C_1(T)$ is a constant non depending on N.

Part (ii) of the result then follows from (3.4), (3.5), (3.6) and (3.7).

Finally, (iii) follows from (ii) and (2.4). □

Proof of Theorem 2.1.b). Let consider $P_{n,N}X^N = P_nX^N + (P_{n,N} - P_n)X^N$. Since for fixed n, we have $\sup_{t\leq T}|(P_{n,N} - P_n)X^N(t)|_0 \to 0$ in probability as $N \to \infty$, by Lemma

3.1 (ii) we obtain relative compactness for $P_{n,N}X^N$. Now from $X^N = P_{n,N}X^N + P_{n,N}^{\perp}X^N$ and Lemma 3.2(iii) we obtain relative compactness for X^N. $\square$

Proof of Theorem 2.1.c).

From Theorem 2.1b) we know that there exist a process X and a subsequence X^{N_k} of X^N such that $X^{N_k} \Rightarrow X$ in $C([0,\infty), L^2([0,1])$. We will denote X^{N_k} by X^N.

Applying Skorohod's representation theorem, we can construct a sequence $X^{N'}$ and a random element X' on some probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ such that $\{X^N\} \overset{\mathcal{D}}{=} \{X'\}$ and $X^N \to X$ in $C([0,\infty), L^2([0,1]))$ with probability 1 (hence with probability 1 $X^N(t) \to X(t)$). Let us denote

$$K_N(t) := X^N(t) - X^N(0) - \int_0^t \Delta_{N,\alpha} X^N(s)\, ds - \int_0^t \nabla_N X^N(s)^2\, ds.$$

Then by (2.5), $K_N(t)$ is an $H(N)$-valued martingale with $\langle K_N \rangle_t = \int_0^t X^N(t)(1 - X^N(t))\, ds$ and it is straightforward to see that $K_N \to K$ in $L^2([0,1])$ where

$$K(t) := X(t) - X(0) - \int_0^t \Delta_\alpha X(s)\, ds - \int_0^t \nabla X(s)^2\, ds.$$

Moreover, since $K_N(t)$ is uniformly integrable ($\sup_N E(|K_N(t)|_0) < \infty$ uniformly for $t \leq T$), $K(t)$ is a $L^2([0,1])$-martingale with $\langle K \rangle_t = \int_0^t X(s)(1 - X(s))\, ds$. Now as in [11] we can construct on a extended probability space a space-time white noise $W(ds, dx)$ such that $K(t) = \int_0^1 \int_0^t \sqrt{X(t)(1 - X(t)} W(ds, dx)$ and hence $X(t)$ is a weak solution of (1.3). $\square$

Proof of Theorem 2.2.

Let consider the equality

$$\begin{aligned} X(t) &= T_\alpha(t) X(0) + \int_0^t T_\alpha(t-s)[\nabla X(s)^2]\, ds \\ &\quad + \int_0^t T_\alpha(t-s) \sqrt{X(t)(1 - X(t))}\, dB(s) \\ &:= T_\alpha(t) X(0) + V(t) + M(t). \end{aligned} \tag{3.8}$$

As in the proof of Theorem 1.2 and Corollary 1.1 in [2] we obtain

$$P\left(\sup_{0 < \leq s < t \leq T} \frac{|M(t) - M(s)|_\beta}{|t - s|^\delta} < \infty \right) = 1$$

for each $0 < \delta < [(\alpha - 2\beta - 1)/(2\alpha)] \wedge 1/2$, $3/2 < \alpha \leq 2$, and $\beta < \frac{\alpha-1}{2}$. The condition that must hold in order to give the result is

$$\sum_{m=1}^{\infty} m^{\alpha(\delta-1)}(1+m^2)^{\beta} < \infty.$$

Now consider the second term in (3.8) and define $V_m(t) = \langle V(t), e_m \rangle$. From (3.6) we have

$$V_m(t) = m \int_0^t e^{-m^\alpha (t-s)} h_m(s)\, ds$$

for some bounded h_m. From

$$V_m(t) - V_m(s) = (e^{-m^\alpha (t-s)} - 1) g V_m(s) + m \int_s^t (e^{-m^\alpha (t-u)} h(u)\, du,$$

we obtain for $0 \leq s < t$ and a constant c,

$$|V_m(t) - V_m(s)| \leq cm \frac{1 - e^{-m^\alpha (t-s)}}{m^\alpha} \leq cm^{\alpha(\delta-1)+1} |t-s|^\delta, \tag{3.9}$$

where in (3.9) we used

$$(1 - e^{-a|t-s|})/a \leq \min\{|t-s|, a^{\delta-1}|t-s|^\delta\}$$

for $a > 0$ and $0 < \delta \leq 1$.

Hence,

$$\begin{aligned} |V_m(t) - V_m(s)|_\beta^2 &= \sum_m ([V_m(t) - V_m(s)]^2 (1+m^2)^\beta \\ &\leq c \sum_{m=1}^{\infty} m^{2\alpha(\delta-1)+2+2\beta} |t-s|^{2\delta}. \end{aligned}$$

Thus for $0 < \delta < [(2\alpha - 2\beta - 3)/(2\alpha)] \wedge 1/2, 3/2 < \alpha \leq 2$ and $\beta < (2\alpha - 3)/2$ we obtain

$$P\left(\sup_{0 < \leq s < t \leq \infty} \frac{|V(t) - V(s)|_\beta}{|t-s|^\delta} < \infty \right) = 1$$

(note that the equality holds also without the probability sign since the estimates are deterministic).

Finally, for the first term in (3.8) we have

$$|(T(t)-T(s))X(0)|_\beta^2 \le C(s_0,\beta,\alpha)|t-s|^2$$

in the same way as in the proof of Corollary 1.1 in [2]. The proof is complete. $\square$

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