

EQUALITY OF A SET MULTIVARIATE LINEAR MODELS

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ABSTRACT

This work derives a procedure for testing the hypothesis about the equality among T multivariate linear models. The results are extended to the case where the errors follow an elliptical distribution

1 Introduction

The general multivariate linear model can be written as follows

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i, \quad i = 1, \dots, p \quad (1)$$

where $\mathbf{y}_i \in \mathfrak{R}^{n_i \times 1}$, $\mathbf{X}_i \in \mathfrak{R}^{n_i \times q_i}$, $\boldsymbol{\beta}_i \in \mathfrak{R}^{q_i \times 1}$ and under the normal theory, $\boldsymbol{\epsilon}_i \sim \mathcal{N}_{n_i}(\mathbf{0}, \Sigma_i)$. Several particular cases of that model have been studied in the literature. For example, when $n_i = N$ and $\Sigma_i = \Sigma$ for every $i = 1, \dots, p$, it is known as *seemingly unrelated regression model* and it was treated by Zellner (1962) (also see Press (1982, Section 8.5.1, p. 239)). Another special models concerned in statistical literature were given by Zellner (1962) and they can be found when:

- $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \dots = \boldsymbol{\beta}_p$ and the matrices $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_p$ are unequal;
- $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \dots = \boldsymbol{\beta}_p$ and $\mathbf{X}_1 = \mathbf{X}_2 = \dots = \mathbf{X}_p$.

Theory and applications of the last two models are exposed in Box and Tiao (1972, Chapter 9, p. 478).

The most well known particular model in the literature can be obtained from (1) by taking $q_i = q$, $n_i = n$, $\mathbf{X}_i = \mathbf{X}$ and $\Sigma_i = \Sigma$, with $i = 1, \dots, p$. Thus (1) becomes

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \quad (2)$$

where, $\mathbf{Y} = (\mathbf{y}_1 \cdots \mathbf{y}_p) \in \mathbb{R}^{n \times p}$, $\boldsymbol{\beta} = (\boldsymbol{\beta}_1 \cdots \boldsymbol{\beta}_p) \in \mathbb{R}^{q \times p}$, $\boldsymbol{\epsilon} = (\boldsymbol{\epsilon}_1 \cdots \boldsymbol{\epsilon}_p) \sim \mathcal{N}_{n \times p}(\mathbf{0}, \mathbf{I}_n \otimes \boldsymbol{\Sigma})$, and \otimes denotes the Kronecker product. Besides, if $q + p \leq n$, then the maximum likelihood estimators for the parameters $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}$ are given by

$$\tilde{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} = \mathbf{X}^- \mathbf{Y} \quad (3)$$

$$\tilde{\boldsymbol{\Sigma}} = \frac{1}{n}(\mathbf{Y} - \mathbf{X}\tilde{\boldsymbol{\beta}})'(\mathbf{Y} - \mathbf{X}\tilde{\boldsymbol{\beta}}), \quad (4)$$

respectively; where \mathbf{X}^- is the Moore-Penrose inverse of \mathbf{X} ; see Roy (1957), Morrison (1982), Press (1982), Muirhead (1982), Seber (1984) and Rencher (1995), among many others. For different situations, it becomes of interest verifying if the multivariate linear models are equal, when those ones are proposed to model the same situation under different conditions. For example: Suppose p dependent variables Y_1, Y_2, \dots, Y_p , which are functions of q independent variables X_1, X_2, \dots, X_q , will be measured in n individuals, and the model to follow has the form (2). Besides, let us suppose that the above situation is presented in T different conditions (they could be T conditions, T different places, T different temperatures, etc.), but, the remaining factors among the different conditions are homogeneous. So, a question to solve talks about if the dependent variables Y_1, Y_2, \dots, Y_p have the same behaviour under the T different conditions and under different levels in the independent variables X_1, X_2, \dots, X_q . Rigorously, this situation can be expressed as follows: let

$$\mathbf{Y}_t = \mathbf{X}_t \boldsymbol{\beta}_t + \boldsymbol{\epsilon}_t, \quad t = 1, 2, \dots, T, \quad (5)$$

be multivariate linear models, where $\mathbf{Y}_t \in \mathbb{R}^{n_t \times p}$, $\mathbf{X}_t \in \mathbb{R}^{n_t \times q}$ of rank q , $\boldsymbol{\beta}_t \in \mathbb{R}^{q \times p}$ and $\boldsymbol{\epsilon}_t \sim \mathcal{N}_{n_t \times p}(\mathbf{0}, \mathbf{I}_{n_t} \otimes \boldsymbol{\Sigma})$, $\boldsymbol{\Sigma} > \mathbf{0}$. It is the objective to test the hypothesis

$$H_0 : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \cdots = \boldsymbol{\beta}_T \quad \text{vs.} \quad (6)$$

H_a : at least one equality is an inequality

In the univariate case, $p = 1$, it was studied by Graybill (1976, Section 8.6.2, pp. 291-297) and Draper and Smith (1981), among others. The present work proposes several statistics for testing the hypothesis (6) under the conditions of the model (5), see Section 2. The paper ends showing an example as application.

2 Test Statistic

By mixing the conditions of the models (1) and (2), in this section are derived several statistics for testing the hypothesis which establishes that the T multivariate linear models are equal.

Theorem 1. *Given the model (5), the likelihood ratio test of $H_0 : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \cdots = \boldsymbol{\beta}_T$ is given by*

$$\Lambda = \frac{|\mathbf{S}_E|}{|\mathbf{S}_E + \mathbf{S}_H|} \quad (7)$$

which is termed Wilks's Λ or it has also been termed Wilks's U . Where

$$\mathbf{S}_E = \sum_{t=1}^T \mathbf{Y}_t' (\mathbf{I}_{n_t} - \mathbf{X}_t \mathbf{X}_t^-) \mathbf{Y}_t \in \mathbb{R}^{p \times p} \quad (8)$$

$$\mathbf{S}_H = \sum_{t=1}^T \mathbf{Y}_t' (\mathbf{X}_t \mathbf{X}_t^-) \mathbf{Y}_t - \left(\sum_{i=1}^T \mathbf{Y}_i' \mathbf{X}_i \right) \left(\sum_{t=1}^T \mathbf{X}_t' \mathbf{X}_t \right)^{-1} \left(\sum_{j=1}^T \mathbf{X}_j' \mathbf{Y}_j \right) \in \mathbb{R}^{p \times p}. \quad (9)$$

We reject H_0 if

$$\Lambda \leq \Lambda_{\alpha, p, \nu_H, \nu_E},$$

where $\nu_H = (T-1)q$, $\nu_E = N - Tq$, $N = \sum_{t=1}^T n_t$. Exact critical values of $\Lambda_{\alpha, p, \nu_H, \nu_E}$ for Wilks's Λ are found in Rencher (1995, Table A.9) or Kres (1983, Table 1).

Proof. If we write

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \mathbf{Y}_T \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} \mathbf{X}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{X}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}_T \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_T \end{bmatrix} \quad \text{and} \quad \mathbf{E} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_T \end{bmatrix}$$

with $\mathbf{Y} \in \mathfrak{R}^{N \times p}$, $\mathbf{X} \in \mathfrak{R}^{N \times Tq}$ and $\mathbf{B} \in \mathfrak{R}^{Tq \times p}$ and noting that $\mathbf{E} \sim \mathcal{N}_{N \times p}(\mathbf{I}_N \otimes \boldsymbol{\Sigma})$, $N = \sum_{t=1}^T n_t$; then the T models (5) can be written as

$$\mathbf{Y} = \mathbf{X}\mathbf{B} + \mathbf{E}, \tag{10}$$

this is a general multivariate linear model of type (2). By noting that the hypothesis $H_0 : \beta_1 = \beta_2 = \cdots = \beta_T$ can be expressed as $\mathbf{C}\mathbf{B} = 0$, with

$$\mathbf{C} = \begin{pmatrix} \mathbf{I}_q & -\mathbf{I}_q & 0 & \cdots & 0 & 0 \\ 0 & \mathbf{I}_q & -\mathbf{I}_q & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \mathbf{I}_q & -\mathbf{I}_q \end{pmatrix} \in \mathfrak{R}^{(T-1)q \times Tq}, \tag{11}$$

of rank $(T-1)q$; then, it is possible to extend the theory of the model (2) to the model (10). Like this, by Rencher (1995, p. 161), Seber (1984, p. 412) or Muirhead (1982, sections 10.1 and 10.2), among many others, likelihood ratio test is given by

$$\Lambda = \frac{|\mathbf{S}_E|}{|\mathbf{S}_E + \mathbf{S}_H|},$$

where

$$\begin{aligned} \mathbf{S}_E &= \mathbf{Y}'(\mathbf{I}_N - \mathbf{X}\mathbf{X}^-)\mathbf{Y} \\ \mathbf{S}_H &= (\mathbf{C}\widehat{\mathbf{B}})'(\mathbf{C}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\widehat{\mathbf{B}}), \end{aligned}$$

besides, by (3), $\widehat{\mathbf{B}} = \mathbf{X}^- \mathbf{Y} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{Y}$. But, note that

$$\widehat{\mathbf{B}} = \begin{bmatrix} \widehat{\beta}_1 \\ \widehat{\beta}_2 \\ \vdots \\ \widehat{\beta}_T \end{bmatrix} = \begin{bmatrix} (\mathbf{X}'\mathbf{X})_1^{-1} & 0 & \cdots & 0 \\ 0 & (\mathbf{X}'\mathbf{X})_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\mathbf{X}'\mathbf{X})_T^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{X}'_1 \mathbf{Y}_1 \\ \mathbf{X}'_2 \mathbf{Y}_2 \\ \vdots \\ \mathbf{X}'_T \mathbf{Y}_T \end{bmatrix}$$

This is, $\widehat{\beta}_t = \mathbf{X}_t^- \mathbf{Y}_t = (\mathbf{X}'_t \mathbf{X}_t)^{-1} \mathbf{X}'_t \mathbf{Y}_t$, $t = 1, 2, \dots, T$. Thus $\widehat{\beta}_t$ is the same as if it was obtained from the t th model $\mathbf{Y}_t = \mathbf{X}_t \beta_t + \epsilon_t$. Now, by (4), and observing that

$$\mathbf{Y}'\mathbf{Y} = \sum_{t=1}^T \mathbf{Y}'_t \mathbf{Y}_t \quad \text{and} \quad \widehat{\mathbf{B}}' \mathbf{X}' \mathbf{Y} = \sum_{t=1}^T \widehat{\beta}'_t \mathbf{X}'_t \mathbf{Y}_t = \sum_{t=1}^T \mathbf{Y}'_t \mathbf{X}'_t \mathbf{X}_t^- \mathbf{Y}_t,$$

it is gotten

$$\mathbf{S}_E = \sum_{t=1}^T \mathbf{Y}'_t (\mathbf{I}_{n_t} - \mathbf{X}_t \mathbf{X}_t^-) \mathbf{Y}_t$$

Under the null hypothesis, the reduced model $\mathbb{Y} = \mathbb{X}(\mathbf{1} \otimes \boldsymbol{\eta}) + \mathbb{E}$ is obtained, where $\boldsymbol{\eta}$ is the common unknown parameter matrix, $\boldsymbol{\eta} = \boldsymbol{\beta}_1 = \dots = \boldsymbol{\beta}_T$ and $\mathbf{1} = (1, \dots, 1)' \in \mathbb{R}^T$. Taking in count that for \mathbb{B} (or $\boldsymbol{\eta}$) its maximum likelihood estimator coincides with its minimum squared estimator, we can proceed as follows. Let

$$\begin{aligned} Q &= \min_{\boldsymbol{\eta}} \text{tr}(\mathbb{E}'\mathbb{E}) \\ &= \min_{\boldsymbol{\eta}} \text{tr} \left(\sum_{i=1}^T \boldsymbol{\epsilon}'_i \boldsymbol{\epsilon}_i \right) \\ &= \min_{\boldsymbol{\eta}} \text{tr} \left(\sum_{i=1}^T (\mathbf{Y}_t - \mathbf{X}_t \boldsymbol{\eta})' (\mathbf{Y}_t - \mathbf{X}_t \boldsymbol{\eta}) \right) \\ &= \min_{\boldsymbol{\eta}} \text{tr} \left(\sum_{i=1}^T \mathbf{Y}'_t \mathbf{Y}_t - 2 \left(\sum_{i=1}^T \mathbf{Y}'_t \mathbf{X}_t \right) \boldsymbol{\eta} + \boldsymbol{\eta} \left(\sum_{i=1}^T \mathbf{X}_t \mathbf{X}_t \right) \boldsymbol{\eta} \right) \end{aligned}$$

where $\text{tr}(\cdot)$ denote the trace. Thus

$$\hat{\boldsymbol{\eta}} = \left(\sum_{i=1}^T \mathbf{X}_t \mathbf{X}_t \right)^{-1} \left(\sum_{i=1}^T \mathbf{X}'_t \mathbf{Y}_t \right)$$

Then

$$\begin{aligned} \mathbf{S}_H &= \mathbf{Y}'\mathbf{Y} - \hat{\boldsymbol{\eta}}' \mathbf{X}'\mathbf{Y} - \mathbf{S}_E \\ &= \sum_{t=1}^T \mathbf{Y}'_t (\mathbf{X}_t \mathbf{X}_t^-) \mathbf{Y}_t - \left(\sum_{i=1}^T \mathbf{Y}'_i \mathbf{X}_i \right) \left(\sum_{t=1}^T \mathbf{X}'_t \mathbf{X}_t \right)^{-1} \left(\sum_{j=1}^T \mathbf{X}'_t \mathbf{Y}_j \right), \end{aligned}$$

and the desired result is obtained. ■

Alternatively we get:

Theorem 2. *Given the model (5), the union-intersection test of $H_0 : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \dots = \boldsymbol{\beta}_T$ is given by*

$$\theta = \frac{\lambda_1}{1 + \lambda_1} \tag{12}$$

which is termed Roy's largest root test. Where λ_1 is the maximum eigenvalue of $(\mathbf{S}_H \mathbf{S}_E^{-1})$, where \mathbf{S}_H and \mathbf{S}_E are given by (9) and (8), respectively. We reject H_0 if $\theta \geq \theta_{\alpha, s, m, h}$. The parameters s , m and h are defined as

$$s = \min(p, \nu_H), \quad m = (|p - \nu_H| - 1)/2, \quad h = (\nu_E - p - 1)/2.$$

As in Theorem 1, $\nu_H = (T - 1)q$, $\nu_E = N - Tq$ and $N = \sum_{t=1}^T n_t$. Exact critical values of $\theta_{\alpha, s, m, h}$ are found in Rencher (1995, Table A.10) or Kres (1983, Tables 2, 4 and 5).

Proof. We proceed as in Theorem 1: by (10) and (11), the multivariate hypothesis can be expressed as

$$H_0 : \mathbf{C}\mathbb{B} = \mathbf{0}$$

which is true if and only if the univariate hypotheses

$$H_{0\mathbf{a}} : \mathbf{C}\mathbb{B}\mathbf{a} = 0$$

hold for all non-null vectors \mathbf{a} . The statistic for all the univariate hypothesis is given by

$$F(\mathbf{a}) = \frac{(N - Tq) \sum_{t=1}^T \mathbf{a}' \mathbf{Y}'_t (\mathbf{X}_t \mathbf{X}_t^-) \mathbf{Y}_t \mathbf{a} - \mathbf{a}' \left(\sum_{i=1}^T \mathbf{Y}'_i \mathbf{X}_i \right) \left(\sum_{t=1}^T \mathbf{X}'_t \mathbf{X}_t \right)^{-1} \left(\sum_{j=1}^T \mathbf{X}'_j \mathbf{Y}_j \right) \mathbf{a}}{(T-1)q \sum_{t=1}^T \mathbf{a}' \mathbf{Y}'_t (\mathbf{I}_{n_t} - \mathbf{X}_t \mathbf{X}_t^-) \mathbf{Y}_t \mathbf{a}}$$

where $N = \sum_{t=1}^T n_t$, see Graybill (1976, Theorem 8.6.4, p. 291). For an univariate test of confidence level γ , $H_{0\mathbf{a}} : \mathbf{C}\mathbb{B}\mathbf{a} = 0$ is accepted if

$$F(\mathbf{a}) \leq F_{\gamma, (T-1)q, N-Tq}$$

where $F_{\gamma, (T-1)q, N-Tq}$ is the upper γ probability point of the central F -distribution with $(T-1)q$ and $N - Tq$ degrees freedom. So, proceeding as in Roy (1957, Section 12.7, pp. 82-83) (also see Morrison (1982, pp. 176-177)), we have for $H_0 = \bigcap_{\mathbf{a}} H_{0\mathbf{a}}$, the critical region of size $\alpha (> \gamma)$ is given by

$$\bigcap_{\mathbf{a}} [F(\mathbf{a}) \leq F_{\gamma, (T-1)q, N-Tq}]$$

which is equivalent to that defined by

$$\max_{\mathbf{a}} F(\mathbf{a}) \leq F_{\gamma, (T-1)q, N-Tq}$$

over all non-null \mathbf{a} . This way we reject H_0 if

$$\theta \geq \theta_{\alpha, s, m, h}$$

with

$$s = \min(p, (T-1)q), \quad m = (|p - (T-1)q| - 1)/2, \quad h = (N - Tq - p - 1)/2, \quad N = \sum_{t=1}^T n_t.$$

where $\theta = \lambda_1 / (1 + \lambda_1)$, λ_1 is the maximum eigenvalue of $(\mathbf{S}_H \mathbf{S}_E^{-1})$ and

$$\begin{aligned} \mathbf{S}_E &= \sum_{t=1}^T \mathbf{Y}'_t (\mathbf{I}_{n_t} - \mathbf{X}_t \mathbf{X}_t^-) \mathbf{Y}_t \\ \mathbf{S}_H &= \sum_{t=1}^T \mathbf{Y}'_t (\mathbf{X}_t \mathbf{X}_t^-) \mathbf{Y}_t - \left(\sum_{i=1}^T \mathbf{Y}'_i \mathbf{X}_i \right) \left(\sum_{t=1}^T \mathbf{X}'_t \mathbf{X}_t \right)^{-1} \left(\sum_{j=1}^T \mathbf{X}'_j \mathbf{Y}_j \right). \end{aligned}$$

also see Rencher (1995, Section 6.1.4, p. 164), among many others. ■

A lot of different test statistics have been proposed for verifying hypothesis of the kind (6). Before to show some of them, let us consider the following notation: given $s = \min(p, \nu_H)$, let $\lambda_1, \dots, \lambda_s$ be the eigenvalues of the matrix $(S_H S_E^{-1})$ such that $\lambda_1 \geq \dots \geq \lambda_s > 0$ and $\theta_1, \dots, \theta_s$ the eigenvalues of the matrix $(S_H(S_E + S_H)^{-1})$, with $1 \geq \theta_1 \geq \dots \geq \theta_s > 0$. Observe that $\theta_i = \lambda_i/(1 + \lambda_i)$, and, $\lambda_i = \theta_i/(1 - \theta_i)$, $i = 1, \dots, s$. Thus, the statistic Λ of Wilks can be written as:

$$\Lambda = \frac{|\mathbf{S}_E|}{|\mathbf{S}_E + \mathbf{S}_H|} = \prod_{i=1}^s \frac{1}{1 + \lambda_i} = \prod_{i=1}^s (1 - \theta_i)$$

from where it is followed that, the range of Λ is $0 \leq \Lambda \leq 1$. Two of these additional test statistics for the hypothesis $H_0 : \beta_1 = \beta_2 = \dots = \beta_T$ are the following:

1. **Pillai Test.** The Pillai statistics is defined as

$$V^{(s)} = \text{tr}[\mathbf{S}_H(\mathbf{S}_E + \mathbf{S}_H)^{-1}] = \sum_{i=1}^s \frac{\lambda_i}{1 + \lambda_i} = \sum_{i=1}^s \theta_i \quad (13)$$

This way we reject H_0 if

$$V^{(s)} \geq V_{\alpha, s, m, h}^{(s)}$$

with

$$s = \min(p, (T - 1)q), \quad m = (|p - (T - 1)q| - 1)/2, \quad h = (N - Tq - p - 1)/2, \quad N = \sum_{t=1}^T n_t.$$

and where the exact critical values of $V_{\alpha, s, m, h}^{(s)}$ are found in Rencher (1995, Table A.11) or Kres (1983, Table 7).

2. **Lawley-Hotelling Test.** The Lawley-Hotelling statistics is given by

$$U^{(s)} = \text{tr}[\mathbf{S}_H \mathbf{S}_E^{-1}] = \sum_{i=1}^s \lambda_i = \sum_{i=1}^s \frac{\theta_i}{1 - \theta_i}. \quad (14)$$

We reject H_0 if

$$U^{(s)} \geq U_{\alpha, s, m, h}^{(s)}$$

with

$$s = \min(p, (T - 1)q), \quad m = (|p - (T - 1)q| - 1)/2, \quad h = (N - Tq - p - 1)/2, \quad N = \sum_{t=1}^T n_t.$$

The upper percentage points, $U_{\alpha, s, m, h}^{(s)}$, are given in Kres (1983, Table 6). A variant of this statistics and its corresponding exact critical values are given in Rencher (1995, p. 167 and Table A.12, respectively).

Finally, note that by the theorems 5.3.3 and 5.3.4 of Gupta and Varga (1993, pp. 185-186), the four above-mentioned test statistics are invariant under the whole family of elliptical distributions, more over, their distribution coincide under normality assumption.

3 Example

The following example was taken from Graybill (1976, p. 295) and it have been modified by adding a new depend variable Y_2 by simulation.

A new food supplement (x unit) was fed to three different breeds of chickens for six weeks to determine the effect on hardness Y_1 and weight Y_2 (gr.) of eggs. A simple linear multivariate model was assumed for each breed.

$$\mathbf{Y}_t = \mathbf{X}_t \boldsymbol{\beta}_t + \boldsymbol{\epsilon}_t, \quad t = 1, 2, 3$$

$\boldsymbol{\epsilon}_t \sim \mathcal{N}_{n_t \times 2}(\mathbf{0}, \mathbf{I}_{n_t} \otimes \boldsymbol{\Sigma})$, $\boldsymbol{\Sigma} \in \mathfrak{R}^{2 \times 2}$, $\boldsymbol{\Sigma} > \mathbf{0}$, with $n_1 = 12$, $n_2 = 8$ and $n_3 = 9$ and

$$\boldsymbol{\beta}_t = \begin{pmatrix} \beta_{01t} & \beta_{02t} \\ \beta_{11t} & \beta_{12t} \end{pmatrix}$$

The problem is to determine if the models are the same for all breeds, that is, to test the hypothesis

$$H_0 : \begin{pmatrix} \beta_{011} & \beta_{021} \\ \beta_{111} & \beta_{121} \end{pmatrix} = \begin{pmatrix} \beta_{012} & \beta_{022} \\ \beta_{112} & \beta_{122} \end{pmatrix} = \begin{pmatrix} \beta_{013} & \beta_{023} \\ \beta_{113} & \beta_{123} \end{pmatrix}$$

The data are given next in Table 1.

Breed 1			Breed 2			Breed 3		
X	Y_1	Y_2	X	Y_1	Y_2	X	Y_1	Y_2
1	8.42	74.2	3	9.86	72.5	2	6.52	70.1
3	14.68	69.1	3	9.54	71.2	5	5.11	72.2
5	21.42	63.5	4	11.96	69.8	7	7.75	69.3
6	25.45	62.8	5	12.46	67.8	8	6.84	68.5
7	27.14	60.0	6	11.38	66.5	10	7.65	66.9
8	30.53	57.1	8	14.69	62.3	15	9.49	63.2
9	34.51	55.2	9	16.48	60.1	16	7.03	61.4
9	34.52	54.9	12	20.11	55.7	18	9.41	59.7
10	33.24	53.6				20	12.01	55.6
11	39.63	50.4						
12	43.98	47.3						
14	47.77	44.4						

By (8) and (9), we have that

$$\mathbf{S}_E = \begin{pmatrix} 39326.040 & 39837.725 \\ 39837.725 & 19800.195 \end{pmatrix} \quad \text{and} \quad \mathbf{S}_H = \begin{pmatrix} 4145.392 & -8404.961 \\ -8404.961 & 23839.162 \end{pmatrix}$$

then, the following results are obtained

It is clear that the four criterions reject the above-mentioned hypothesis.

Table 1: Four criteria to proof the null hypothesis

Criteria	Statistic	α Critical value
Wilks ^a	5.856e-006	0.515922
Roy	0.99977	0.415
Pillai	1.97383	0.532
Lawley-Hotelling	11721.5216	2.16811 ^b

^aRemember that for this tests, the decision rule is: statistics \leq critical value

^bUsing an F approximation, see equation (6.30) in Rencher (1995, p. 167).

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