Selfdecomposability of Fractional Lévy Fields

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Abstract

In this article we consider fractional Lévy fields, which are obtained as integrals of a deterministic kernel with respect to a Lévy random measure. Actually the kernel is such that if the Lévy random measure is a Brownian random measure the fractional Lévy field is a fractional Brownian motion. We show that the selfdecomposability of the driving Lévy random measure is a sufficient but not a necessary condition to have selfdecomposability of one dimensional margins of the fractional Lévy field. Moreover we show that there is an equivalence between selfdecomposability as a process of the driving Lévy random measure and of the fractional Lévy process.

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Many fractional fields have been introduced in the literature to generalize fractional Brownian motion including fractional stable processes (See [11]) and more recently fractional Lévy fields (cf. [3, 4, 6, 8]), whose marginal distributions are infinitely divisible. On the one hand most studies on fractional processes are concerned with sample paths properties or integration with respect to these processes (See [10]). On the other hand various subclasses of infinitely divisible distribution are known (see for instance [7, 5, 2, 9]). The aim of this article is to investigate properties of the distribution of some of these fractional fields. In this paper we ask when the distributions of fractional Lévy fields are selfdecomposable. Let us be more specific. Generically fractional Lévy fields $Y_t = (Y_t)_{t \in \mathbb{R}}$ are obtained as integrals of a driving Lévy random measure $M(ds), s \in \mathbb{R}$, so that

$$Y_t \overset{def}{=} \int_{\mathbb{R}} f_t(s) M(ds),$$

when the integral of the deterministic kernel $f_t(s)$ is properly defined. (Precise assumptions are given later.) For instance, moving average can be defined as

$$Y_t \overset{def}{=} \int_{\mathbb{R}} \left( (t-s)_{+}^{\beta} - (-s)_{+}^{\beta} \right) M(ds),$$

where $\beta \in (0, 1/2)$. We will show that if $X_1 = \int_{[0,1]} M(ds)$ is selfdecomposable, then $Y_t$ is selfdecomposable. Actually it is the consequence of a general fact for all marginals of $\int f_t(s) M(ds)$. It turns out that this sufficient condition is not necessary and an example where $Y_t$ is not selfdecomposable and $X_1$ is selfdecomposable is exhibited. Another question is selfdecomposability of $(Y_t)_{t \in \mathbb{R}}$ as a process. We can find definition of selfdecomposability of processes in [1]. In the same article the authors show that Ornstein-Uhlenbeck process

$$O_t \overset{def}{=} e^{-\gamma t} V_0 + \int_0^t e^{-\gamma (t-s)} dX_s, \quad t > 0,$$

where $V_0$ is equal in distribution to $\int_0^{+\infty} e^{-\gamma s} dX_s$, and independent of $X = (X_s)_{s \geq 0}$, is selfdecomposable as a process if and only if $X$ is selfdecomposable as a process. We will show that this necessary and sufficient condition holds for moving average fractional Lévy process. Hence the answer to our main question is different when we take a marginal point of view or a process point of view. Please note that our proof of the equivalence for moving average fractional Lévy process is different from the one in [1].

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References


