Exercises for Section 7.2

Evaluate each of the integrals in Exercises 1–6 by making the indicated substitution, and check your answers by differentiating.

1. $\int 2x(x^2+4)^{3/2}\,dx;\ u = x^2+4.$
2. $\int (x+1)(x^2+2x-4)^{-4}\,dx;\ u = x^2+2x-4.$
3. $\int \frac{2y^7+1}{(y^8+4y-1)^2}\,dy;\ x = y^8+4y-1.$
4. $\int \frac{x}{1+x^4}\,dx;\ u = x^2.$
5. $\int \frac{\sec^2\theta}{\tan^3\theta}\,d\theta;\ u = \tan\theta.$
6. $\int \tan x\,dx;\ u = \cos x.$

Evaluate each of the integrals in Exercises 7–22 by the method of substitution, and check your answer by differentiating.

7. $\int (x+1)\cos(x^2+2x)\,dx$
8. $\int u\sin(u^2)\,du$
9. $\int \frac{x^3}{\sqrt{x^4+2}}\,dx$
10. $\int \frac{x}{(x^2+3)^2}\,dx$
11. $\int \frac{t^{1/3}}{(t^{4/3}+1)^{3/2}}\,dt$
12. $\int \frac{x^{1/2}}{(x^{3/2}+2)^2}\,dx$
13. $\int 2r\sin(r^2)\cos^3(r^2)\,dr$
14. $\int e^{\sin x}\cos x\,dx$
15. $\int \frac{x^3}{1+x^8}\,dx$
16. $\int \frac{dx}{\sqrt{1-4x^2}}$
17. $\int \sin(\theta+4)\,d\theta$
18. $\int \frac{1}{x^2}\sin\frac{1}{x}\,dx$
19. $\int (5x^4+1)(x^5+x)^{100}\,dx$
20. $\int (1+\cos s)\sqrt{s+\sin s}\,ds$
21. $\int \left(\frac{t+1}{\sqrt{t^2+2t+3}}\right)dt$
22. $\int \frac{dx}{x^2+4}$

Evaluate the indefinite integrals in Exercises 23–36.

23. $\int t\sqrt{t^2+1}\,dt.$
24. $\int t\sqrt{t+1}\,dt.$
25. $\int \cos^3\theta\,d\theta.$ [*Hint*: Use $\cos^2\theta+\sin^2\theta = 1$.]
26. $\int \cot x\,dx.$
27. $\int \frac{dx}{x\ln x}.$
28. $\int \frac{dx}{\ln(x^x)}.$
29. $\int \sqrt{4-x^2}\,dx.$ [*Hint*: Let $x = 2\sin u$.]
30. $\int \sin^2 x\,dx.$ (Use $\cos 2x = 1-2\sin^2 x$.)
31. $\int \frac{\cos\theta}{1+\sin\theta}\,d\theta.$
32. $\int \sec^2 x(e^{\tan x}+1)\,dx.$
33. $\int \frac{\sin(\ln t)}{t}\,dt.$
34. $\int \frac{e^{2s}}{1+e^{2s}}\,ds.$
35. $\int \frac{\sqrt[3]{3+1/x}}{x^2}\,dx.$
36. $\int \frac{1}{x^3}\left(1-\frac{1}{x^2}\right)^{1/3}dx.$
37. Compute $\int \sin x\cos x\,dx$ by each of the following three methods: (a) Substitute $u = \sin x$, (b) substitute $u = \cos x$, (c) use the identity $\sin 2x = 2\sin x\cos x$. Show that the three answers you get are really the same.
38. Compute $\int e^{ax}\,dx$, where a is constant, by each of the following substitutions: (a) $u = ax$; (b) $u = e^x$. Show that you get the same answer either way.

★39. For which values of m and n can $\int \sin^m x\cos^n x\,dx$ be evaluated by using a substitution $u = \sin x$ or $u = \cos x$ and the identity $\cos^2 x + \sin^2 x = 1$?

★40. For which values of r can $\int \tan^r x\,dx$ be evaluated by the substitution suggested in Exercise 39?