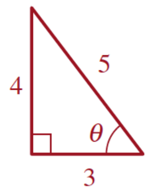
Revisar tarea.

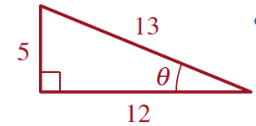
1.



$\operatorname{sen}\theta = \frac{4}{5}$

FIGURA 8.2.6 Triángulo del problema 1

2.

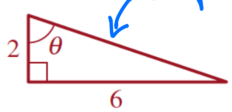


$\operatorname{sen}\theta = \frac{5}{13}$

FIGURA 8.2.7 Triángulo del problema 2

$$\operatorname{sen}\theta = \frac{\text{cateto opuesto}}{\text{hipotenusa}}$$

3.

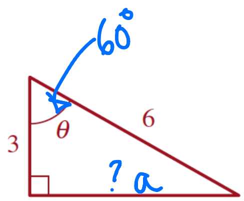


$h = \sqrt{4+36} = \sqrt{40}$
$= 2\sqrt{10}$

$$\operatorname{sen}\theta = \frac{6}{\sqrt{40}} = \frac{6}{2\sqrt{10}} = \frac{3}{\sqrt{10}} < 1$$

FIGURA 8.2.8 Triángulo del problema 3

4.

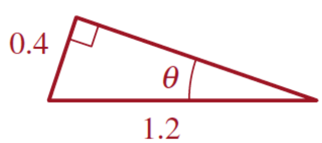


$$\operatorname{sen}\theta = \frac{a}{6} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

FIGURA 8.2.9 Triángulo del problema 4

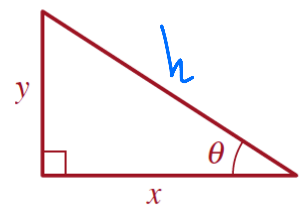
$$a^2 + 3^2 = 6^2$$
$$a^2 + 9 = 36 \Rightarrow a^2 = 27 \Rightarrow a = \sqrt{27} = \sqrt{3}\sqrt{9} = 3\sqrt{3}$$

7.



$$\operatorname{sen}\theta = \frac{0.4}{1.2} = \frac{4}{12} = \frac{1}{3}$$

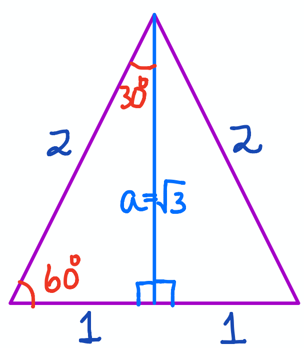
9.



$$\operatorname{sen}\theta = \frac{y}{h} = \frac{y}{\sqrt{x^2+y^2}}$$

$$h = \sqrt{x^2+y^2}$$

Prueba que $\operatorname{sen} 60^\circ < 2 \operatorname{sen} 30^\circ$



$$a^2 + 1 = 2^2 = 4$$

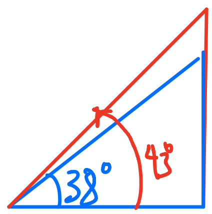
$$\Rightarrow \quad a^2 = 3 \quad \Rightarrow \quad a = \sqrt{3}$$

$$\operatorname{sen} 60^\circ = \frac{\sqrt{3}}{2}$$

$$\operatorname{sen} 30^\circ = \frac{1}{2}$$

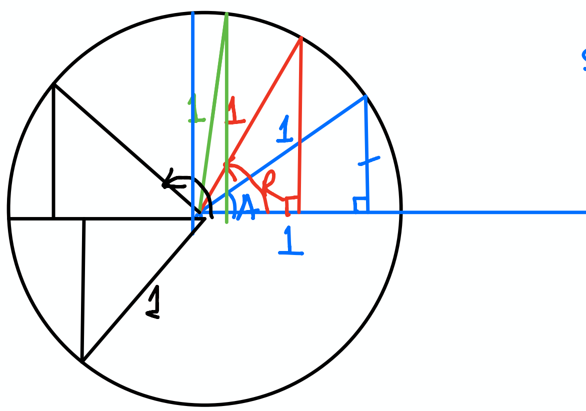
$$2 \operatorname{sen} 30^\circ = 1 > \frac{\sqrt{3}}{2} = \operatorname{sen} 60^\circ$$

$$1^2 = 1 \overset{?}{>} \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} \qquad á$$



$\text{sen } 43° = \dfrac{\text{cateto rojo}}{\text{hip. roja}}$?

$\text{sen } 38° = \dfrac{\text{cateto azul}}{\text{hip. azul.}}$



$\text{sen } A = \dfrac{\text{cat op. azul}}{1}$

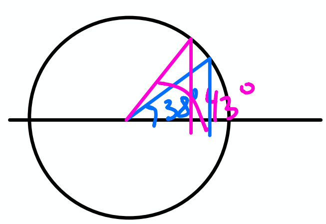
$\wedge$

$\text{sen } R = \dfrac{\text{cat. op. rojo}}{1}$

$\wedge$

$\text{sen } V = \dfrac{\text{cat op. verde}}{1}$

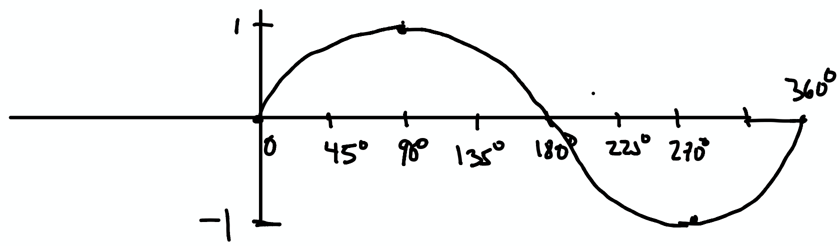
$\text{Sen } 38° < \text{sen } 43°$



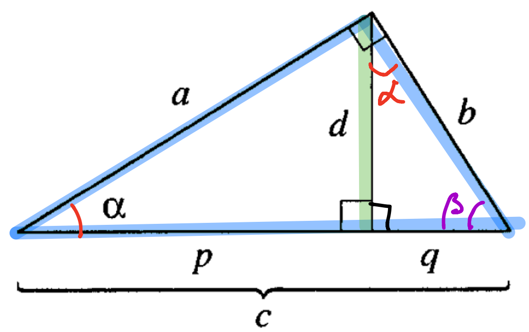
$\text{Sen } 0° = 0$

$\text{Sen } 90° = 1$

sen θ



1. The diagram below shows a right triangle with an altitude drawn to the hypotenuse. The small letters stand for the lengths of certain line segments.



$\alpha + \beta = 90^\circ$

a) Find a ratio of the lengths of two segments equal to $\sin \alpha$.

b) Find another ratio of the lengths of two segments equal to $\sin \alpha$.

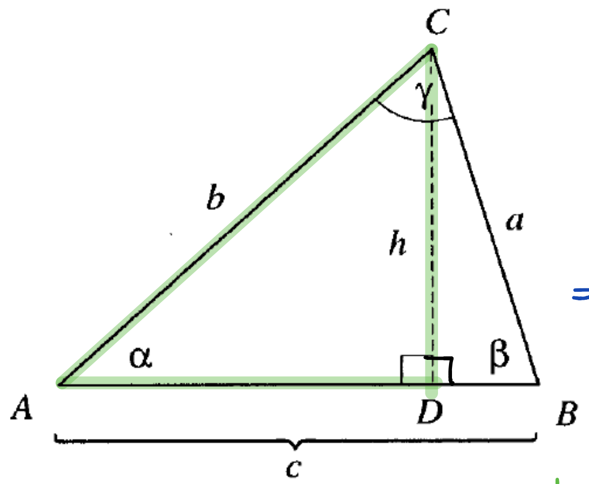
c) Find a third ratio of the lengths of two segments equal to $\sin \alpha$.

a) $\sin \alpha = \frac{d}{a}$

b) $\sin \alpha = \frac{b}{c}$ ✓

c) $\text{sen}\,\alpha = \frac{q}{b}$

2. The three angles of triangle ABC below are acute (in particular, none of them is a right angle), and CD is the altitude to side AB. We let $CD = h$, and $CA = b$.



$\text{sen}\,\beta = \frac{h}{a}$

$\Rightarrow h = a\,\text{sen}\,\beta$

a) Find a ratio equal to $\sin\alpha$. $\text{sen}\,\alpha = \frac{h}{b}$

b) Express h in terms of $\sin\alpha$ and b. $h = b\,\text{sen}\,\alpha$

c) We know that the area of triangle ABC is $hc/2$. Express this area in terms of b, c, and $\sin\alpha$. $\text{Area} = \frac{c \cdot b\,\text{sen}\,\alpha}{2}$

d) Express the area of triangle ABC in terms of a, c, and $\sin\beta$.

e) Express the length of the altitude from A to BC in terms of c and $\sin\beta$. (You may want to draw a new diagram, showing the altitude to side BC.)

$\text{Area} = \frac{c \cdot a\,\text{sen}\,\beta}{2}$ ✓

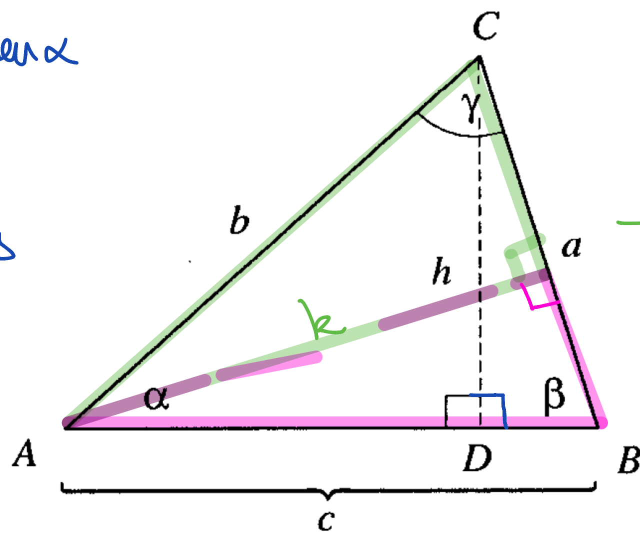
$a \operatorname{sen}\beta = b \operatorname{sen}\alpha$

$\Downarrow$

$\dfrac{a}{\operatorname{sen}\alpha} = \dfrac{b}{\operatorname{sen}\beta}$

$\|$

$\dfrac{b}{\operatorname{sen}\beta} = \dfrac{c}{\operatorname{sen}\gamma}$



$h = b \operatorname{sen}\alpha$

$\|$

$a \operatorname{sen}\beta$

$\operatorname{sen}\gamma = \dfrac{k}{b}$

$\operatorname{sen}\beta = \dfrac{k}{c}$

$b \operatorname{sen}\gamma = c \operatorname{sen}\beta$

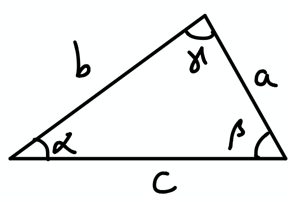
3. a) Using the diagram above, write two expressions for h: one using side b and $\sin \alpha$ and one using side a and $\sin \beta$.

   b) Using the result to part (a), show that $a \sin \beta = b \sin \alpha$.

   c) Using the result of part (e) in problem 2 above, show that $c \sin \beta = b \sin \gamma$.

   d) Prove that $\dfrac{a}{\sin \alpha} = \dfrac{b}{\sin \beta} = \dfrac{c}{\sin \gamma}$. This relation is true for any acute triangle (and, as we will see, even for any obtuse triangle). It is called the *Law of Sines*.

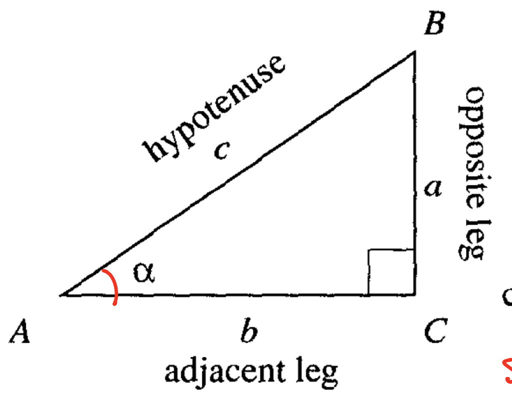
$\Uparrow$



El coseno

Definition: In a right triangle with acute angle α, the ratio of the leg adjacent to angle α to the hypotenuse is called the *cosine* of angle α, abbreviated $\cos\alpha$.

Tengo un ángulo α, lo mido en un Δ rectángulo ABC.

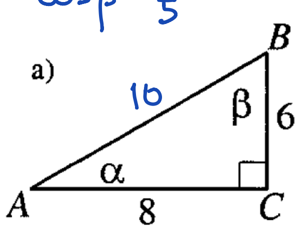


$$\cos\ \alpha = \frac{b}{c} = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{\text{cat. adyacente}}{\text{hipotenusa}}$$

$$\operatorname{sen}\alpha = \frac{a}{c}$$

2. Find the cosines of angles α and β in each triangle below.

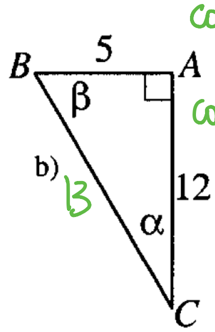
$\cos\beta = \frac{3}{5} = \operatorname{sen}\alpha$



$\cos\alpha = \frac{8}{10} = \frac{4}{5} = \operatorname{sen}\beta$

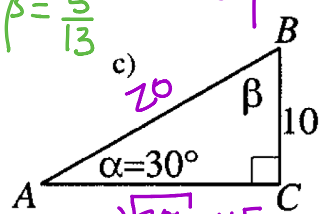
$\cos\alpha = \frac{12}{13}$

$\cos\beta = \frac{5}{13}$



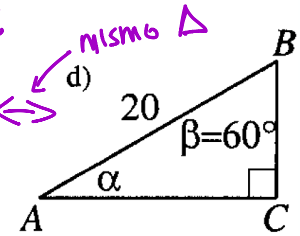
$\cos\alpha = \frac{\sqrt{300}}{20} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$

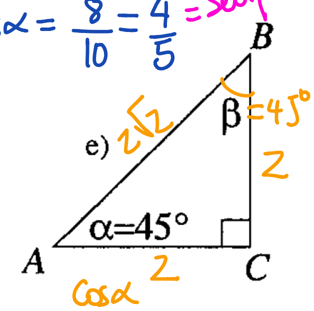
$\cos\beta = \frac{1}{2}$



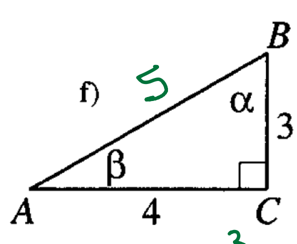
$\sqrt{300} = 10\sqrt{3}$

$\Leftrightarrow$ mismo Δ





$\cos\alpha = \cos\beta = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

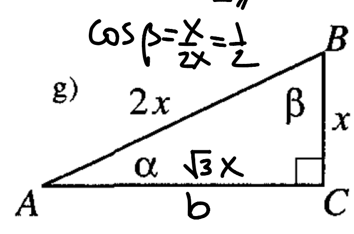


$\cos\alpha = \frac{3}{5}$

$\cos\beta = 4/5$

$\cos\alpha = \frac{\sqrt{3}\cancel{x}}{2\cancel{x}} = \frac{\sqrt{3}}{2}$

$\cos\beta = \frac{x}{2x} = \frac{1}{2}$

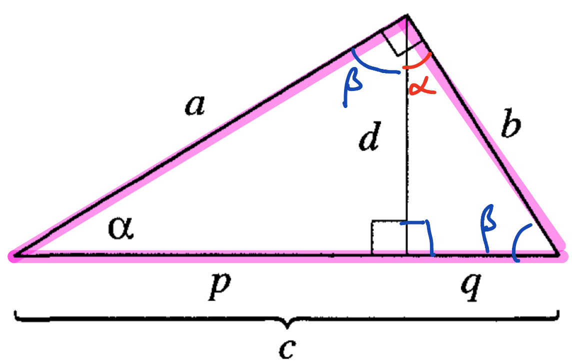


$b^2 + x^2 = 4x^2 \Rightarrow b^2 = 3x^2$

$b = \sqrt{3}x$

También podemos mirar los cosenos de los ángulos en los Δ's de la tarea 4.

3. The diagram below shows a right triangle with an altitude drawn to the hypotenuse. The small letters stand for the lengths of certain line segments.



a) Find a ratio of the lengths of two segments equal to $\cos\alpha$.

b) Find another ratio of the lengths of two segments equal to $\cos\alpha$.

c) Find a third ratio of the lengths of two segments equal to $\cos\alpha$.

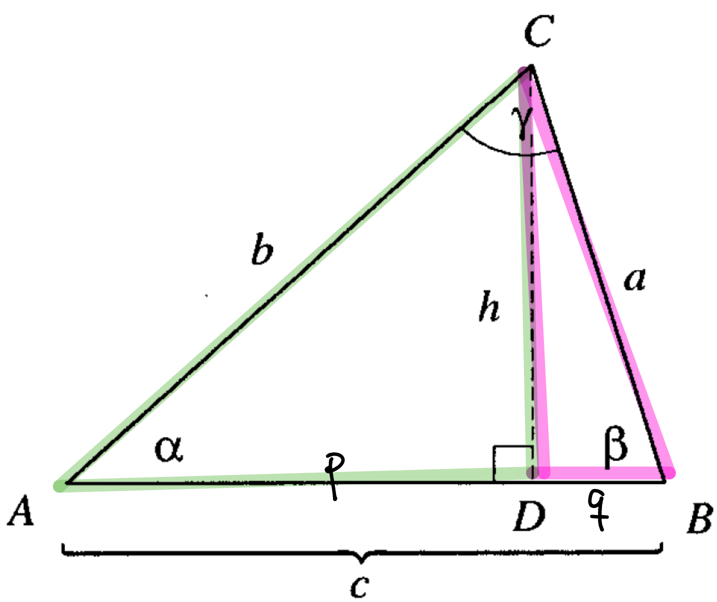
a) $\cos\alpha = p/a$ $\qquad \cos\beta = \frac{q}{b}$

b) $\cos\alpha = \frac{a}{c}$ $\qquad \cos\beta = \frac{b}{c}$

c) $\cos\alpha = \frac{d}{b}$ $\qquad \cos\beta = \frac{d}{a}$

$d = b\cos\alpha = a\cos\beta$

$$\frac{a}{\cos\alpha} = \frac{b}{\cos\beta}$$ ✓ $\alpha+\beta=90^\circ$



$$\cos\alpha = \frac{p}{b} \qquad \cos\beta = \frac{q}{a} \Rightarrow \begin{array}{l} p = b\cos\alpha \\ q = a\cos\beta \end{array}$$

Pitágoras para el ΔADC:

$$p^2+h^2=b^2 \Rightarrow h^2=b^2-p^2$$

Pitágoras para el ΔBDC

$$q^2+h^2=a^2 \Rightarrow h^2 = a^2-q^2$$

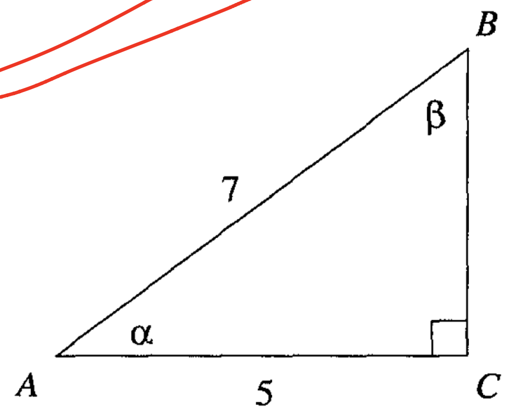
$$b^2-p^2=a^2-q^2 \Rightarrow b^2-b^2\cos^2\alpha = a^2-a^2\cos^2\beta$$

$$b^2(1-\cos^2\alpha) = a^2(1-\cos^2\beta)$$

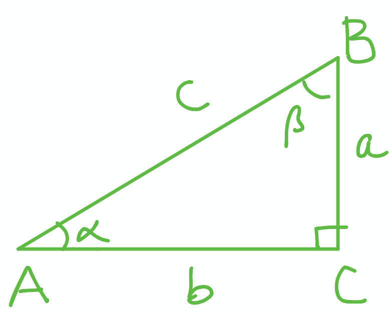
$$\boxed{\frac{a^2}{b^2} = \frac{1-\cos^2\alpha}{1-\cos^2\beta}}$$

Example 14 In the following diagram, $\cos\alpha = 5/7$. What is the numerical value of $\sin\beta$?

$\text{sen}\,\beta = \frac{5}{7}$



OjO: Mira un Δ rectángulo ABC

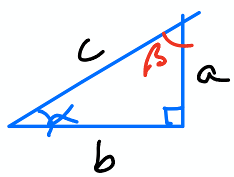


$\text{sen}\,\alpha = \frac{a}{c} = \cos\beta$

$\text{sen}\,\beta = \frac{b}{c} = \cos\alpha$

Teorema: Si $\alpha+\beta=90^\circ \Rightarrow \operatorname{sen}\alpha = \cos\beta$.

Prueba. Mira un triángulo rectángulo cualquiera que tenga a α como ángulo: Entonces el otro ángulo agudo del Δ es β.

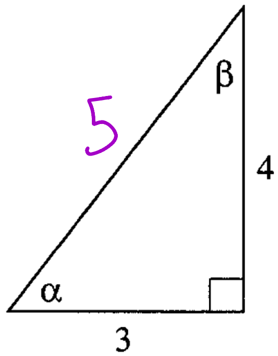


$$\operatorname{sen}\alpha = \frac{a}{c} = \cos\beta$$

Exercises

1. Show that $\sin 29^\circ = \cos 61^\circ$. $29^\circ + 61^\circ = 90^\circ$ ✓
2. If $\sin 35^\circ = \cos x$, what could the numerical value of x be? $X = 55^\circ$
3. Show that we can rewrite the theorem of the above section as: $\sin\alpha = \cos(90-\alpha)$. ✓

In the diagram below, find the numerical value of the following expressions:



1. $\sin^2 \alpha$
2. $\sin^2 \beta$
3. $\cos^2 \alpha$
4. $\cos^2 \beta$
5. $\sin^2 \alpha + \cos^2 \alpha$
6. $\sin^2 \alpha + \cos^2 \beta$
7. $\cos^2 \alpha + \sin^2 \beta$

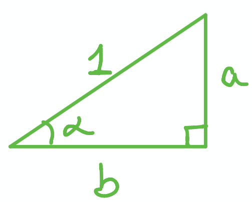
$\text{Sen}\,\alpha = \frac{4}{5}$ $\qquad$ $\text{sen}^2\alpha = \frac{16}{25}$

$\text{Cos}\,\alpha = \frac{3}{5}$ $\qquad$ $\cos^2\alpha = \frac{9}{25}$

$\text{sen}^2\alpha + \cos^2\alpha = 1$

$\cos\alpha = b$

$\text{Sen}\,\alpha = a$



Pitágoras: $a^2 + b^2 = 1$

$\parallel$

$\text{Sen}^2\alpha + \cos^2\alpha$ ✓