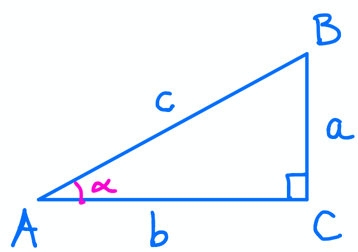
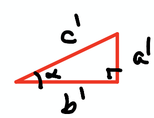
Breve repaso: Comienzas con un ángulo agudo α. Lo metes en cualquier triángulo rectángulo.



$$\operatorname{sen}\alpha = \frac{a}{c}$$

$$\cos\alpha = \frac{b}{c}$$

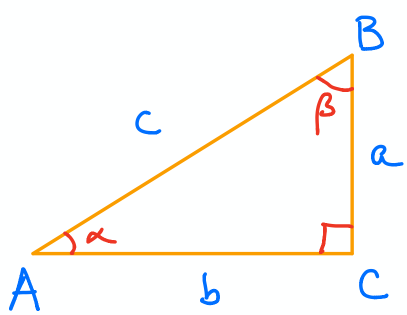
Y esta definición <u>NO</u> depende del triángulo rectángulo en el cual metimos a α. (porque cualquier otro Δ rectángulo con ángulo α será similar:



$$a' = ka$$
$$b' = kb$$
$$c' = kc$$

porque son similares

$$\Rightarrow \frac{a'}{c'} = \frac{\cancel{k}a}{\cancel{k}c} = \frac{a}{c}$$



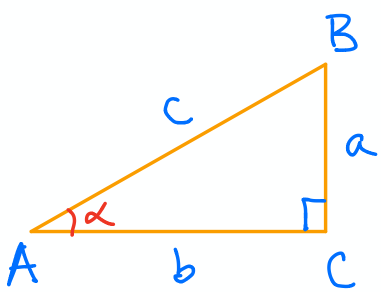
$$\cos\alpha = \frac{b}{c} = \operatorname{sen}\beta$$

$$\operatorname{sen}\alpha = \frac{a}{c} = \cos\beta$$

Teorema Si α es un ángulo agudo, entonces

$$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

Demo. Mete α en un triángulo rectángulo $\triangle ABC$



$$\cos\alpha = \frac{b}{c}, \quad \operatorname{sen}\alpha = \frac{a}{c}$$

Pitágoras

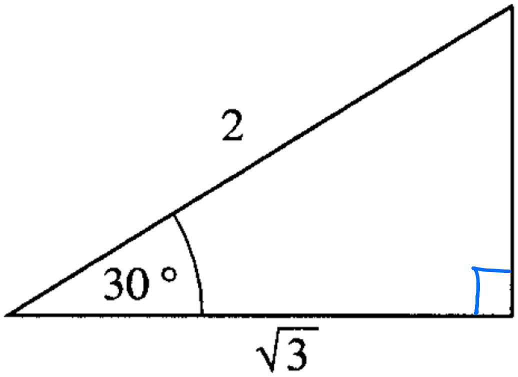
$$\cos^2\alpha + \operatorname{sen}^2\alpha = \frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{b^2+a^2}{c^2} = \frac{c^2}{c^2} = 1 \quad \blacksquare$$

4. Find the value of $\cos\alpha$ if α is an acute angle and $\sin\alpha = 5/13$.

5. Find the value of $\cos\alpha$ if α is an acute angle and $\sin\alpha = 5/7$.

6. If α and β are acute angles in the same right triangle, show that $\sin^2\alpha + \sin^2\beta = 1$.

Example 15 Find $\cos 30°$.

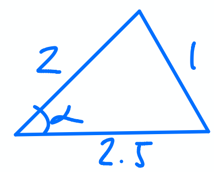


¿Cómo sé que este △ es rectángulo?

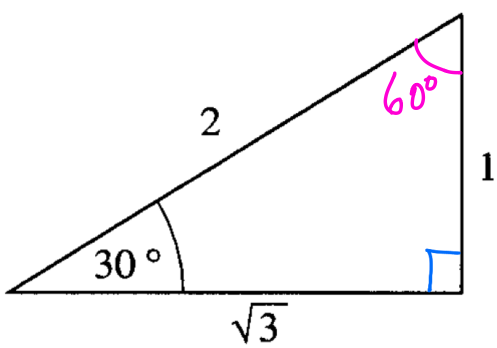
$$2^2 \stackrel{?}{=} 1^2 + (\sqrt{3})^2$$

$$4 = 1 + 3 \checkmark$$ sí

$$\cos 30° = \frac{\sqrt{3}}{2} \checkmark$$

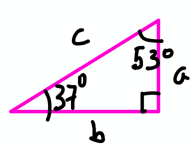


Example 16 Show that $\cos 60° = \sin 30°$.



$$\cos 60° = \frac{1}{2} = \text{sen}\, 30° \checkmark$$

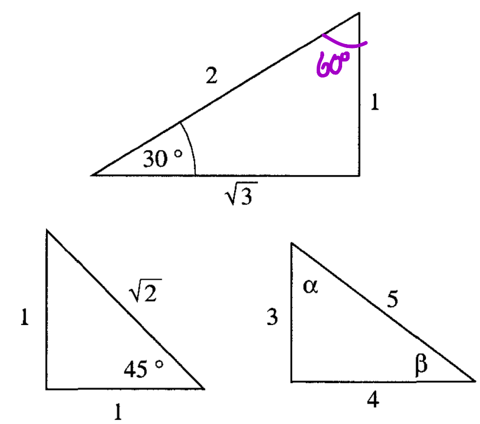
Prueba que $\cos 37° = \text{sen}\, 53°$



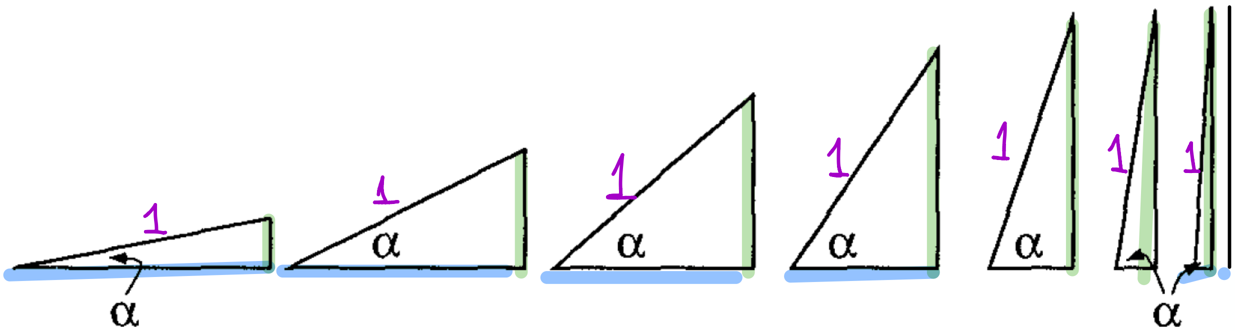
$$\cos 37° = \frac{b}{c} = \text{sen}\, 53°$$

1. Fill in the following table. You may want to use the model triangles given in the diagram below.

angle x	$\sin x$	$\cos x$
$30°$	$1/2$	$\sqrt{3}/2$
$45°$	$1/\sqrt{2}$	$1/\sqrt{2}$
$60°$	$\sqrt{3}/2$	$1/2$
α	$4/5$	$3/5$
β	$3/5$	$4/5$



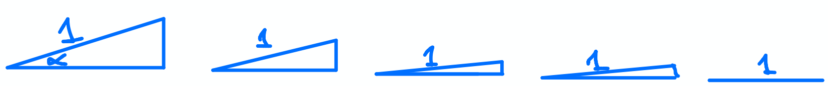
(The angles α and β are angles in a 3-4-5 right triangle.)



Definition $\boxed{\sin 90^\circ = 1.}$

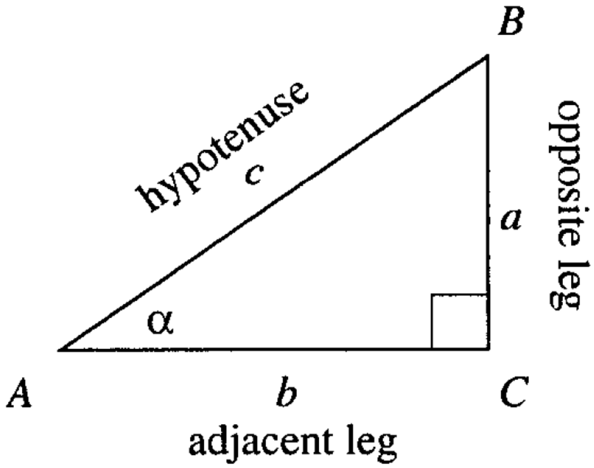
Definition $\boxed{\cos 90^\circ = 0.}$

¿Cómo debo definir sen 0°? $= 0$



¿Y cos 0°? $= 1$

Las otras razones trigonométricas.



$\operatorname{sen}\alpha = \frac{a}{c}$ seno

$\cos\alpha = \frac{b}{c}$ coseno

$\tan\alpha = \frac{a}{b}$ tangente

$\cot\alpha = \frac{b}{a}$ cotangente

$\sec\alpha = \frac{c}{b}$ secante

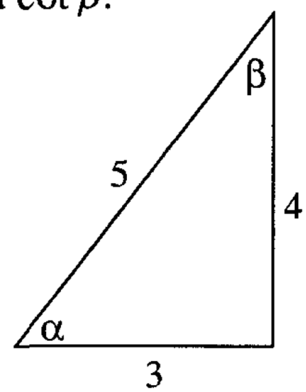
$\csc\alpha = \frac{c}{a}$ cosecante

Definición 8.2.1 Funciones trigonométricas

Las funciones trigonométricas de un ángulo agudo θ en un triángulo rectángulo son

$$\begin{aligned} \operatorname{sen}\theta &= \frac{\text{op}}{\text{hip}} & \cos\theta &= \frac{\text{ady}}{\text{hip}} \\ \tan\theta &= \frac{\text{op}}{\text{ady}} & \cot\theta &= \frac{\text{ady}}{\text{op}} \\ \sec\theta &= \frac{\text{hip}}{\text{ady}} & \csc\theta &= \frac{\text{hip}}{\text{op}}. \end{aligned} \tag{1}$$

1. For the angles in the figure below, find $\cos\alpha$, $\cos\beta$, $\sin\alpha$, $\sin\beta$, $\tan\alpha$, $\tan\beta$, $\cot\alpha$ and $\cot\beta$.



$\cos\alpha = \frac{3}{5}$

$\cos\beta = \frac{4}{5}$

$\text{sen}\,\alpha = \frac{4}{5}$

$\text{sen}\,\beta = \frac{3}{5}$

$\tan\alpha = \frac{4}{3}$ $\qquad$ $\tan\beta = \frac{3}{4}$

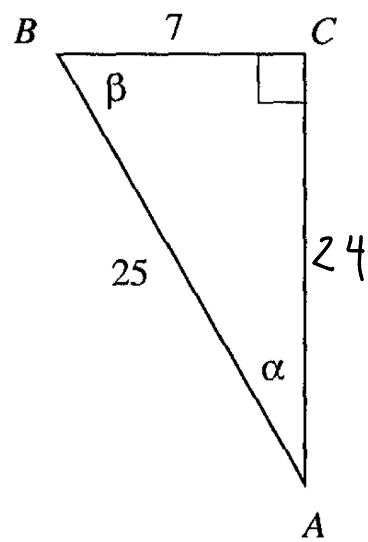
$\cot\alpha = \frac{3}{4}$ $\qquad$ $\cot\beta = \frac{4}{3}$

2. Did you assume that the triangle in the figure above is a right triangle? Why is this assumption correct?

Si, porque $5^2 = 4^2 + 3^2$ ✓

5. In the diagram below, find the numerical value of $\cos\alpha$, $\cos\beta$, $\cot\alpha$ and $\cot\beta$.

$\cot\alpha = \dfrac{\text{cat ady}}{\text{cat. op.}}$



$\cos\alpha = \dfrac{24}{25}$

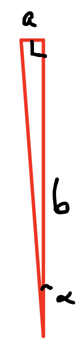
$\cos\beta = \dfrac{7}{25}$

$\cot\alpha = \dfrac{24}{7} > 1$

$\cot\beta = \dfrac{7}{24}$

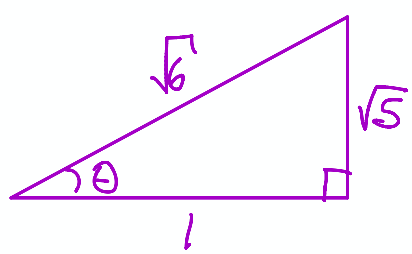
$\cot\alpha = \dfrac{b}{a}$

Ojo: la $\cot\alpha$ puede ser tan grande como quiera:



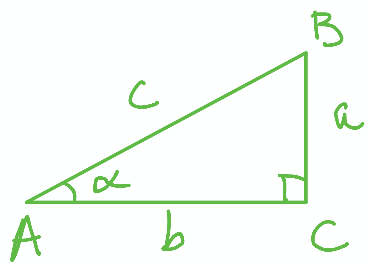
Si θ es un ángulo agudo y $\tan\theta = \sqrt{5}$, calcule el valor de $\cos\theta$.

$\tan\theta = \dfrac{\text{cat op.}}{\text{cat ady}}$



$\cos\theta = \dfrac{1}{\sqrt{6}}$

Ojo:



$$\cos\alpha = \frac{b}{c} \qquad \sec\alpha = \frac{c}{b}$$

$$\operatorname{sen}\alpha = \frac{a}{c}$$

$$\tan\alpha = \frac{a}{b}$$

$$\frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a\cdot \cancel{c}}{b\,\cancel{c}} = \frac{a}{b} = \tan\alpha$$

$$\sec\alpha = \frac{1}{\cos\alpha} \qquad \frac{1}{\frac{c}{b}} = \frac{b}{c}$$

$$\frac{\cos^2\alpha + \operatorname{sen}^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$$

dividan todo entre $\cos^2\alpha$

$$\frac{\cos^2\alpha}{\cos^2\alpha} + \frac{\operatorname{sen}^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$$

$$\boxed{1 + \tan^2\alpha = \sec^2\alpha}$$

identidad trigonométrica

para todo α agudo

De vuelta al ejercicio...

Si $\tan\theta = \sqrt{5}$ calcula $\cos\theta$

Solución algebraica:

$$\sec^2\alpha = 1 + \tan^2\alpha = 1 + 5 = 6$$

$$\Rightarrow \sec\alpha = \sqrt{6} \qquad \Rightarrow \cos\alpha = \frac{1}{\sec\alpha} = \frac{1}{\sqrt{6}}$$

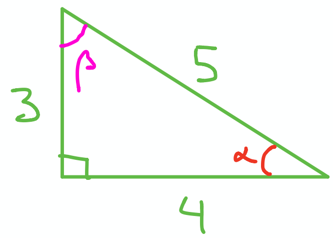
Otra identidad trigonométrica:

$$\frac{\cos^2\alpha + \operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha} = \frac{1}{\operatorname{sen}^2\alpha}$$

$$\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} + 1 = \frac{1}{\operatorname{sen}^2\alpha}$$

$$\boxed{\cot^2\alpha + 1 = \csc^2\alpha}$$

Resolviendo triángulos rectángulos.



$$\operatorname{sen}\alpha = \frac{3}{5} = .6$$

calculadora

$$\Rightarrow \alpha = 36.8^\circ$$

$$\beta = 53.2^\circ$$

Resolvimos el $\Delta\ \{3, 4, 5\}$