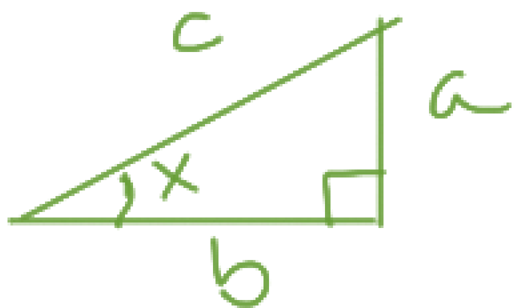
5. Prueba las siguientes identidades para un ángulo agudo $\alpha$:

*a*) $\frac{\tan x}{\operatorname{sen} x} = \frac{1}{\cos x}$.

*b*) $\cos^2 x = \frac{1}{1+\tan^2 x}$.

a) $$\frac{\tan x}{\operatorname{sen} x} = \frac{\frac{\operatorname{sen} x}{\cos x}}{\operatorname{sen} x} = \frac{\cancel{\operatorname{sen} x}}{\cos x \cdot \cancel{\operatorname{sen} x}} = \frac{1}{\cos x}$$

$$\tan x = \frac{\text{cat. op}}{\text{cat. ady}}$$




$$\tan x = \frac{a}{b}$$

$$\operatorname{sen} x = \frac{a}{c} \qquad \frac{\operatorname{sen} x}{\cos x} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{b} = \tan x$$

$$\cos x = \frac{b}{c}$$

$$\boxed{\cos^2 x = \frac{1}{1+\tan^2 x}}$$

$\Longrightarrow \cos^2 x + \operatorname{sen}^2 x = 1$

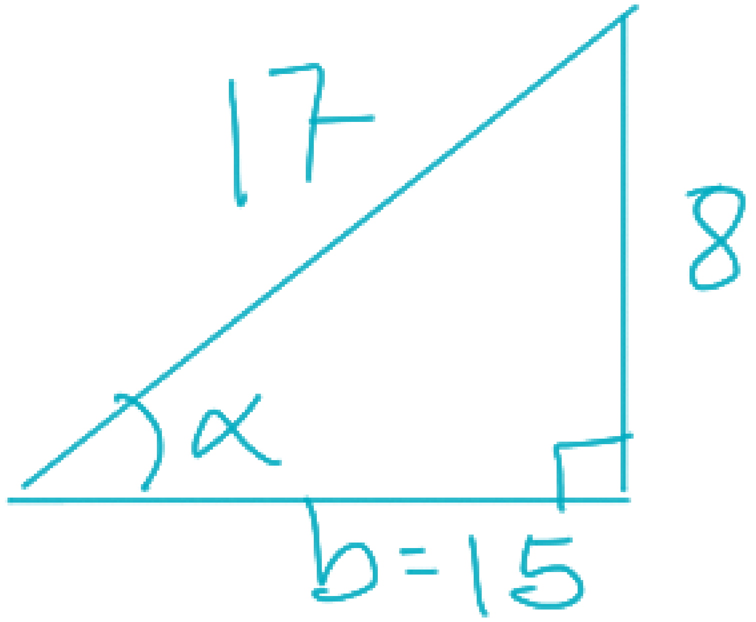
dividiendo entre $\cos^2 x$

$$1 + \tan^2 x = \frac{\cos^2 x}{\cos^2 x} + \frac{\operatorname{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\Downarrow$$

$$\frac{\operatorname{sen}^2 x}{\cos^2 x} = \left(\frac{\operatorname{sen} x}{\cos x}\right)^2 = \tan^2 x \qquad \frac{1}{1+\tan^2 x} = \cos^2 x \ \checkmark$$

**3.** Supón que $\operatorname{sen}\alpha = 8/17$. Calcula el valor de $\cos\alpha$, $\tan\alpha$ y $\cot\alpha$.




$$\cos\alpha = \frac{15}{17} \qquad \tan\alpha = \frac{8}{15}$$

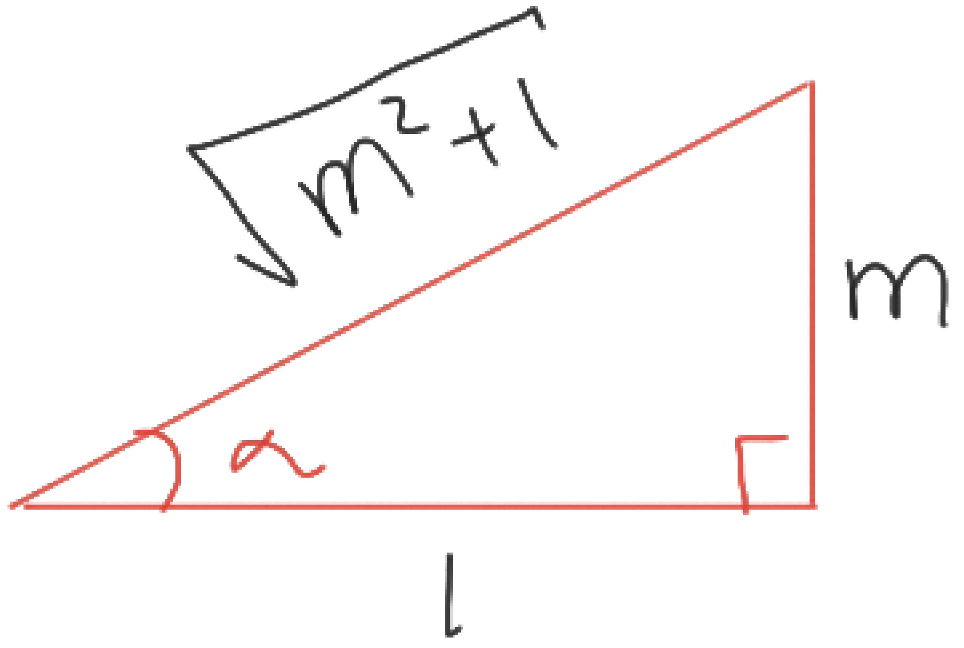
$$\cot\alpha = \frac{15}{8}$$

Calculo b usando Pitágoras:

$$b^2 = 17^2 - 8^2 = 289 - 64 = 225 \quad \Rightarrow b = \sqrt{225} = 15$$

$$\begin{array}{r} 17 \\ 17 \\ \hline 119 \end{array}$$

4. Supón que $\tan\alpha = m$. Expresa en términos de $m$ el valor de $\cos\alpha$, $\operatorname{sen}\alpha$ y $\cot\alpha$.

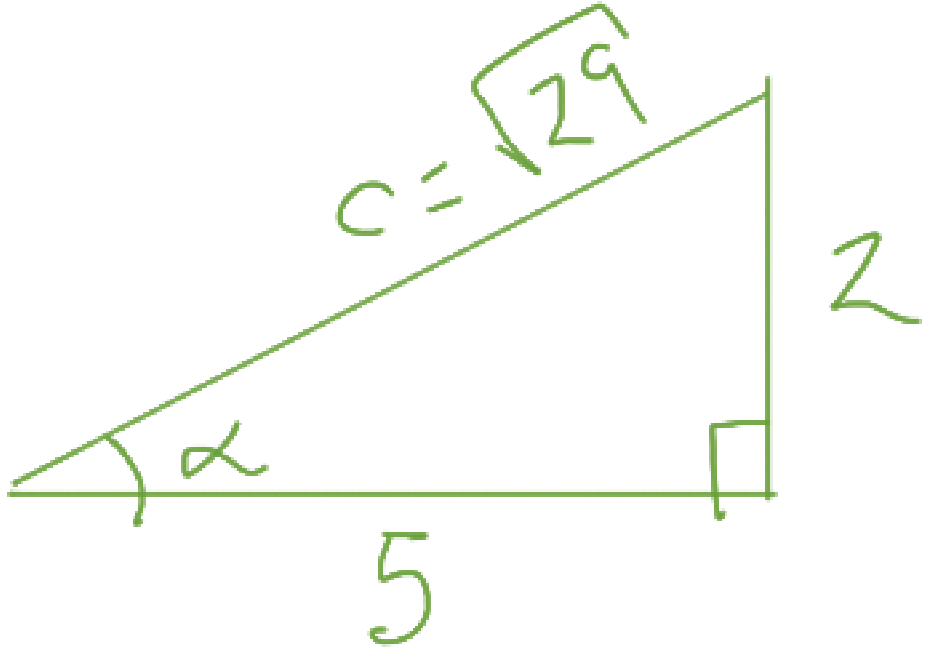



$$\cos\alpha = \frac{1}{\sqrt{1+m^2}}$$

$$\operatorname{sen}\alpha = \frac{m}{\sqrt{m^2+1}}$$

$$\cot\alpha = \frac{1}{m}$$

6. Si $\tan\alpha = 2/5$, calcula el valor de $\frac{\operatorname{sen}\alpha - 2\cos\alpha}{\cos\alpha - 3\operatorname{sen}\alpha}$.




$$\operatorname{sen}\alpha = \frac{2}{\sqrt{29}}$$

$$\cos\alpha = \frac{5}{\sqrt{29}}$$

$$c^2 = 5^2 + 2^2 = 29$$

$$\frac{\frac{2}{\sqrt{29}} - 2 \cdot \frac{5}{\sqrt{29}}}{\frac{5}{\sqrt{29}} - 3 \cdot \frac{2}{\sqrt{29}}}$$

$$= \frac{\frac{2}{\sqrt{29}} - \frac{10}{\sqrt{29}}}{\frac{5}{\sqrt{29}} - \frac{6}{\sqrt{29}}} = \frac{\frac{-8}{\cancel{\sqrt{29}}}}{\frac{-1}{\cancel{\sqrt{29}}}}$$

$$= 8$$

*a*) Haz una tabla (usando tu teléfono o calculadora) para cada $\alpha = 0°, 10°, 20°, ..., 90°$, con los números $\cos\alpha + \operatorname{sen}\alpha$. También calcula $\cos 45° + \operatorname{sen} 45°$.

Conjetura: $\operatorname{sen}\alpha + \cos\alpha \le \sqrt{2}$

| $0°$ | $10°$ | $20°$ | $30°$ | $40°$ | $45°$ | $50°$ | $60°$ | $70°$ | $80°$ | $90°$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.16 | 1.28 | 1.37 | 1.408 | 1.414 | 1.408 | 1.37 | 1.28 | 1.16 | 1 |

$\cos 50° = \operatorname{sen} 40°$

$\operatorname{sen} 50° = \cos 40°$

$\cos 0° = 1$

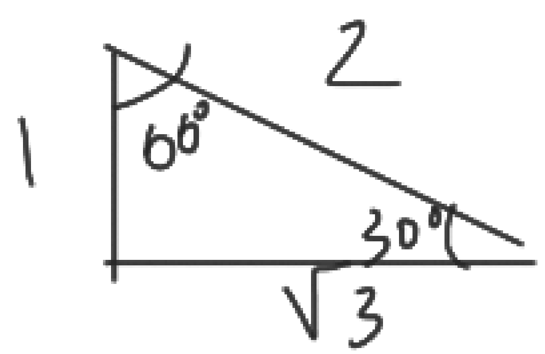
$\operatorname{sen} 0° = 0$

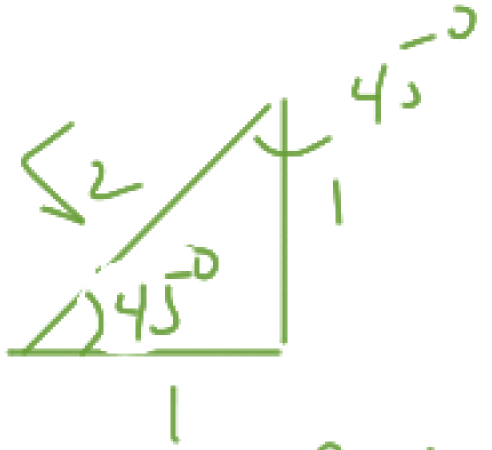
$\cos 90° = 0$

$\operatorname{sen} 90° = 1$

$\cos 30° = \frac{\sqrt{3}}{2} = .87$

$\operatorname{sen} 30° = \frac{1}{2} = .5$




$\cos 45° = \frac{1}{\sqrt{2}}$

$\operatorname{sen} 45° =$

2. Using your calculator, find

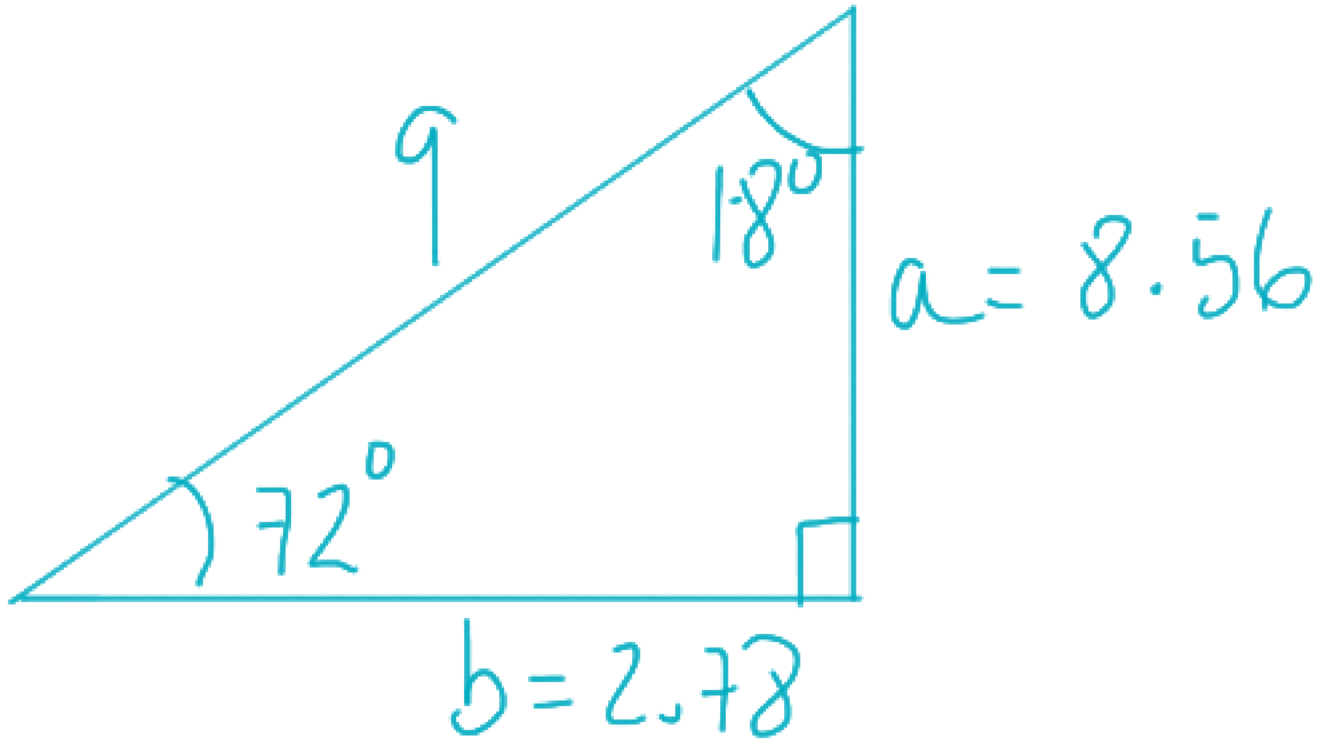
a) arcsin 1.

b) arccos 0.7071067811865.

a) $90^{\circ}$

b) $45^{\circ}$ $0.76 \times 180/\pi = 43.54$

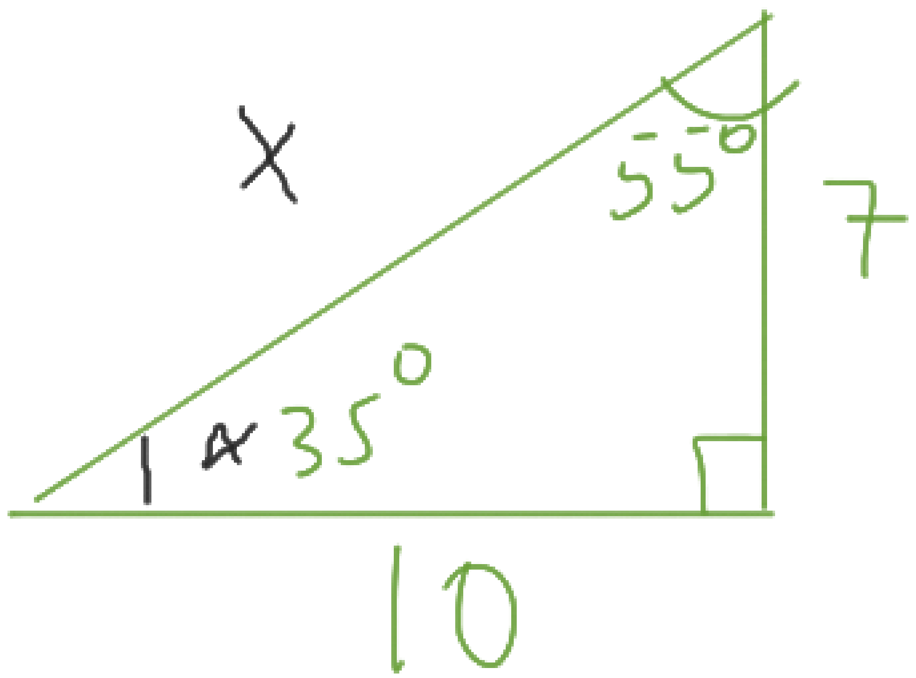
1. Find the legs of a right triangle with hypotenuse 9 and an acute angle of 72 degrees.




$$\operatorname{sen} 72^\circ = \frac{a}{9} \Rightarrow a = 9 \cdot \operatorname{sen} 72^\circ$$
$$\approx 8.56$$

$$\cos 72^\circ = \frac{b}{9} \Rightarrow b = 9 \cdot \cos 72^\circ$$
$$\approx 2.78$$

2. The two legs of a right triangle are 7 and 10. Find the hypotenuse and the two acute angles.




$$\text{Sen}\,\alpha = \frac{7}{\sqrt{149}} = 0.57$$

$$\alpha = 35^\circ$$

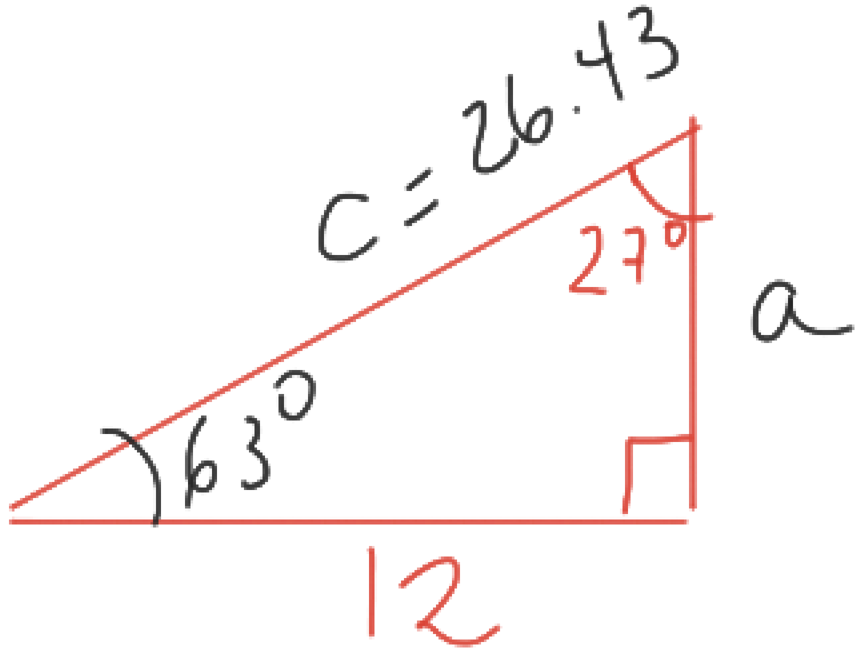
$$90^\circ - 35^\circ = 55^\circ$$

$$x^2 = 7^2 + 10^2$$

$$x^2 = 49 + 100$$

$$x = \sqrt{149}$$

3. A right triangle has a leg of length 12. If the acute angle opposite this leg measures 27 degrees, find the other leg, the other acute angle, and the hypotenuse.




$$\text{sen } 27^\circ = \frac{12}{c} \Rightarrow c = \frac{12}{\text{sen } 27^\circ}$$

$$\approx 26.43$$

Pitágoras:

$$a^2 + 12^2 = (26.43)^2$$

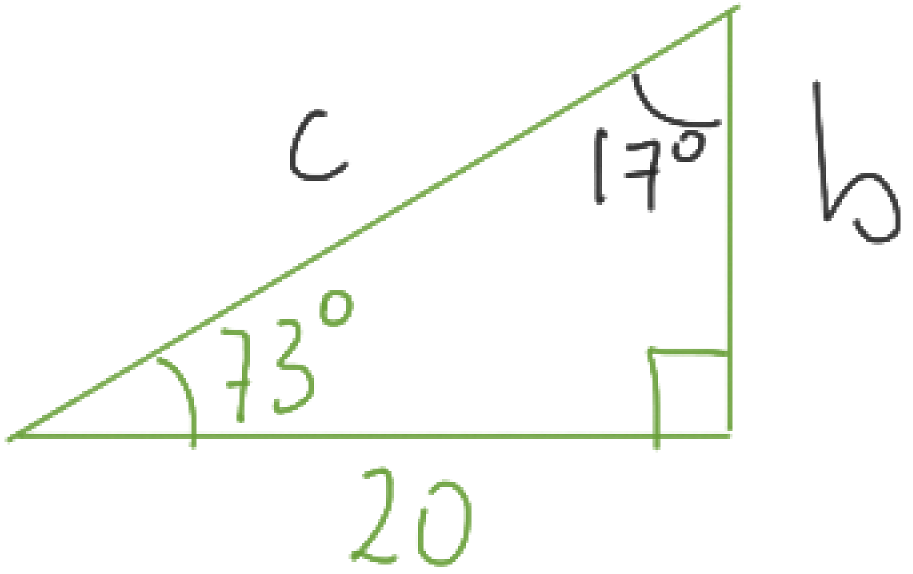
$$a^2 \approx (26.43)^2 - 144 \approx 554.5$$

$$a = 23.6$$

$$\tan 63^\circ = \frac{a}{12}$$

$$\Rightarrow a = 12 \cdot \tan 63^\circ \approx 23.6$$

4. A right triangle has a leg of length 20. If the acute angle adjacent to this leg measures 73 degrees, find the other leg, the other acute angle, and the hypotenuse.




$$\tan 73^\circ = \frac{b}{20}$$

$$\Rightarrow b = 20 \cdot \tan 73^\circ \approx 65.4$$

$$c = \sqrt{(20)^2 + (65.4)^2}$$

$$\cos 73^\circ = \frac{20}{c} \Rightarrow c = \frac{20}{\cos 73^\circ} = 68.4$$