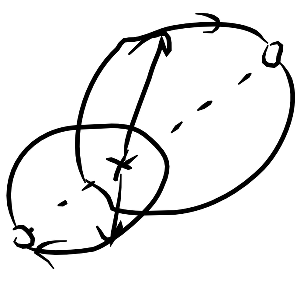
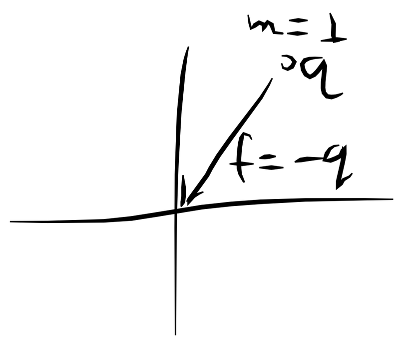
Fuerzas centrales



$q = q_2 - q_1$ $\qquad$ $\ddot{q} = -f(r)\,q$

Hooke (planar) :



$\ddot{q} = -q, \; q = x + iy \in \mathbb{C}$
$\dot{q} = u + iv \in \mathbb{C}$

$E = \frac{|\dot q|^2}{2} + \frac{|q|^2}{2}$

$C = \dot q \cdot iq$

$\ddot{x} = -x$

$x^2 + u^2$, $y^2 + v^2$

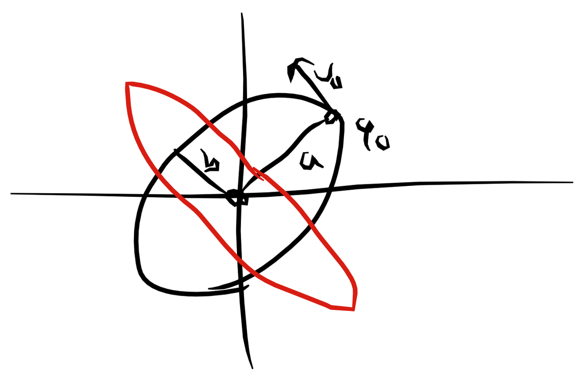
$\dot q^2 + q^2 = cst \in \mathbb{C}$

¿Cuantos integrales de movimiento?
$2E = x^2 + y^2 + u^2 + v^2$
$C = xv - yu$
$F = x^2 + u^2 - y^2 - v^2$
$I = xy + uv$

$\frac{d}{dt} q \times \dot q = 0$

* en este caso, sabemos la solución general : $q(t) = q_o \cos t + v_o \sin t$ *

ell. centrat 0

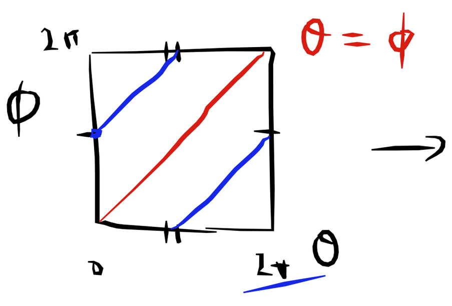


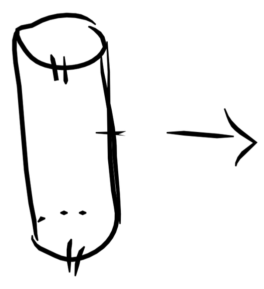
$2E = a^2 + b^2$

$C = ab$

Usando las integrales :

$x^2 + u^2 = c_1^2 \qquad y^2 + v^2 = c_2^2$

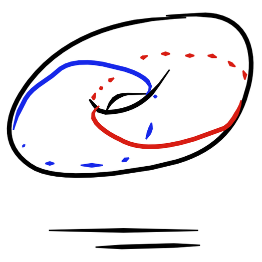




$\Rightarrow x = c_1 \cos\theta,\ u = c_1 \sin\theta,\ y = c_2 \cos\phi,\ v = c_2 \sin\phi$

$C = xv - yu = c_1 c_2 (\cos\theta \sin\phi - \sin\theta \cos\phi) = c_1 c_2 \sin(\theta - \phi)$
$\Rightarrow \phi = \theta + \delta \quad (\delta = cst.)$

$x = c_1 \cos\theta,\ y = c_2 \cos(\theta + \delta)$ (órbita en el plano)



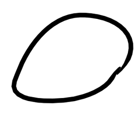
$\subset \mathbb{C}^2$

La topologia de unas superficies en $\mathbb{C}^2$ por fijando valores de las integrales :

$2(E+C) = (x+v)^2 + (y-u)^2$
$2(E-C) = (x-v)^2 + (y+u)^2$

$|C| < E$

$|C| = E$



$q = \sqrt{E}\, e^{\pm it}$

$|C| > E$

$\emptyset$

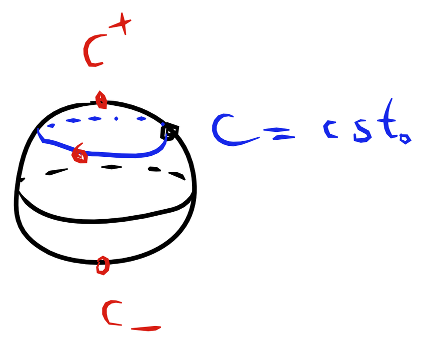
$z := x + iu,\ w := y + iv$

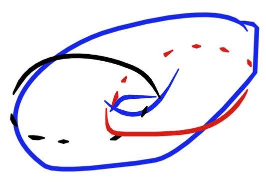
$2E = |z|^2 + |w|^2$
$I = Re(\bar z w)$
$C = Im(\bar z w)$
$F = |z|^2 - |w|^2$

$\mathbb{C}^2 \xrightarrow{\text{HOPF}} \mathbb{R}^3 \; \frac{(2I, 2C, F)}{2E} \in S^2$

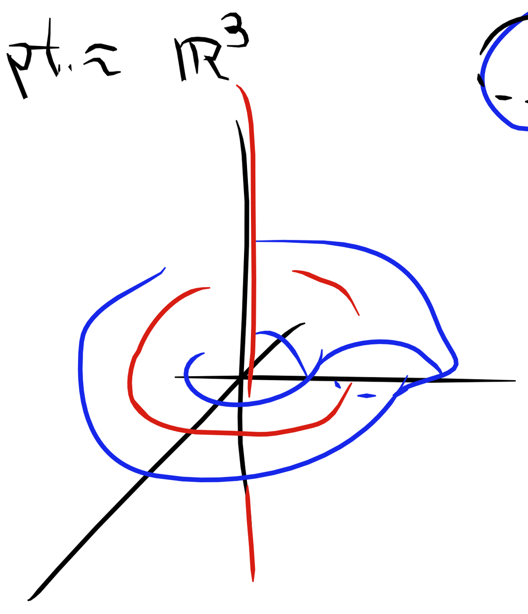
$2E = 1$

$S^3 \longrightarrow S^2$





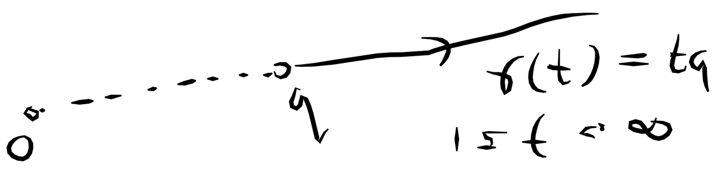
$S^3 \setminus pt. \cong \mathbb{R}^3$



El problema de Kepler :

$\ddot q = -\frac{q}{|q|^3},\quad q \in \mathbb{C}\setminus 0$

$E = \frac{|\dot q|^2}{2} - \frac{1}{|q|}$
$C = \dot q \cdot iq$

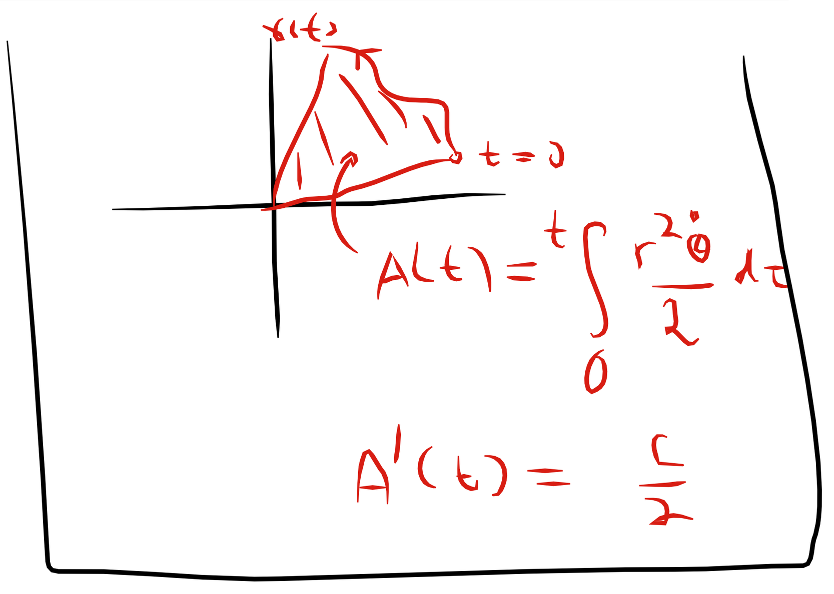


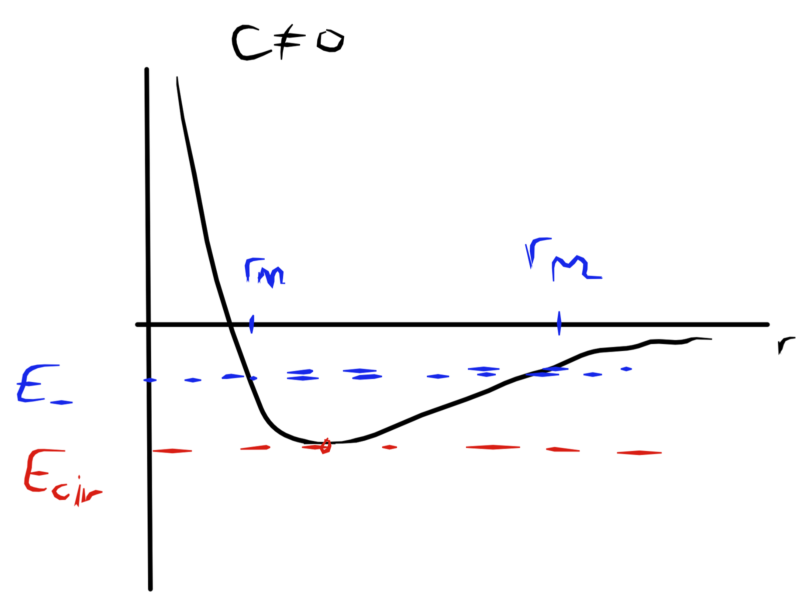
en coordenadas polares : $q = r e^{i\theta}$
$E = \frac{\dot r^2 + r^2 \dot\theta^2}{2} - \frac{1}{r}$
$C = r^2 \dot\theta$

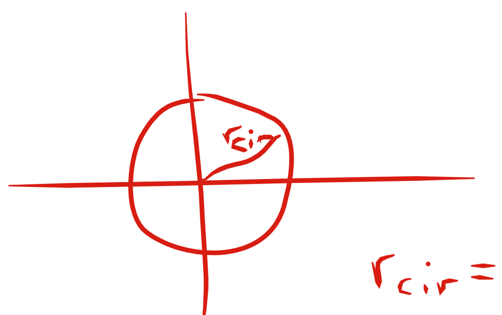
$E = \frac{\dot r^2}{2} + \underbrace{\frac{C^2}{2r^2} - \frac{1}{r}}$

$E \geq V_{eff}(r; C)$

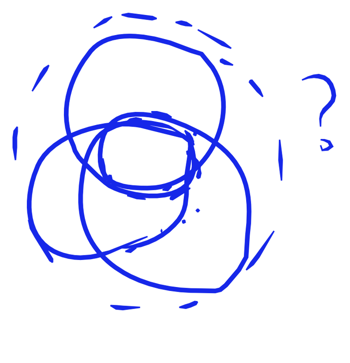
C es el ritmo al que se barre el área desde el origen







$r_{cir} = C^2$
$E_{cir} = -\frac{1}{2C^2}$



$\Phi_{kep} = \pi$