

MIDTERM (ELECTRICITY AND MAGNETISM 2022)

1. Consider two solid balls with radii R_1, R_2 and constant charge densities $\rho, -\rho$ respectively. Supposing the balls intersect, determine the electric field in their intersection.

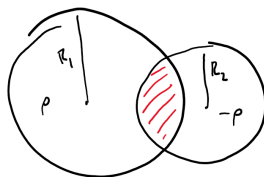


Figure 1. Two intersecting homogeneous balls (with total charges $Q_1 = \frac{4\pi R_1^3}{3}\rho$ and $Q_2 = -\frac{4\pi R_2^3}{3}\rho$).

2. Let Ω be a neutral conductor with an inner cavity and outer boundary a sphere. If a point charge q is placed inside the cavity of Ω , determine the resulting electric field exterior to Ω .



Figure 2. A conductor with a point charged placed in its inner cavity.

3. Consider a grounded conducting 'wedge': $\Omega = \{z \leq 0\} \cup \{y \leq 0\}$. Determine the electric field produced by this conductor and a point charge q in Ω^c . What is the resulting total charge on Ω ?

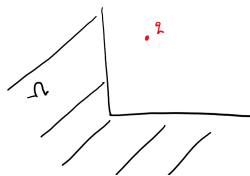


Figure 3. A conducting wedge in the presence of a point charge.

4. Determine the capacitance of a parallel plate capacitor filled by two dielectric 'bands' with permittivities ϵ_1, ϵ_2 :

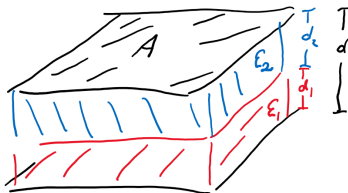


Figure 4. A parallel plate capacitor filled with two dielectrics (assume $d \ll A$ and use uniform field approximations).

5. Consider a conductor Ω_o with charge Q_o and generating the electric field \vec{E}_o in Ω_o^c . Show that if Ω_o^c is filled with a homogeneous and isotropic dielectric of permittivity ϵ , the electric field produced by the charged conductor and this dielectric is given by:

$$\vec{E} = \frac{\epsilon_o}{\epsilon} \vec{E}_o, \text{ in } \Omega_o^c$$

and vanishes in Ω_o .