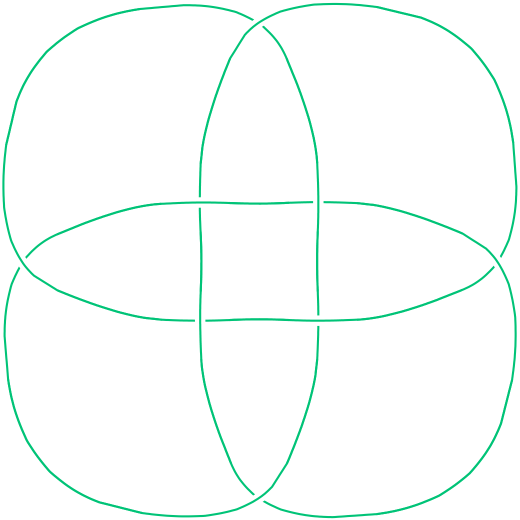
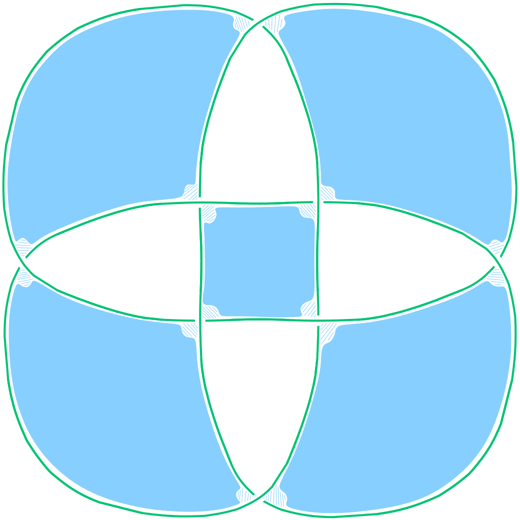
Superficies en la naturaleza

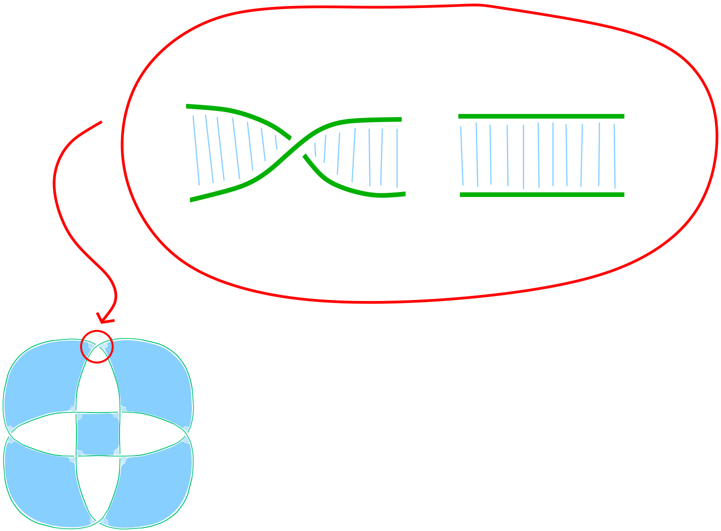
Comenzamos con un diagrama:

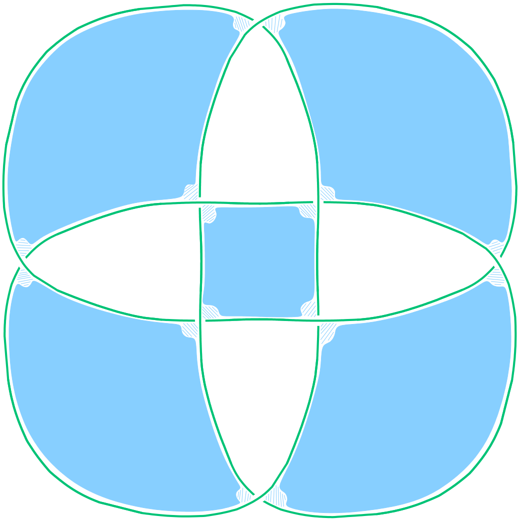


Coloreamos las regiones complementarias como tablero de ajedrez:

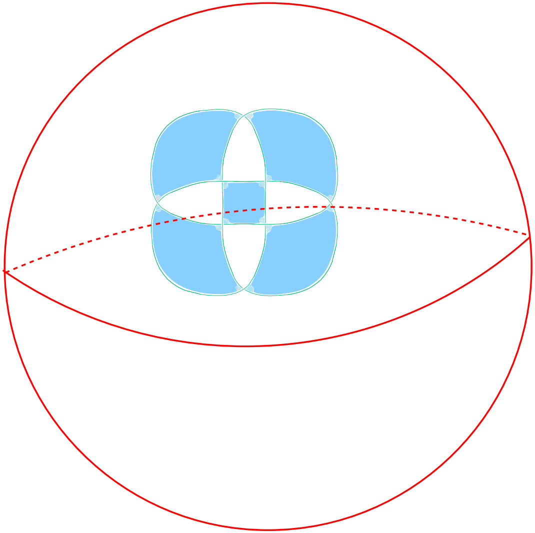


En los puntos de cruce:

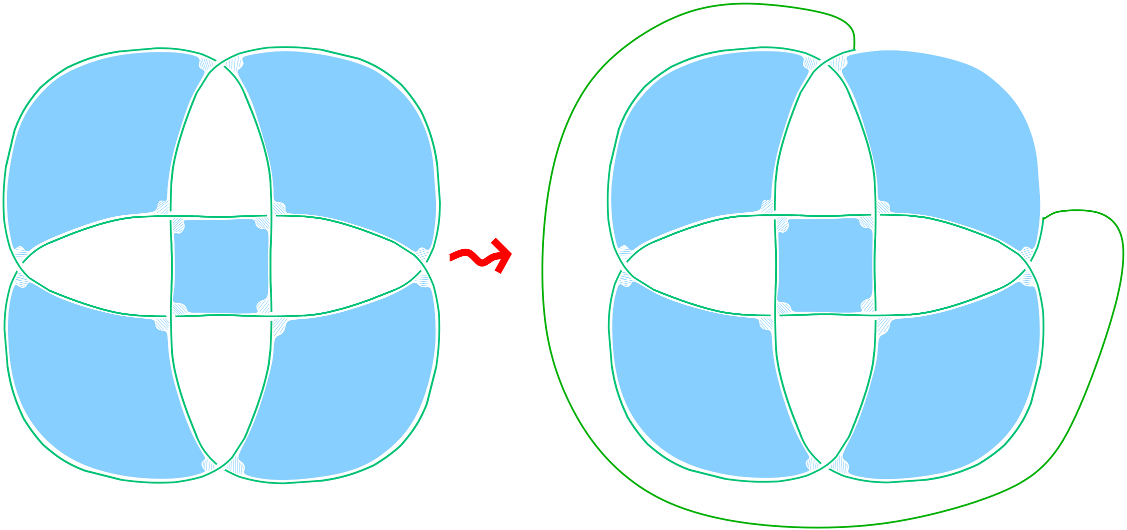


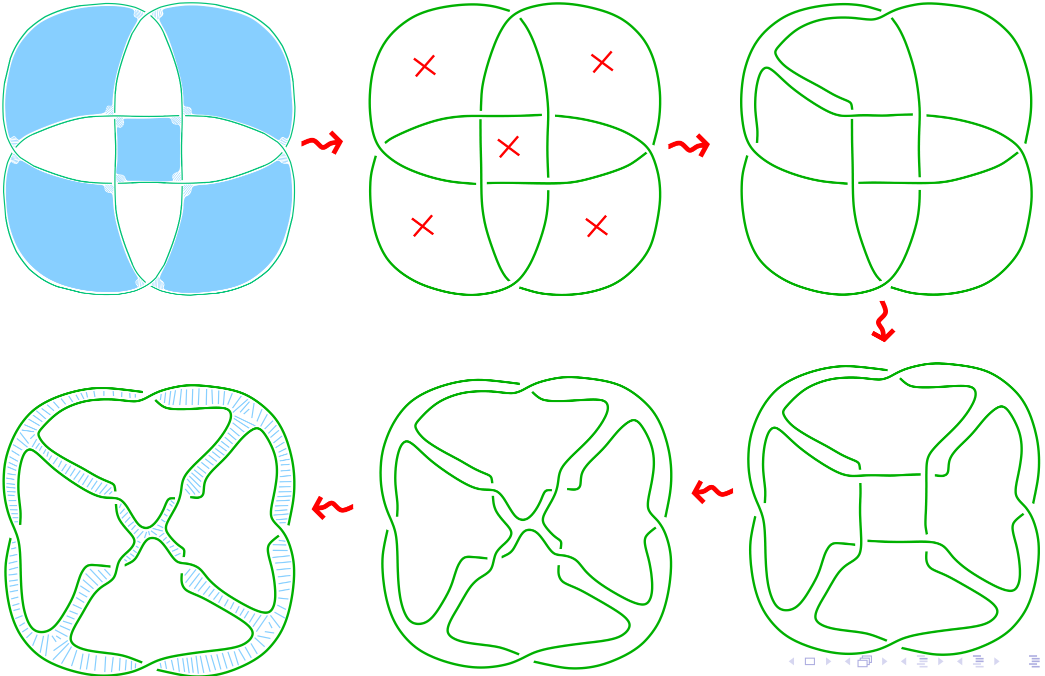


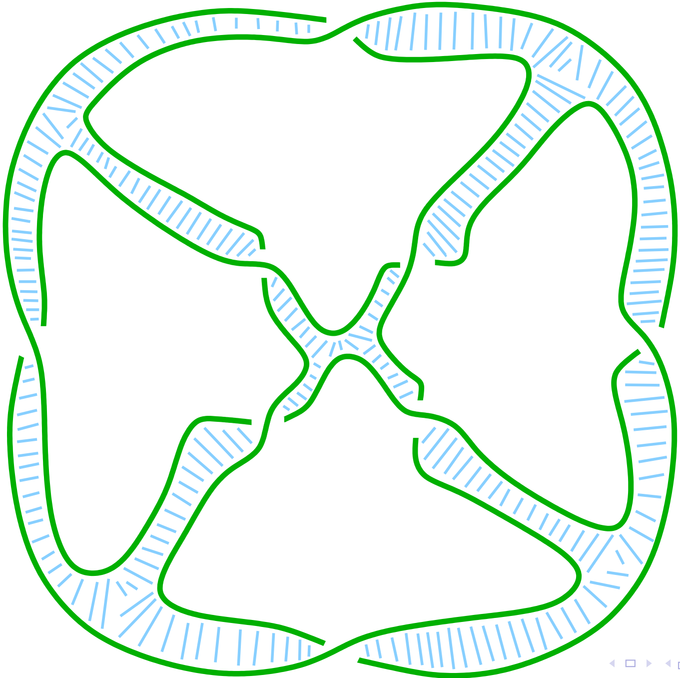
Obtuvimos una superficie *negra*, pero también una blanca:



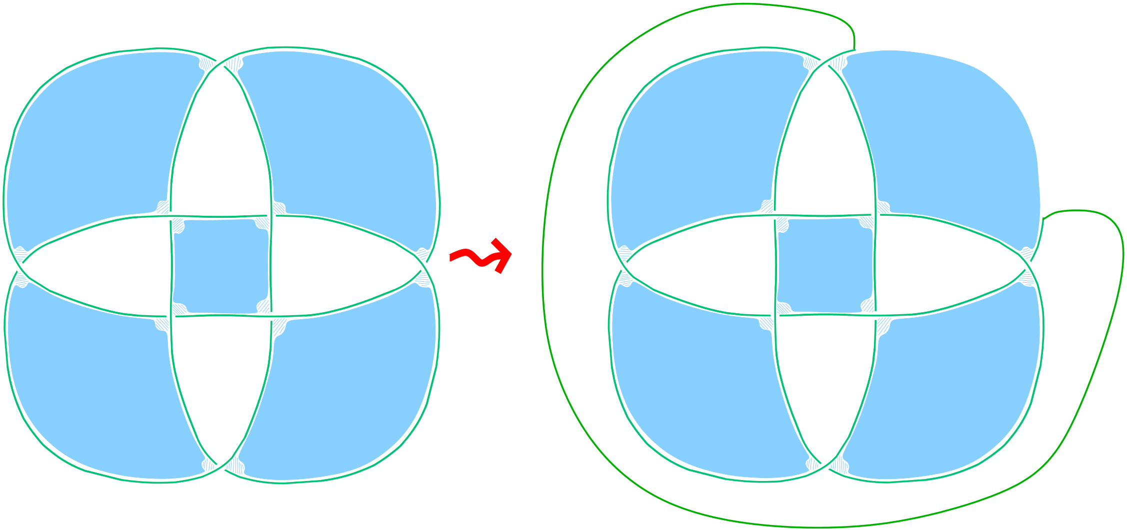
O bien:

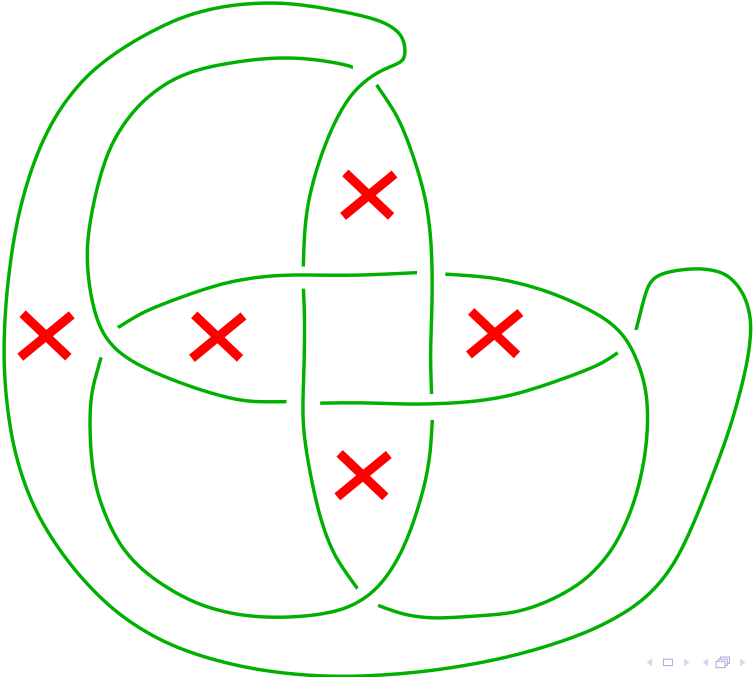


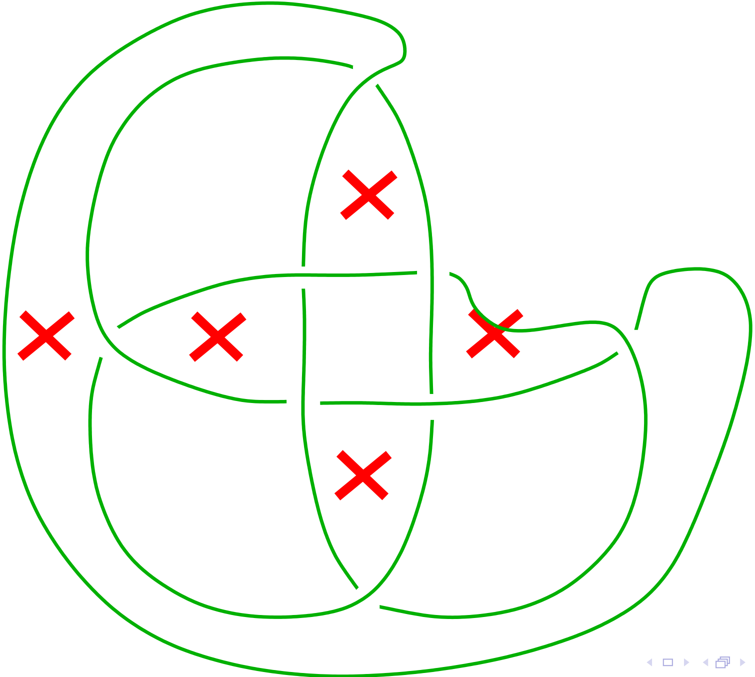


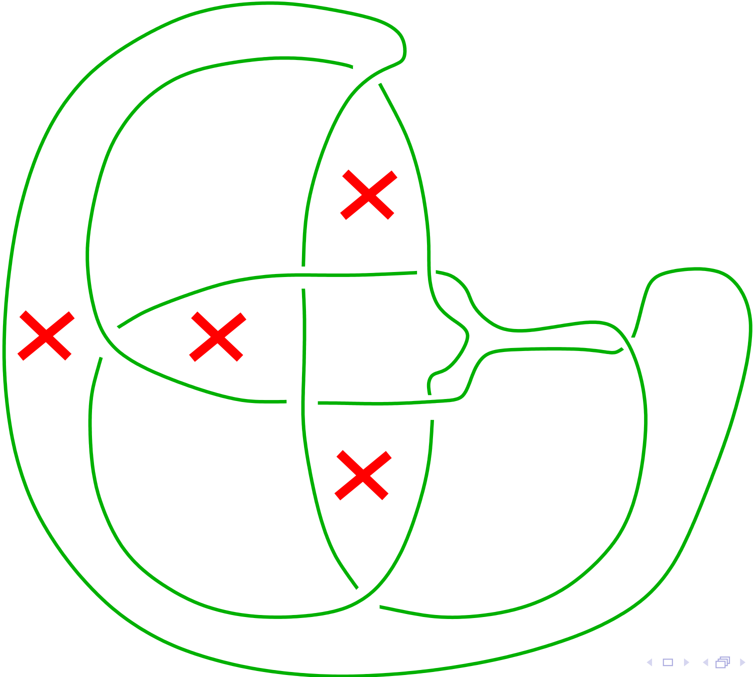


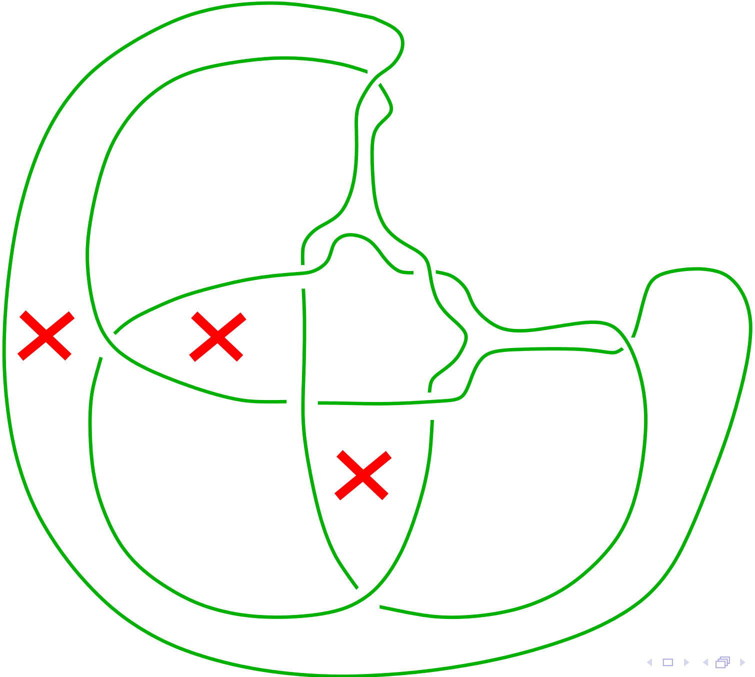
Para la “blanca”:

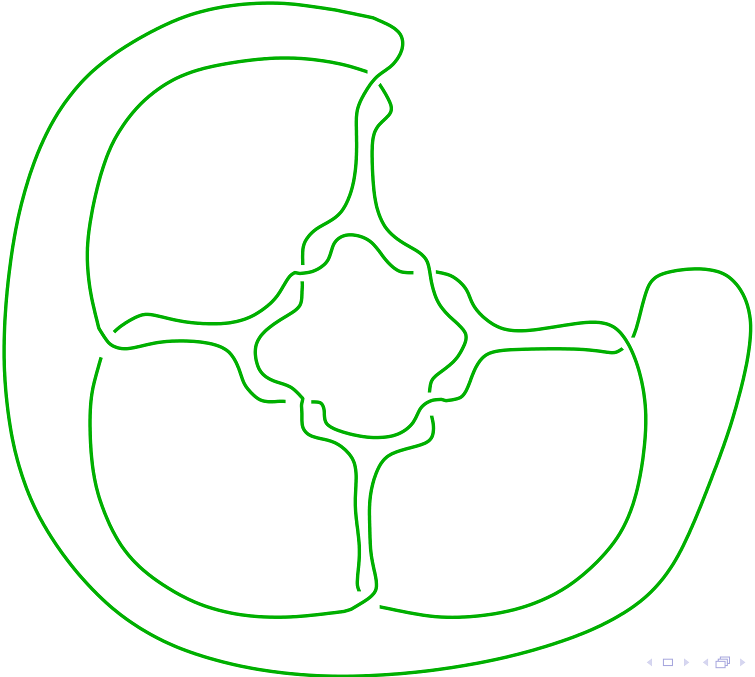


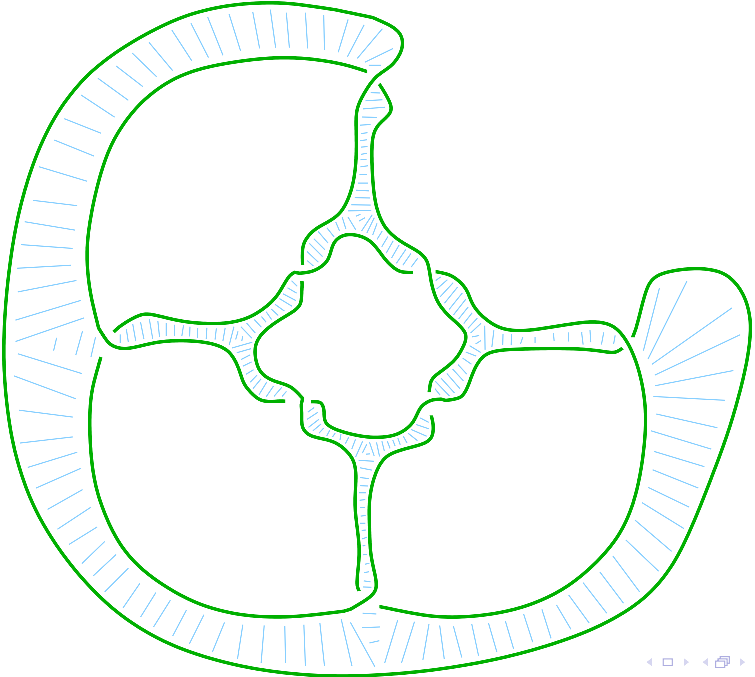




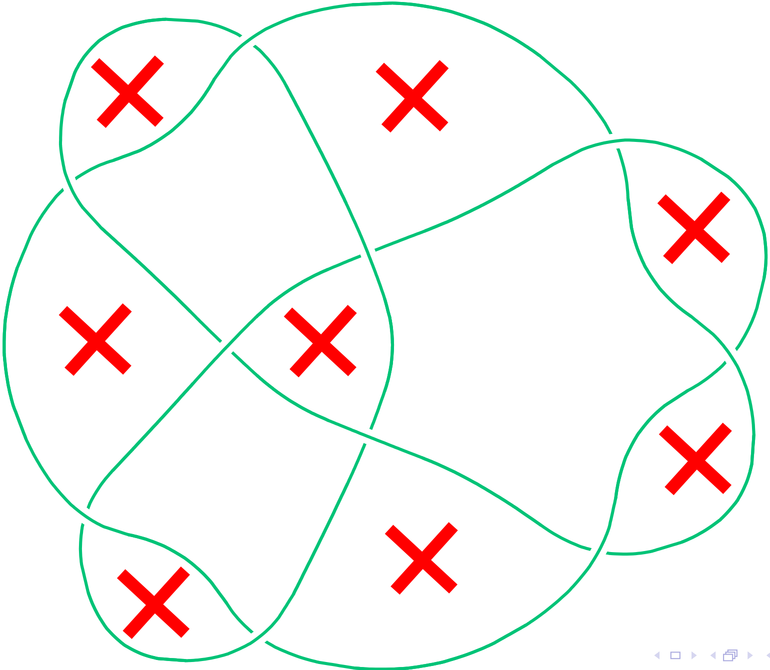


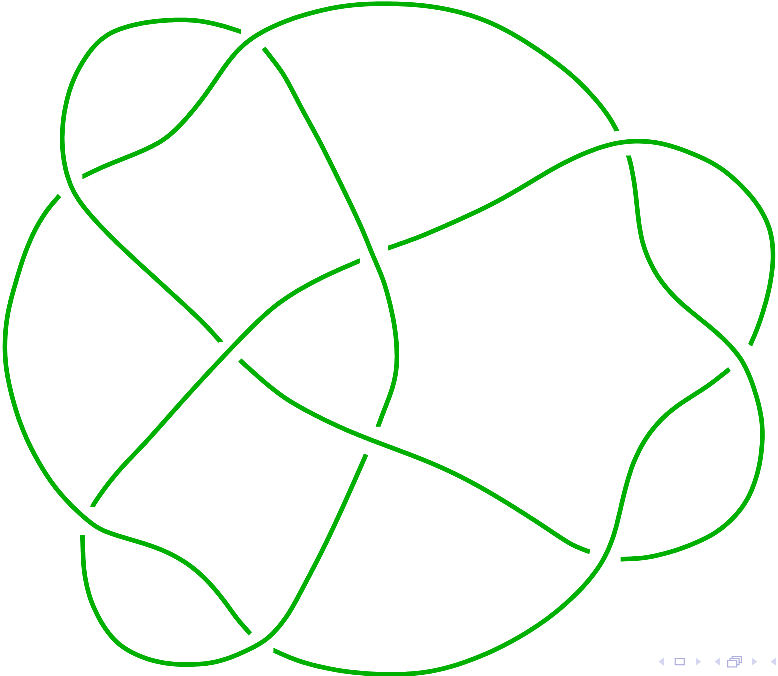


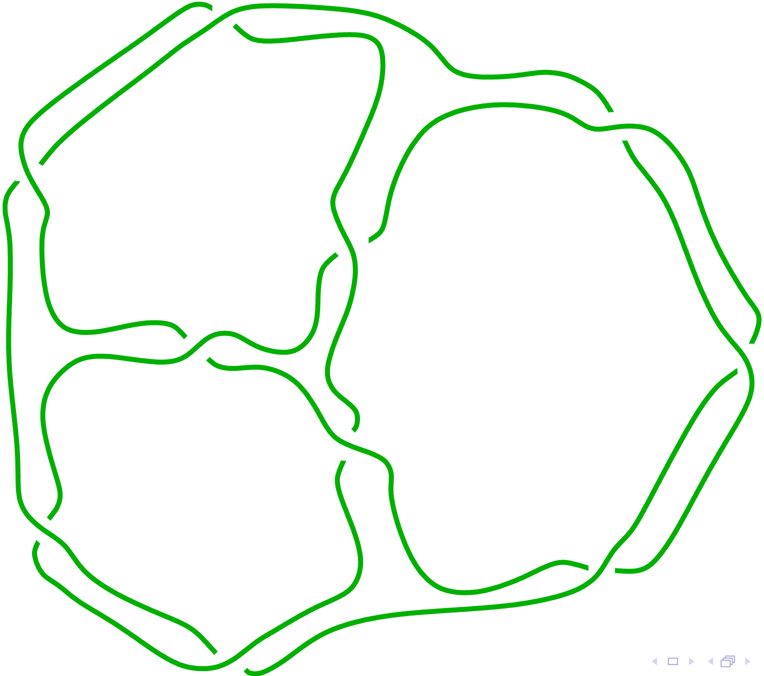


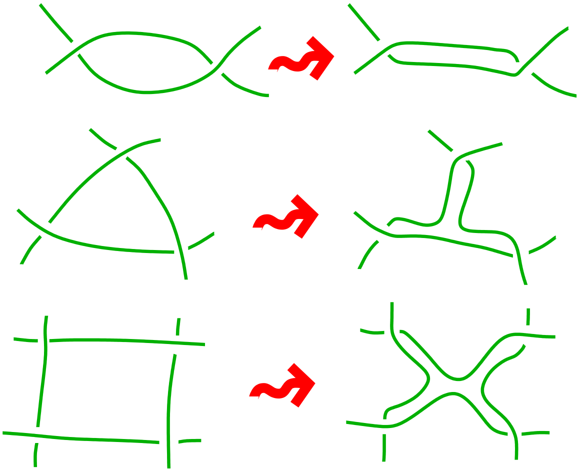


Otro nudo:

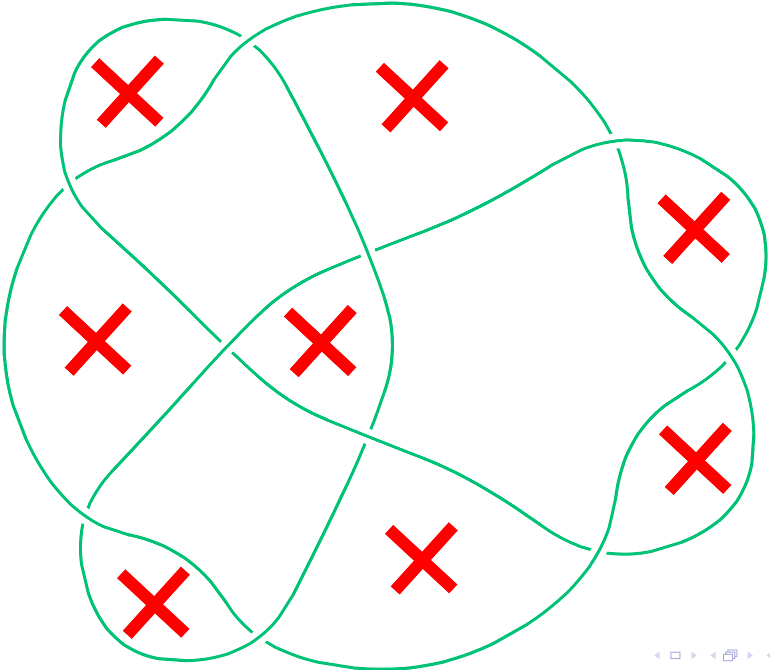


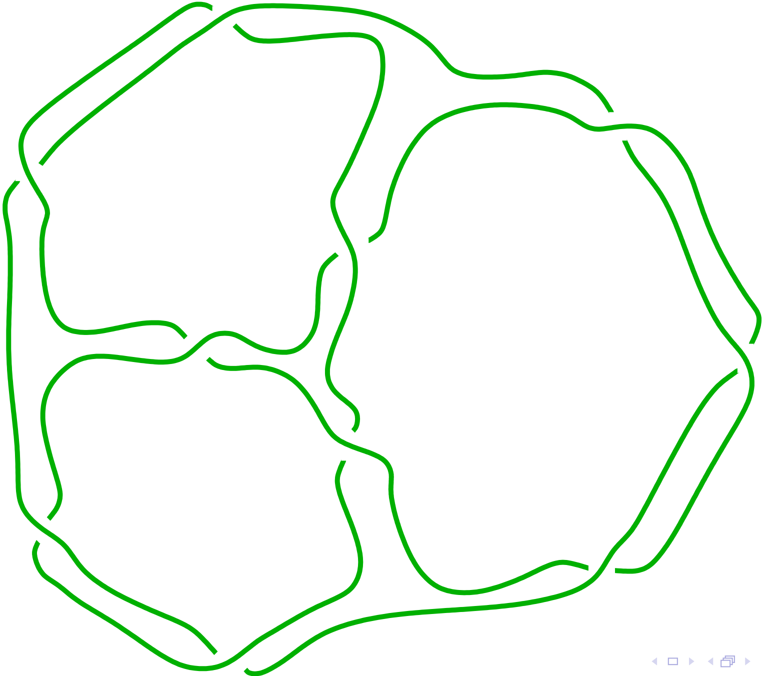






Etcétera



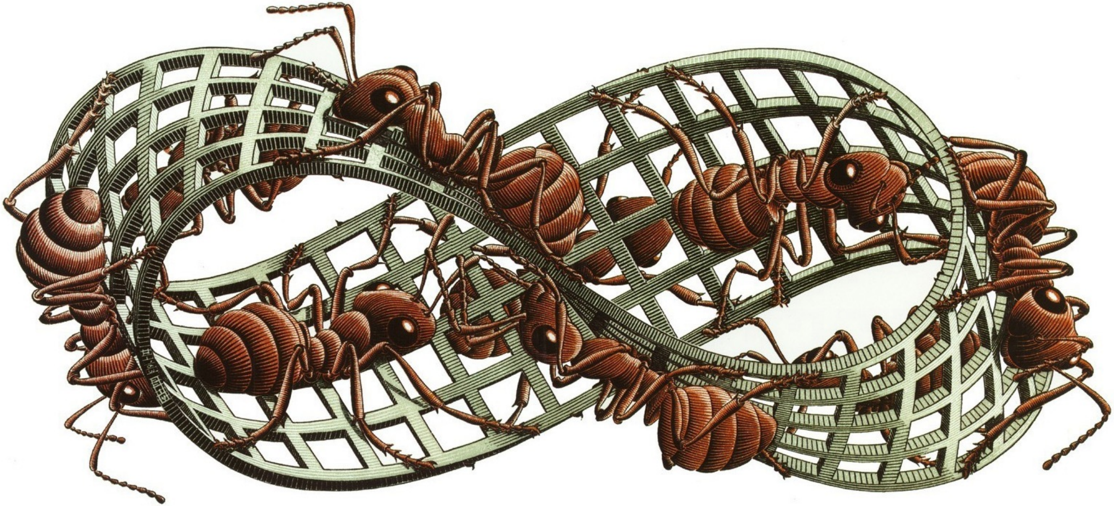


Möbius

A. F. Möbius.

Bueno, más bien

La cinta de Möbius

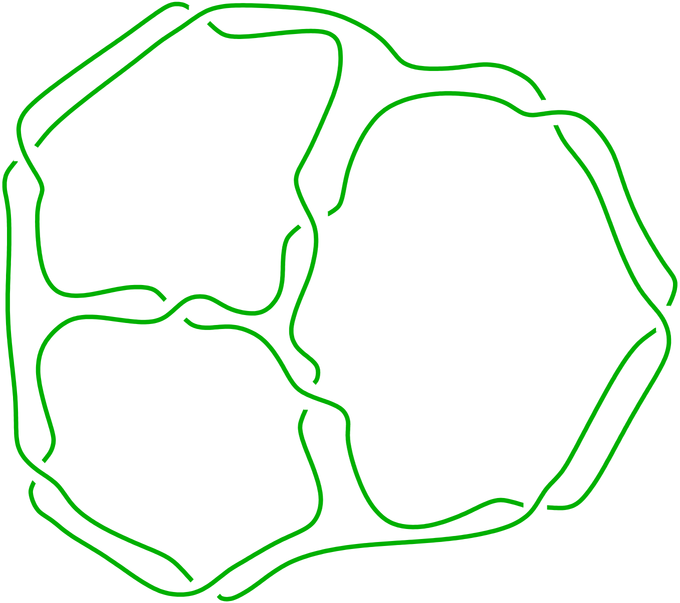


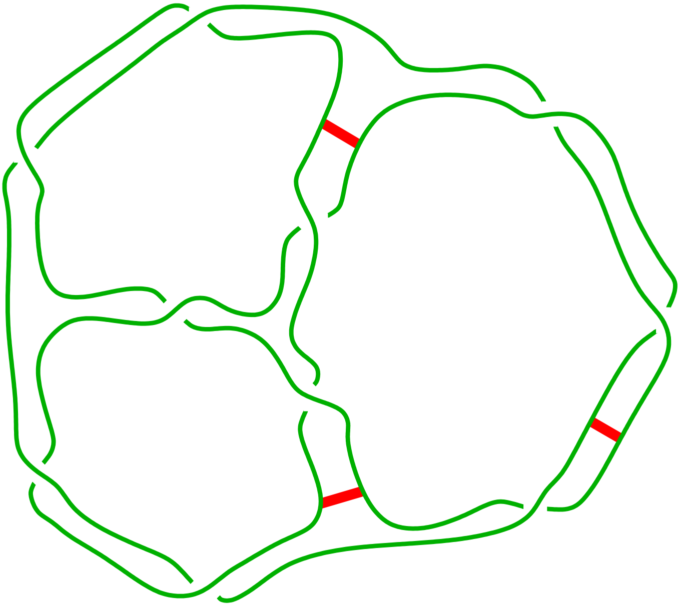
Sea X una superficie $(n+1)$-conexa (X conexa, compacta, con frontera).

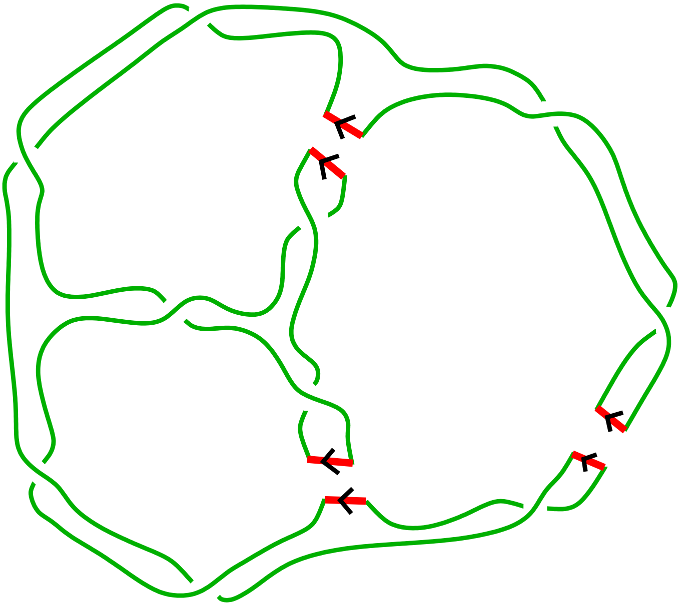
Entonces existe un sistema de arcos propiamente encajados en X,

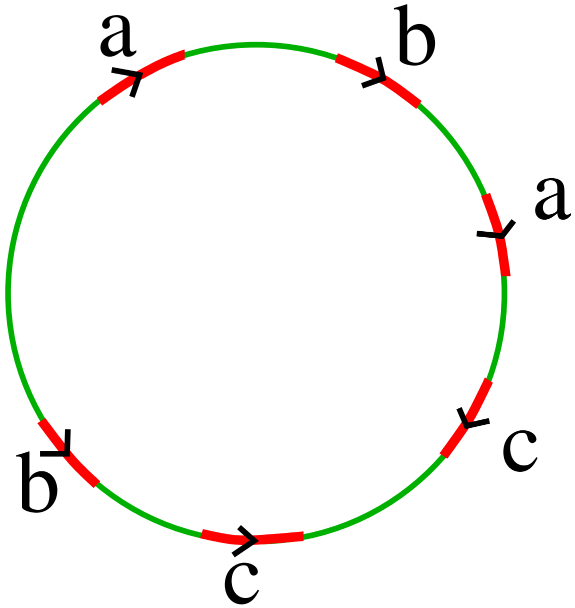
$$\alpha_1, \ldots, \alpha_n \subset X,$$

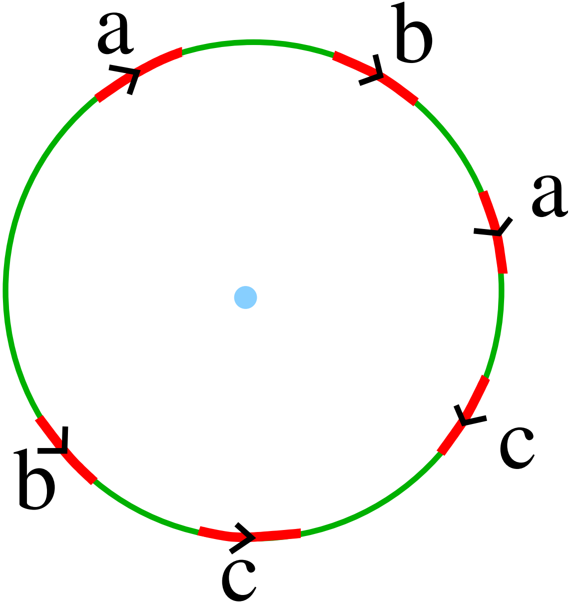
tales que
$X - \alpha_1 \cup \cdots \cup \alpha_n$ es un disco.



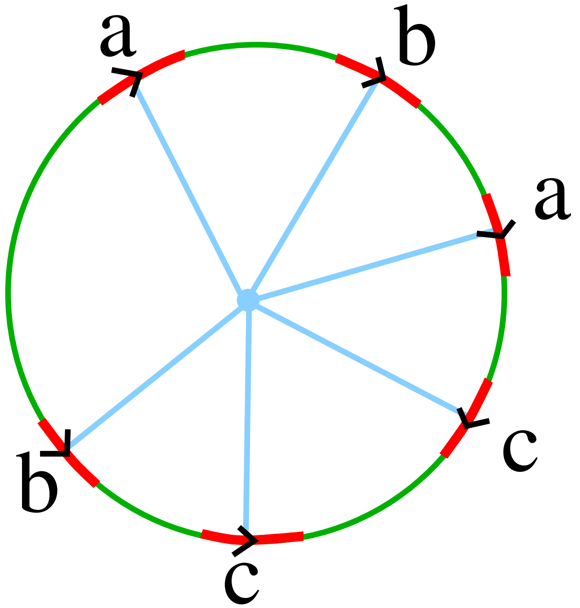


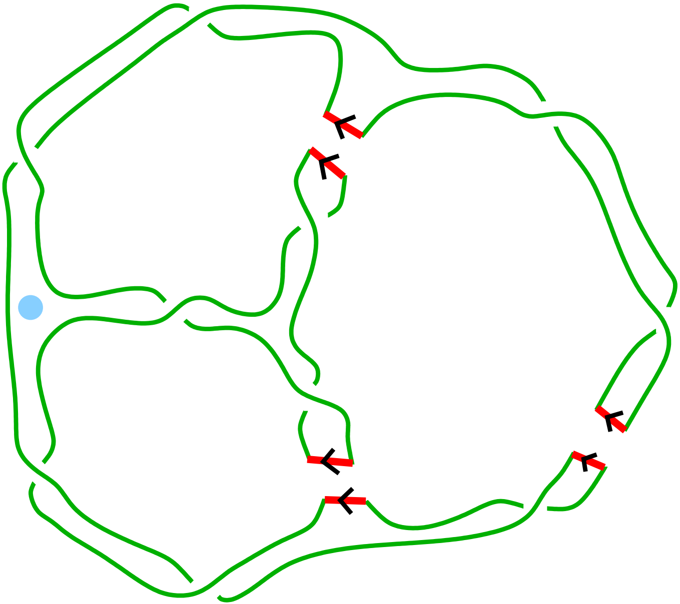


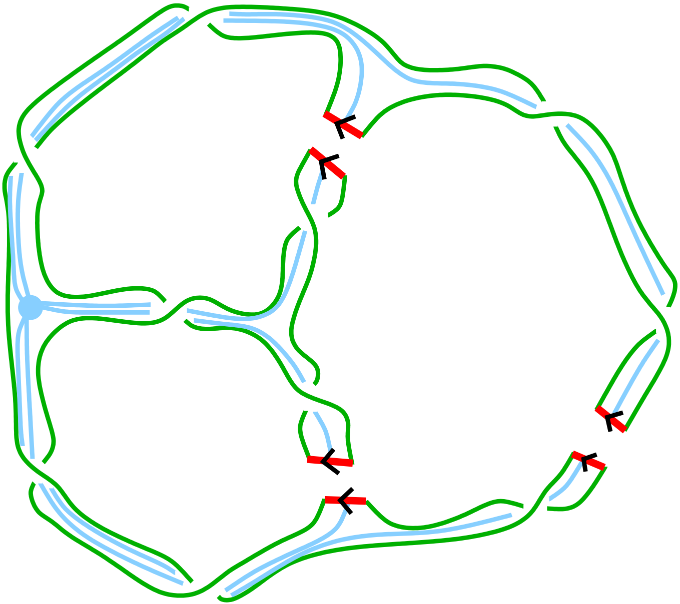

a
b
a
c
c
b

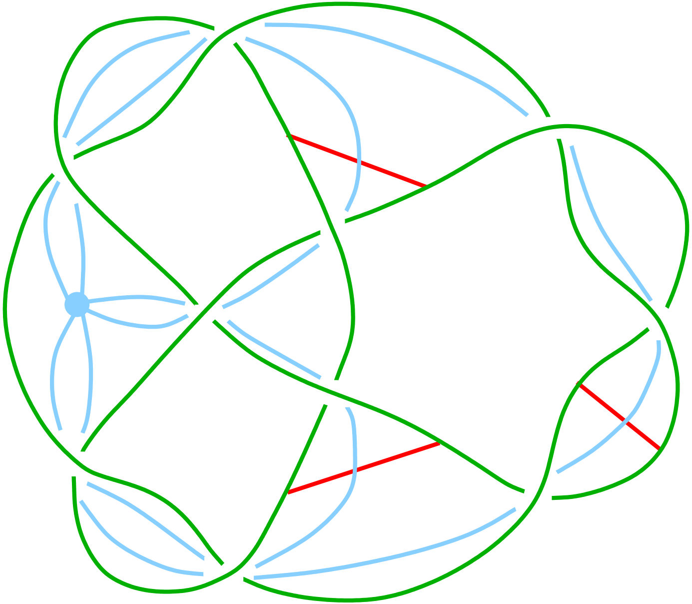


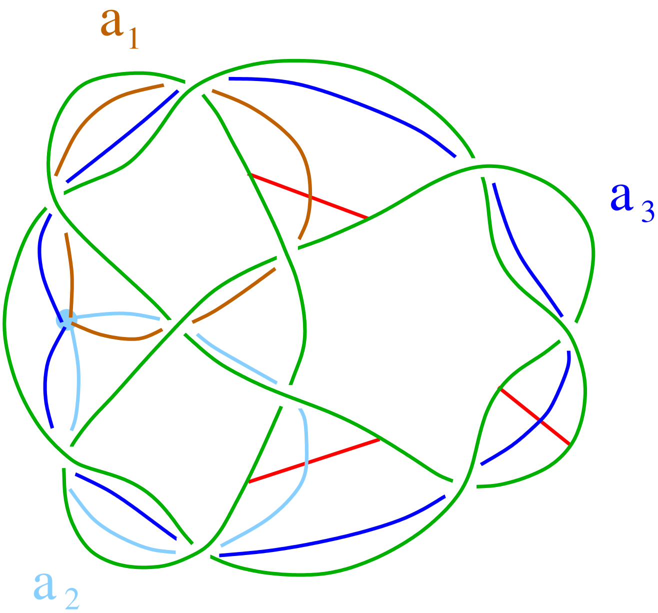
a
b
a
c
c
b


a
b
a
c
c
b









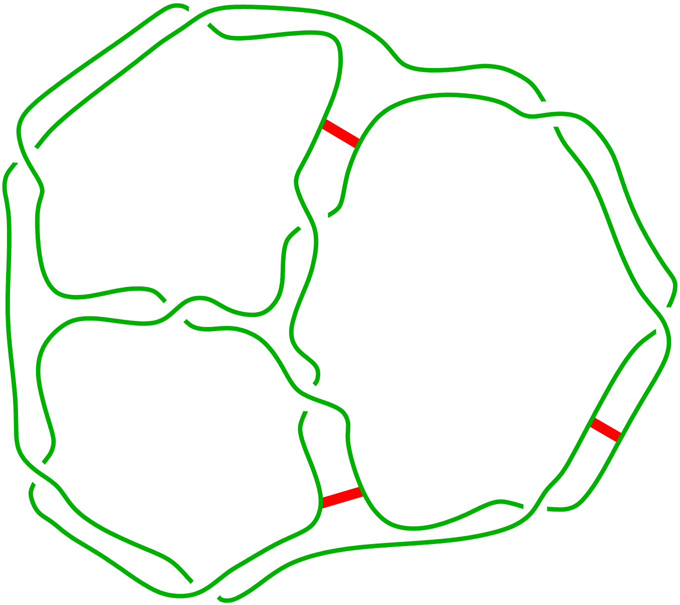
a_1
a_3
a_2

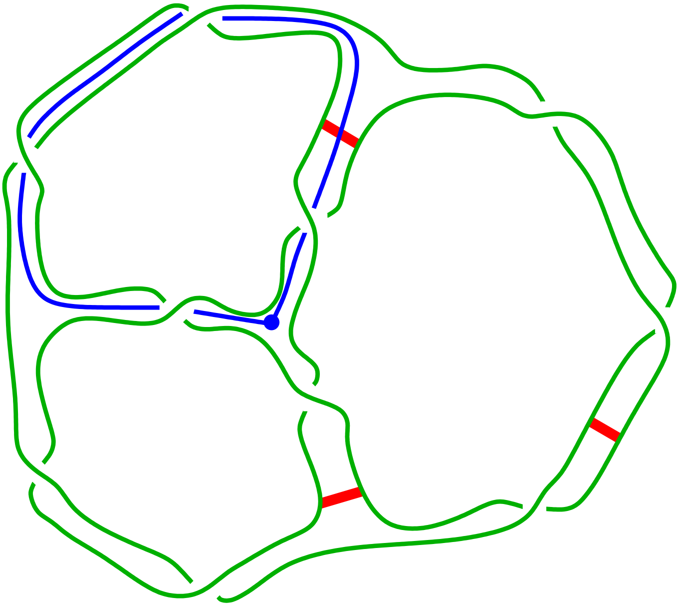
Obtuvimos una cuña de circunferencias

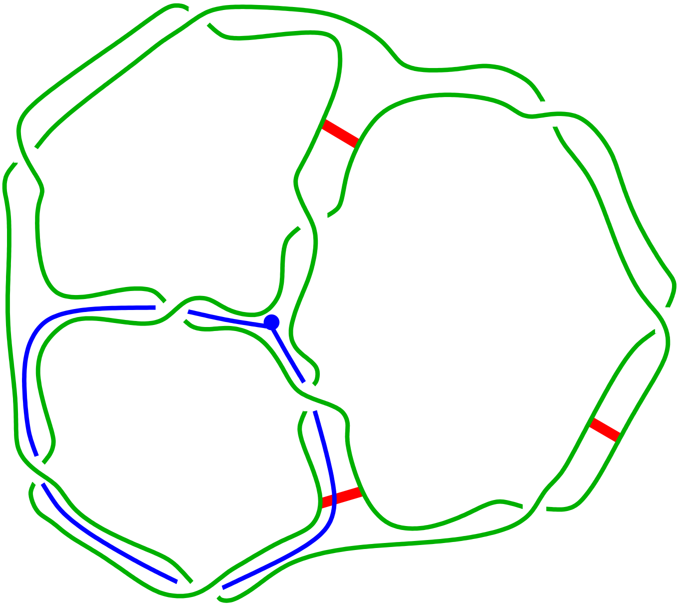
$$a_1 \vee a_2 \vee \cdots \vee a_n$$

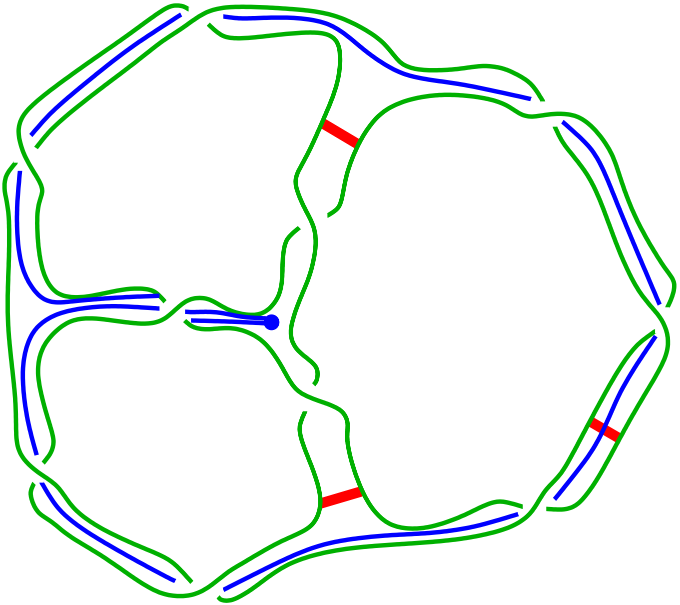
Resulta que:

La superficie X es no orientable
si y sólo si
para alguna $i = 1, \ldots, n$, una vecindad de a_i es
una cinta de Möbius.





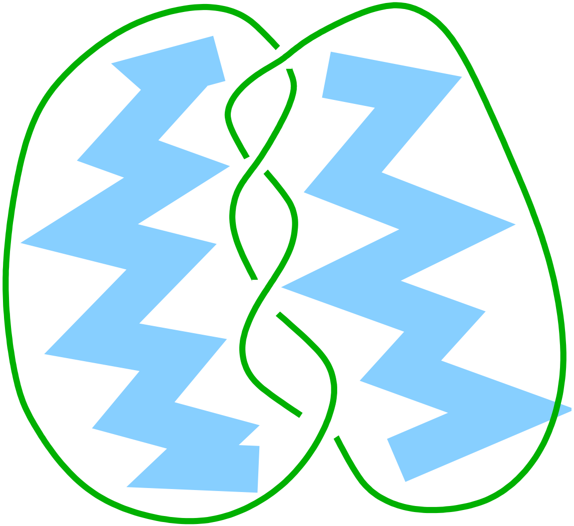




(Nota. La cuña $a_1 \vee \cdots \vee a_n$ se llama una *espina* de la superficie X.)

(Más Nota. La cuña $a_1 \vee \cdots \vee a_n$ representa un sistema de generadores de $\pi_1(X)$.)

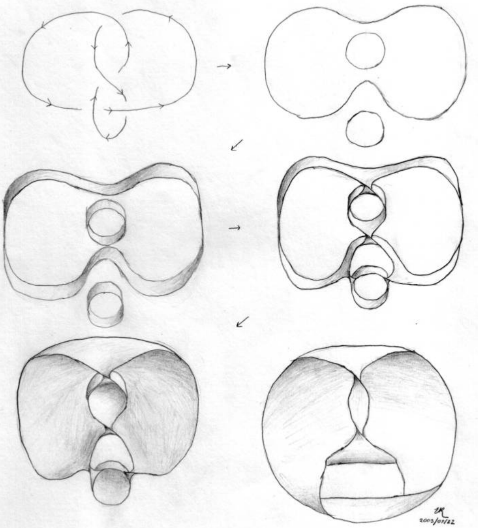
A veces:



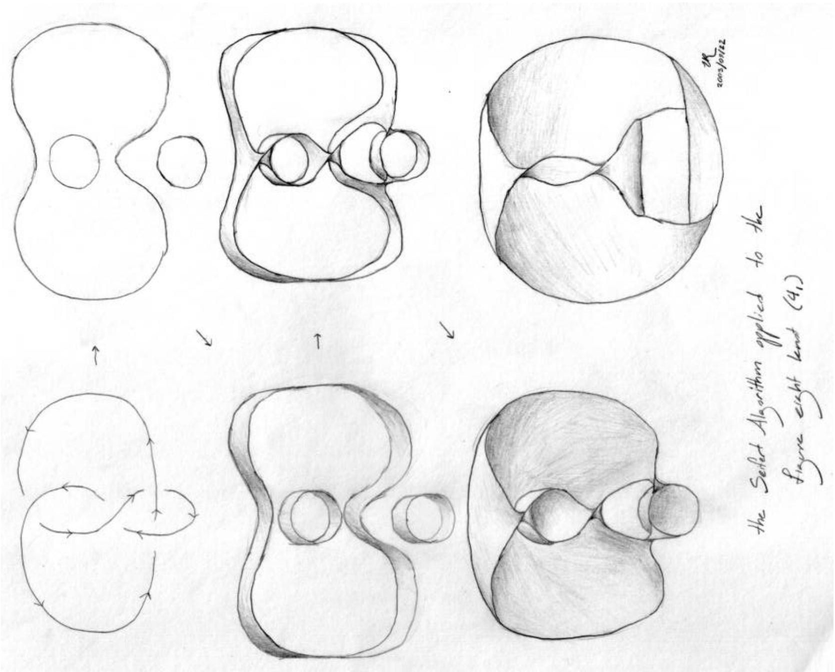
Seifert

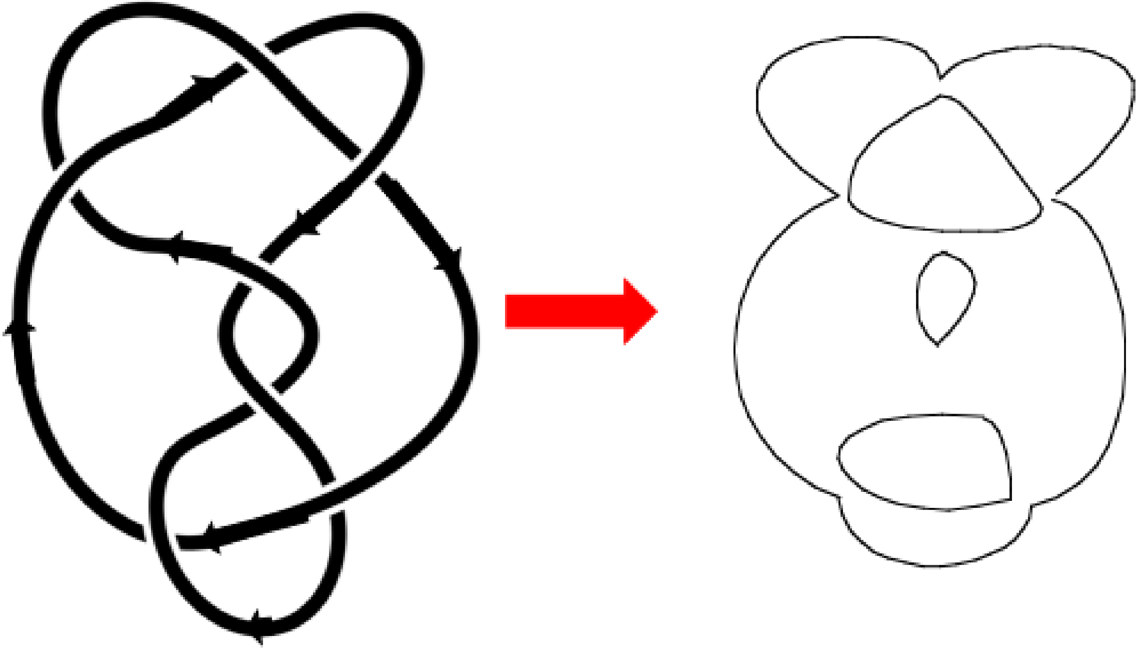
Bueno, más bien

El algoritmo de Seifert

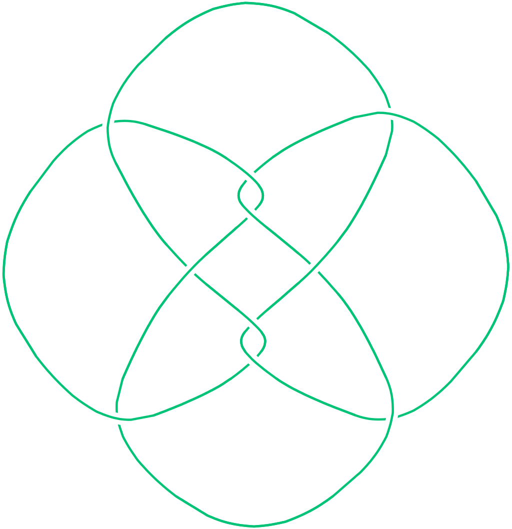


the Seifert Algorithm applied to the figure eight knot (4_1)

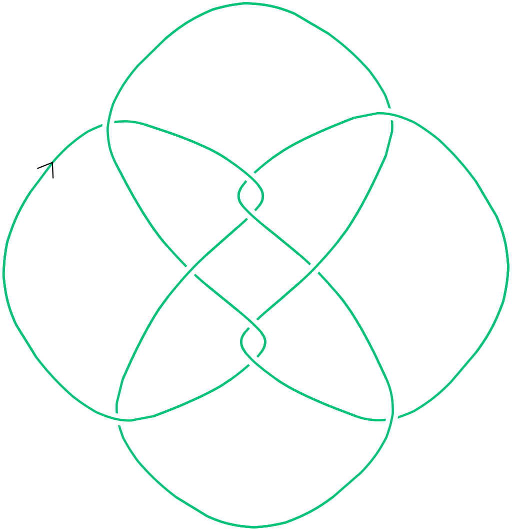

the Seifert Algorithm applied to the
figure eight knot (4_1)



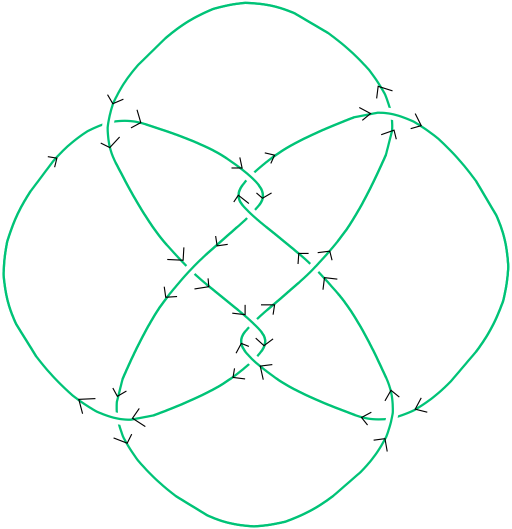
Comenzamos con un diagrama



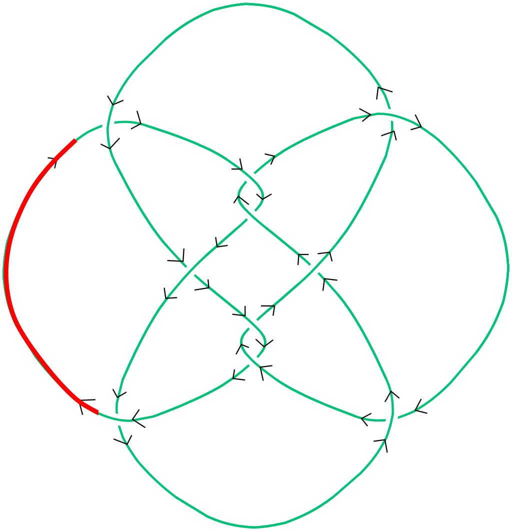
orientamos el nudo



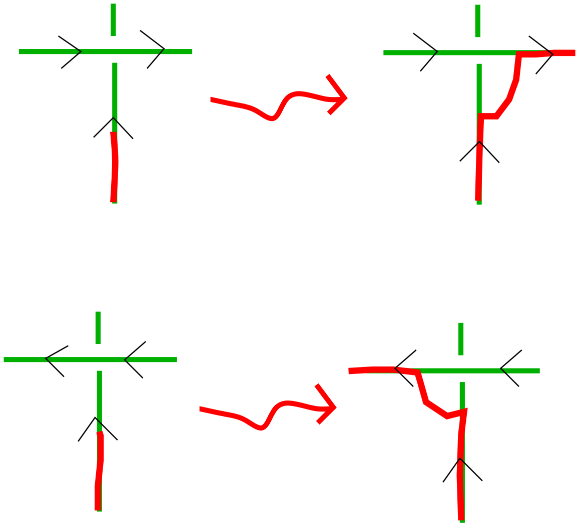
al principio vale la pena poner flechitas en todos los cruces:



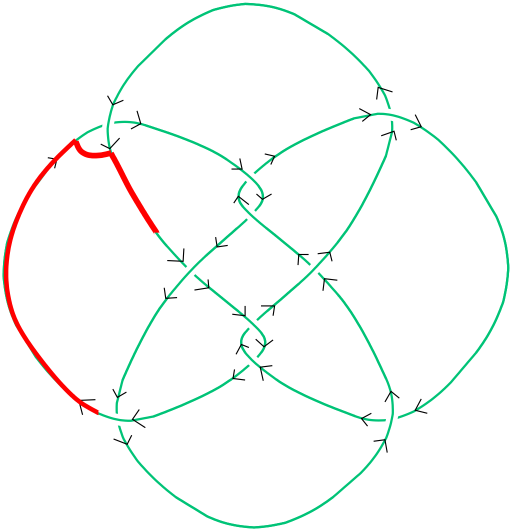
recorremos alguno de los arcos:



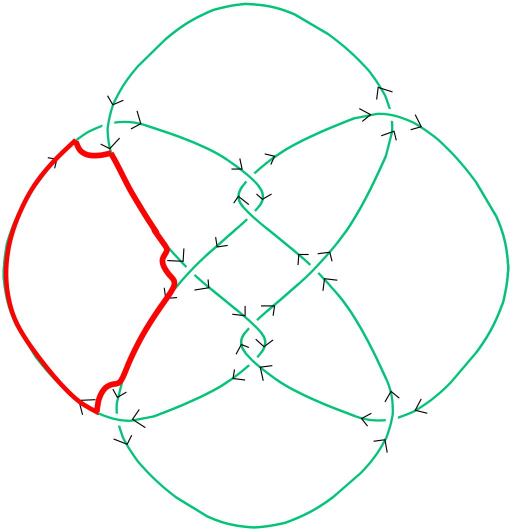
al llegar a un cruce, brincamos:



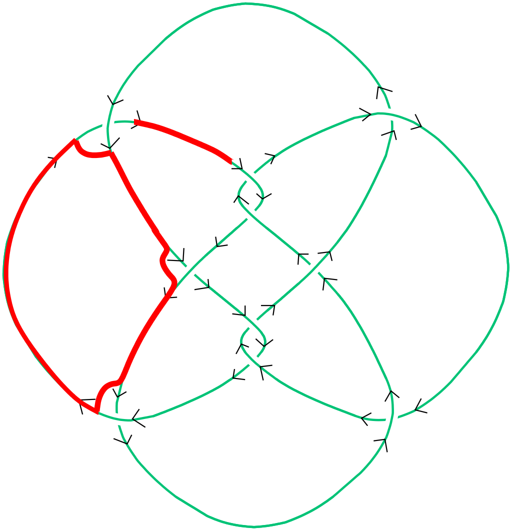
y recorremos el arco que sigue:



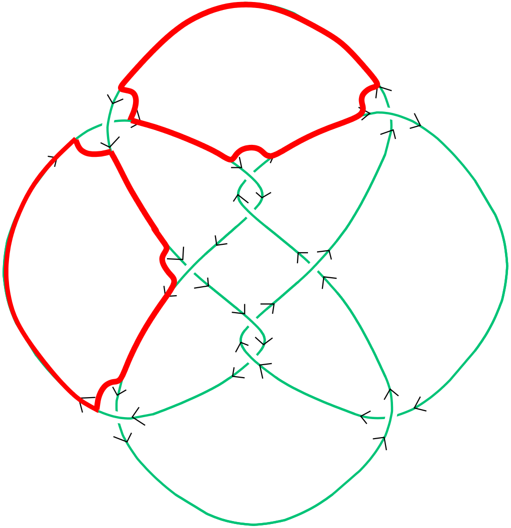
y le seguimos hasta regresar al primer arco:



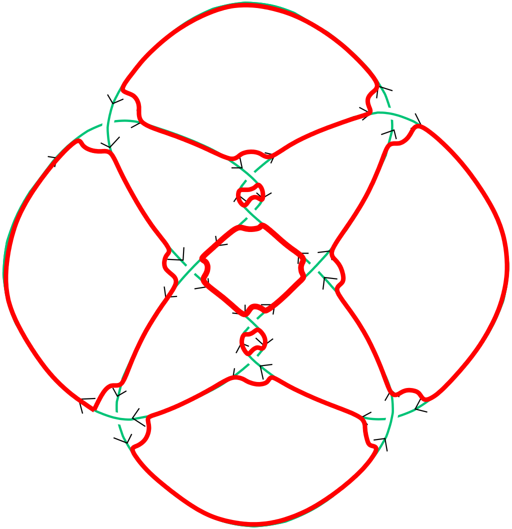
si quedan arcos sin recorrer, recorremos uno más:



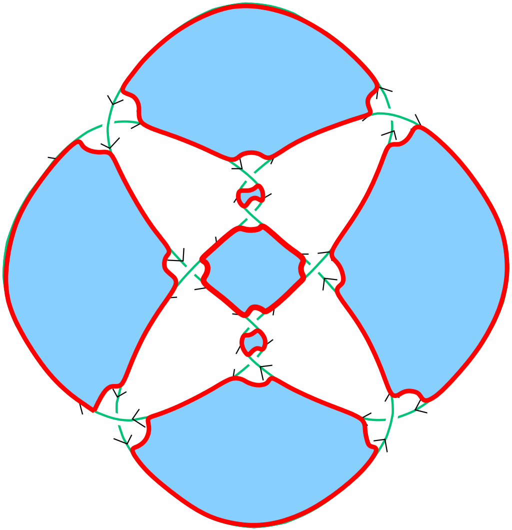
hasta cerrar otro ciclo:



GOTO1:

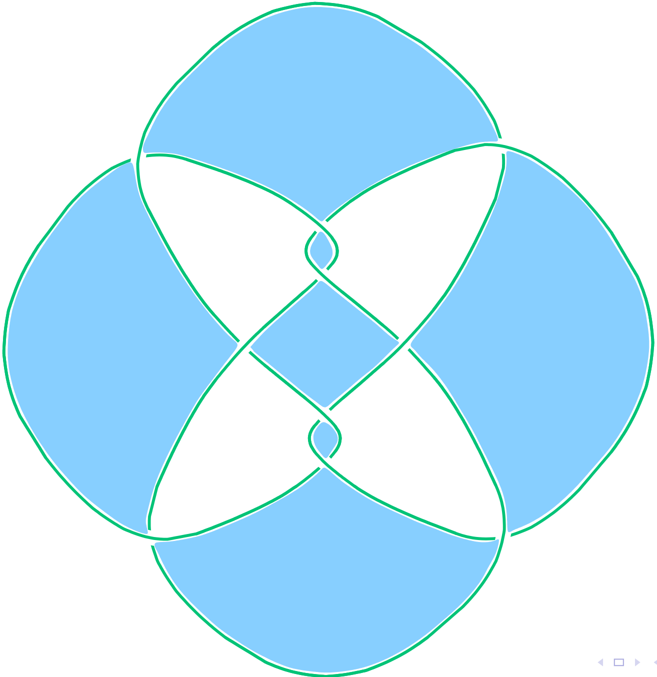


En cada ciclo ponemos un disco:



Y en cada cruce ponemos...

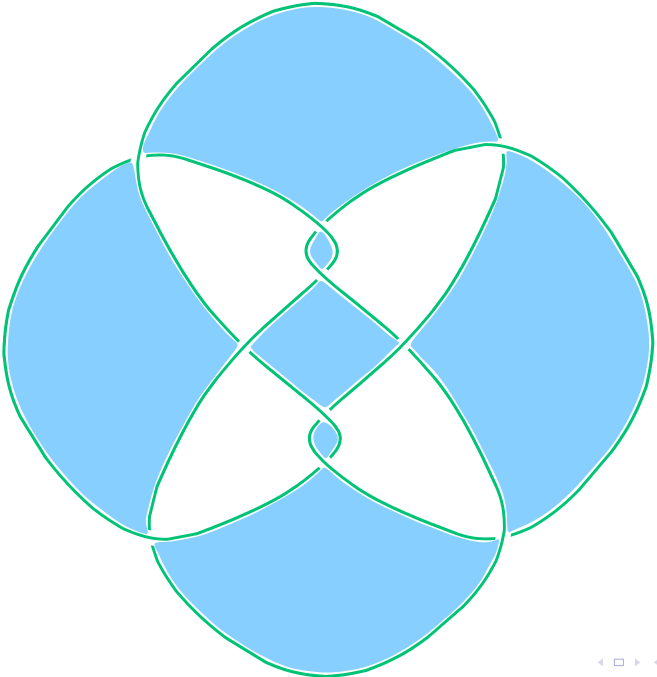
¡Una bandita torcida!



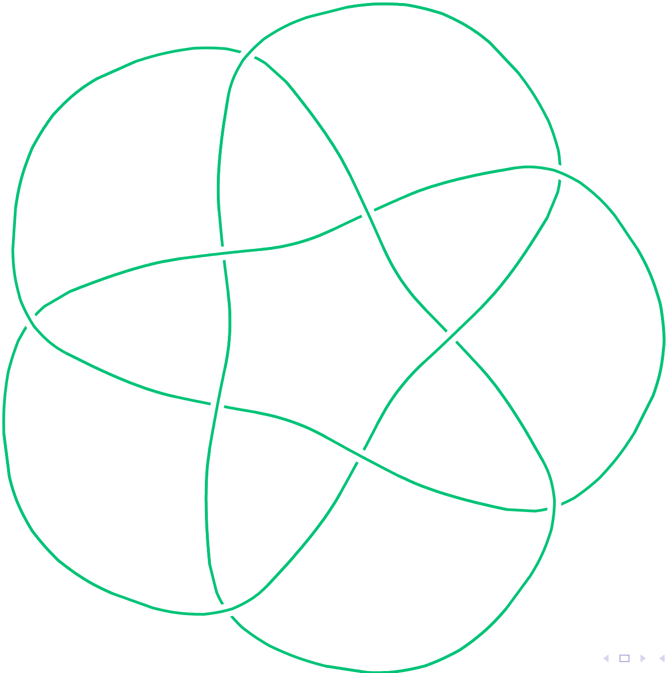
(tuvimos suerte)

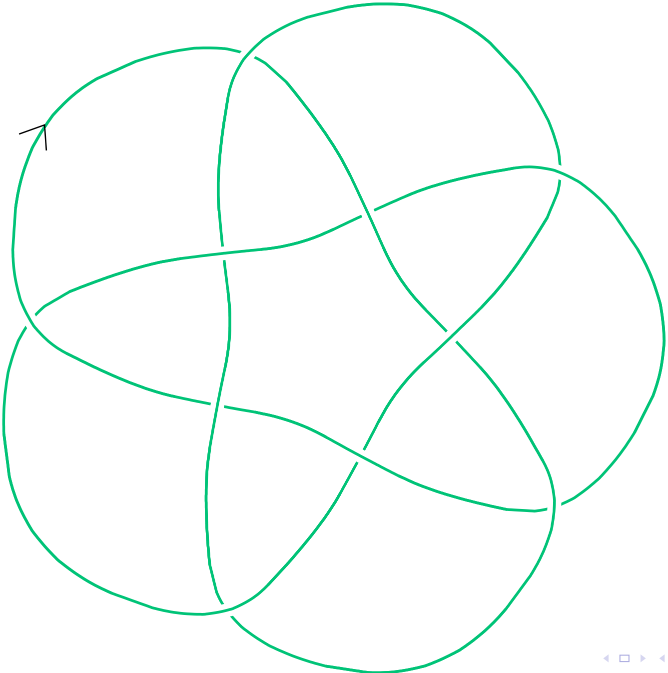
(o sea, salió fácil)

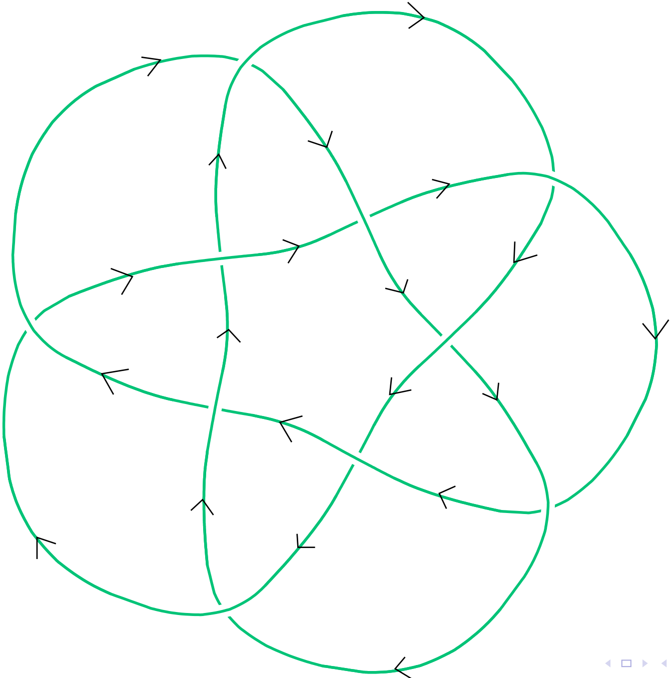
(es decir, nos salió la superficie negra que es fácil)

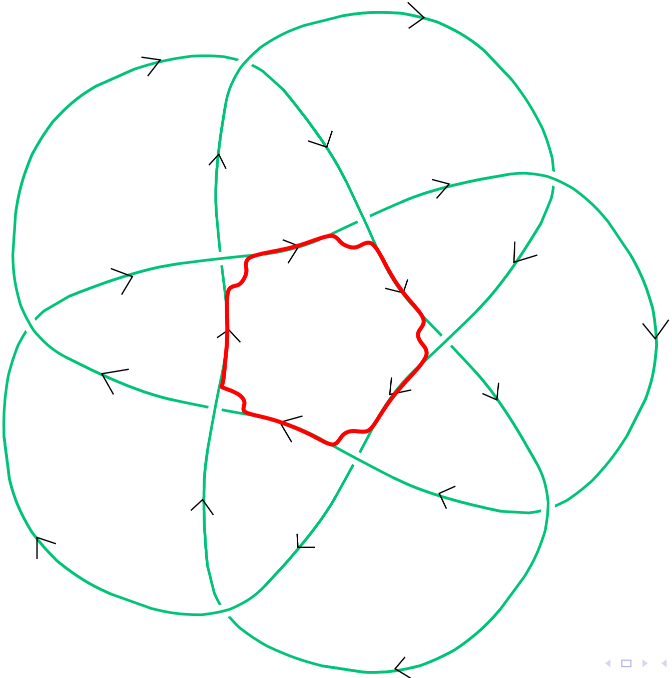


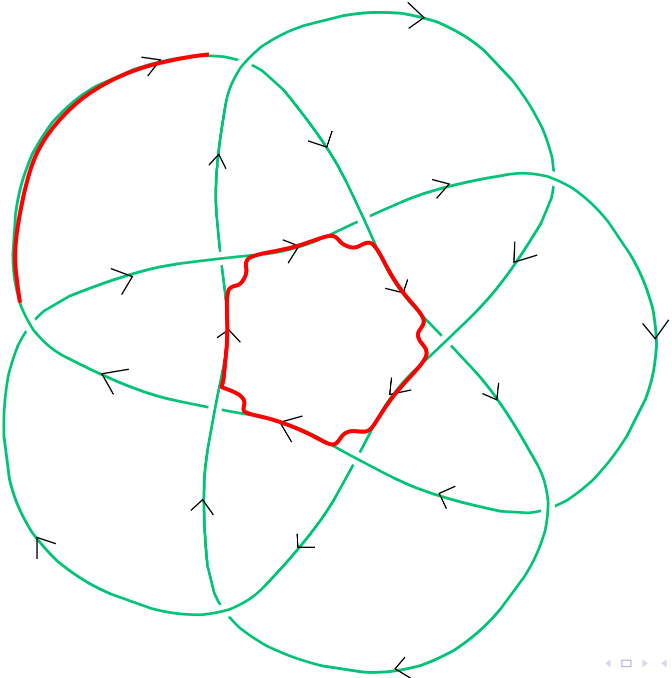
(pero, claro, hay más nudos:)

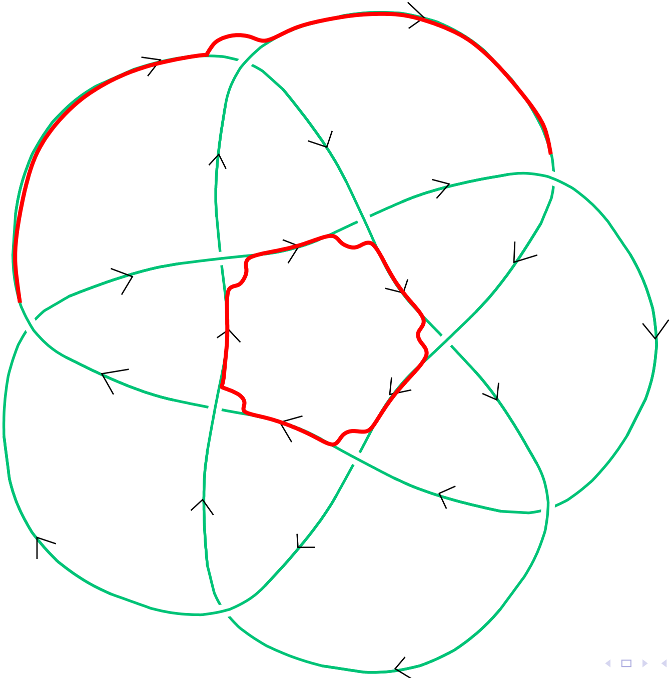


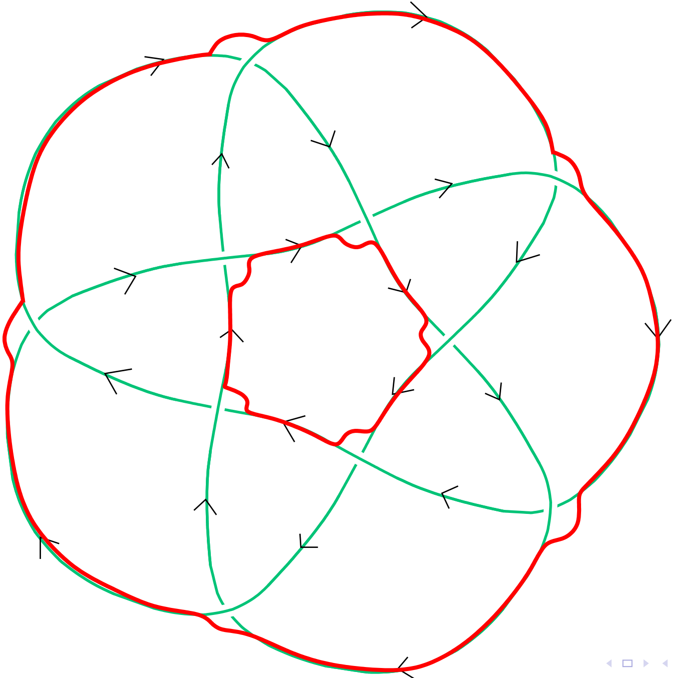


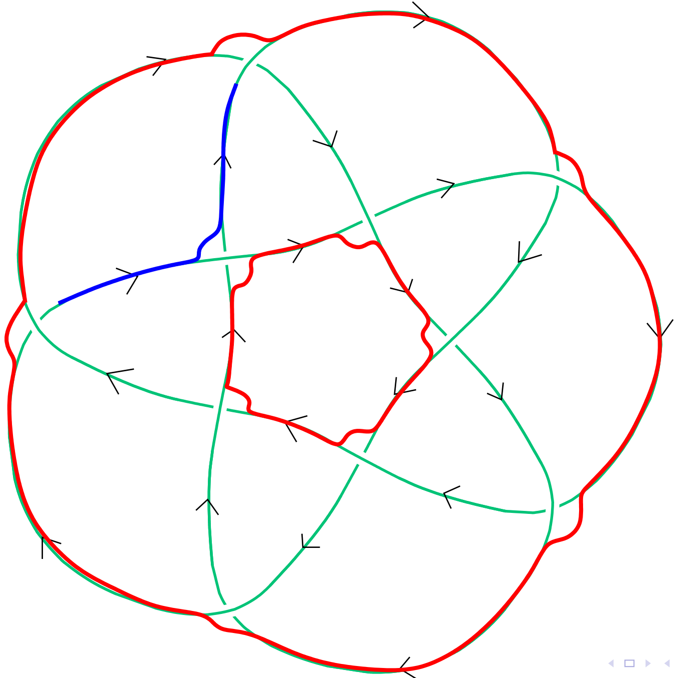


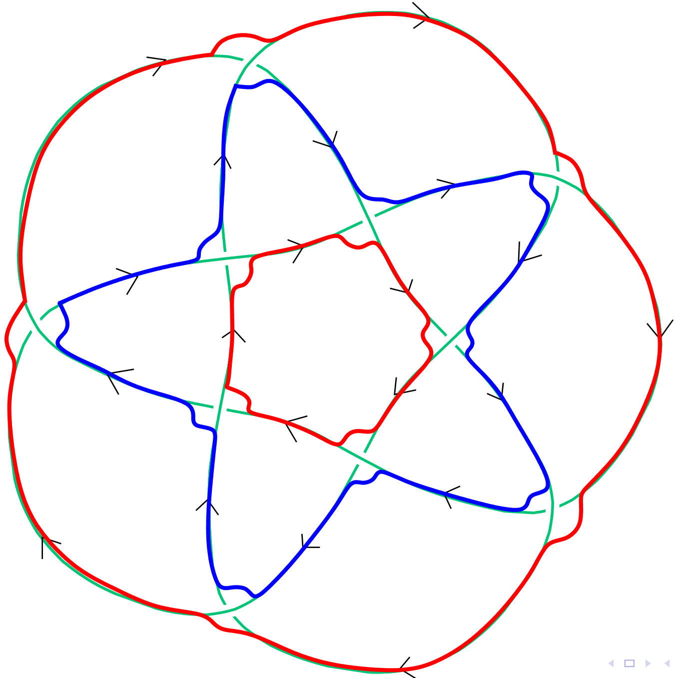




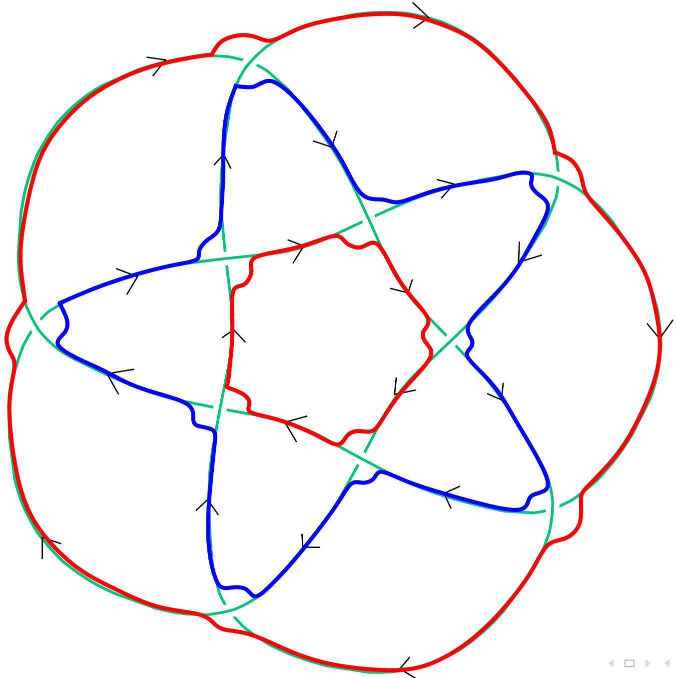






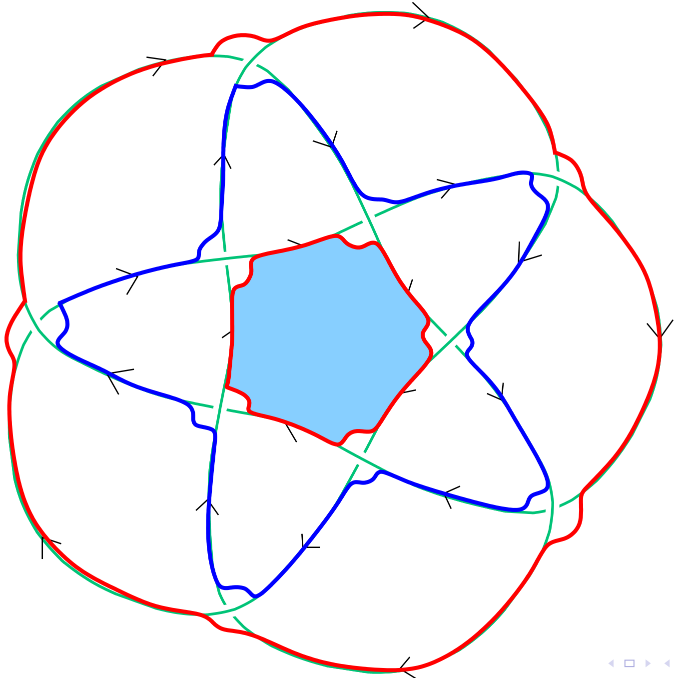


Los ciclos quedaron “anidados”.

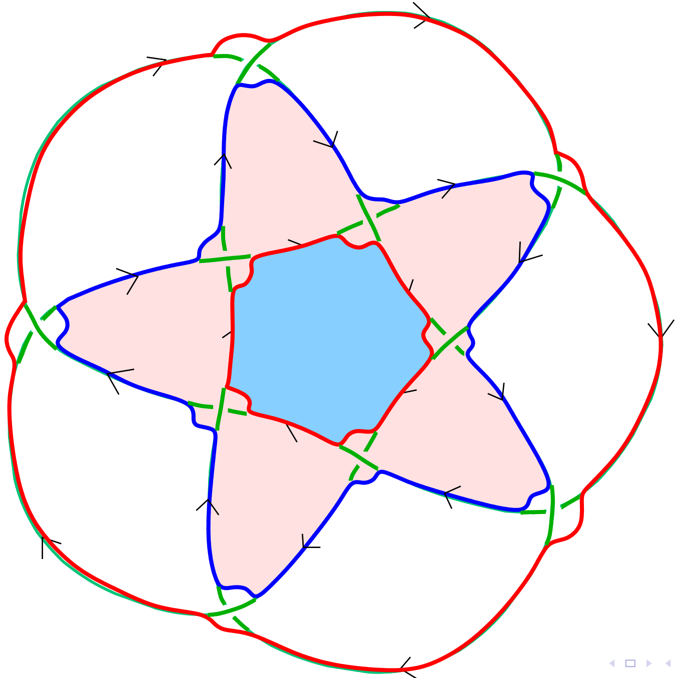


De todos modos:
En cada ciclo ponemos un disco

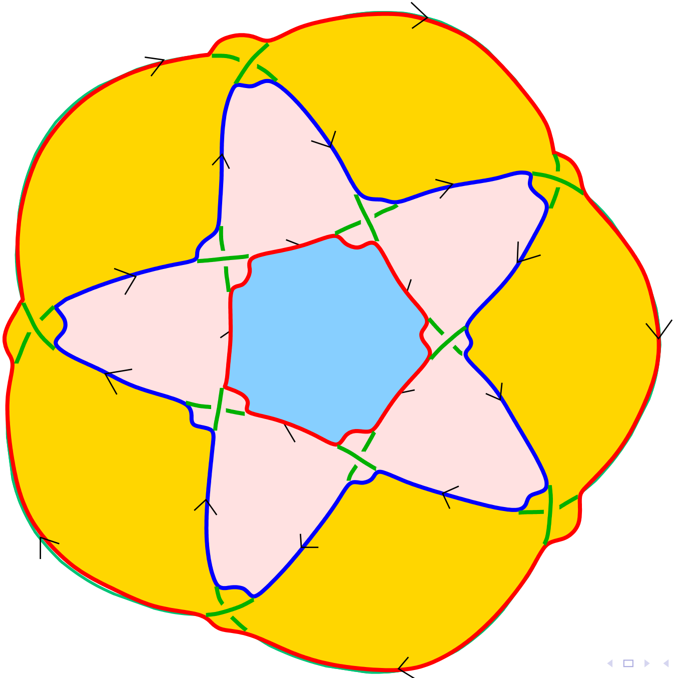
Pero el disco del ciclo de más adentro lo ponemos hasta arribotota.



el disco del ciclo que sigue lo ponemos un poco más abajo

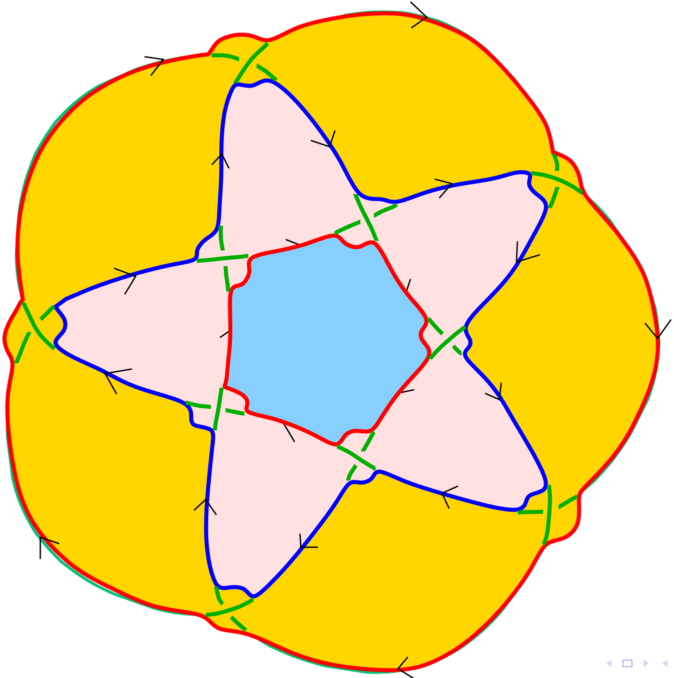


Y así, hasta que el disco del ciclo de hasta afuera lo ponemos hasta abajote:

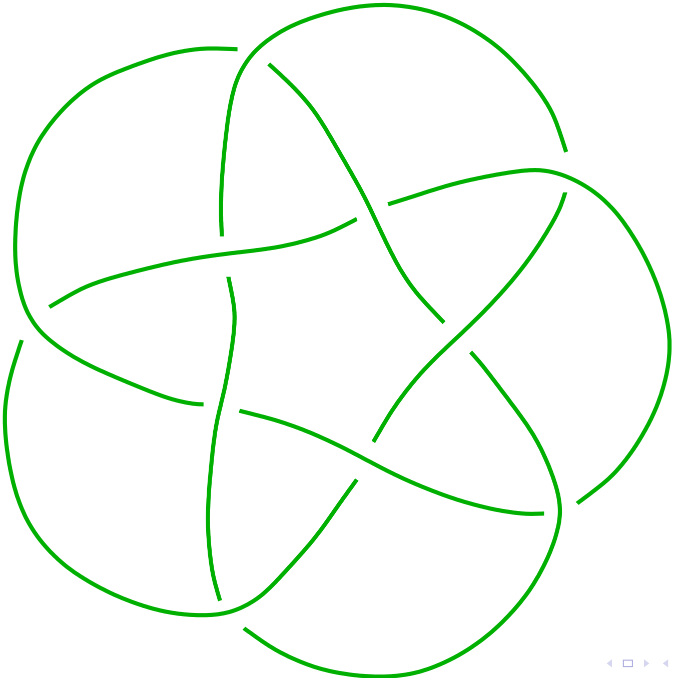


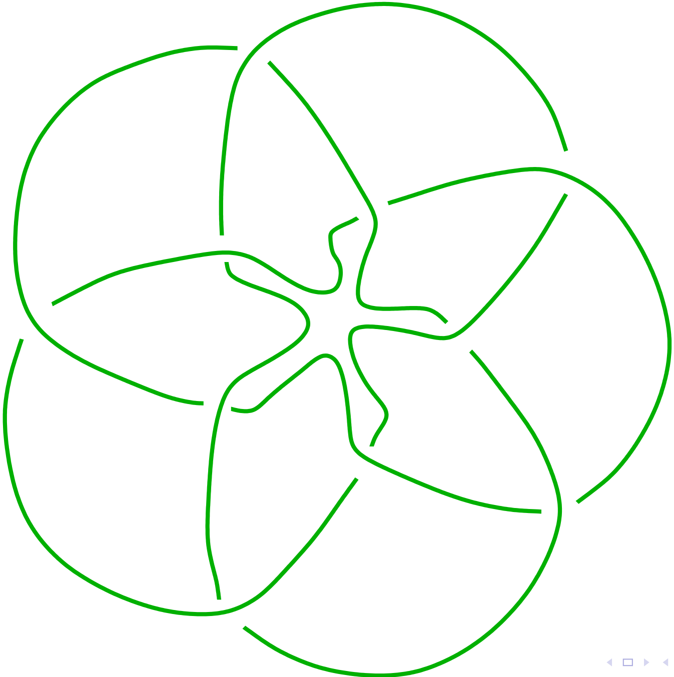
Y en cada cruce ponemos...

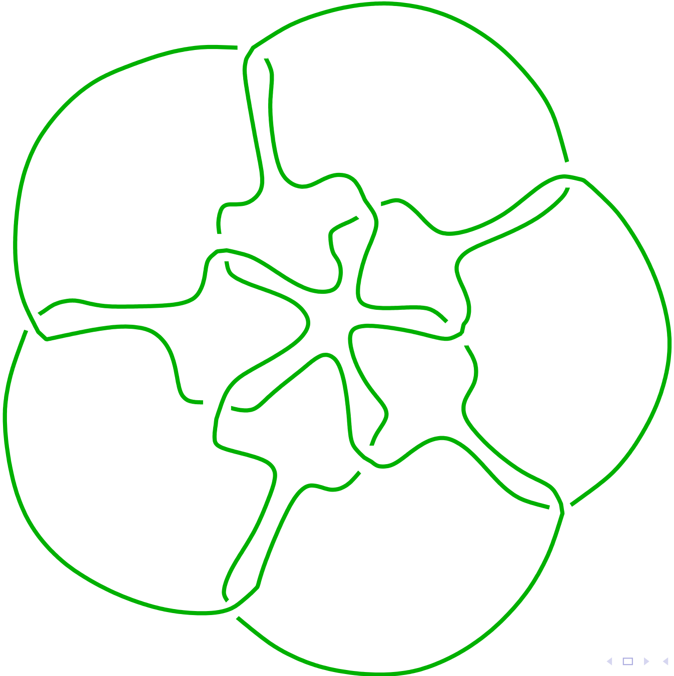
¡Una bandita torcida!

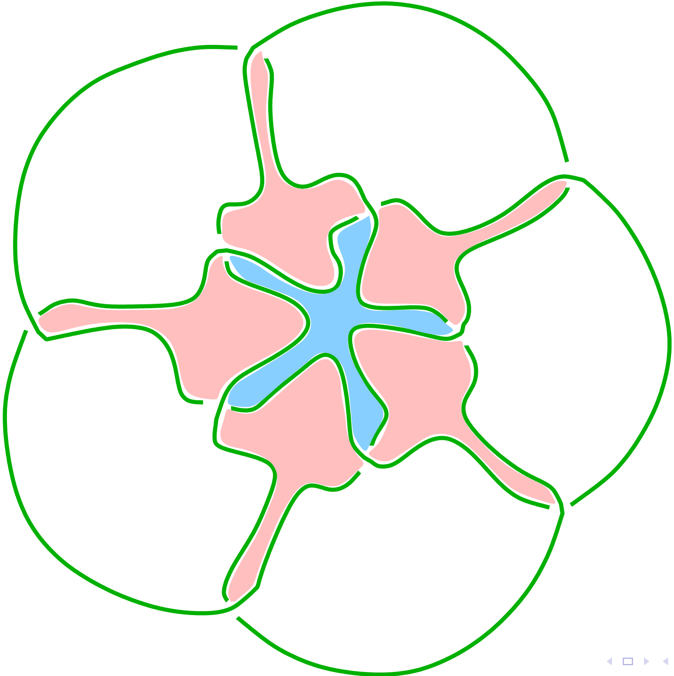


(este fue el nudo **10a121** de su lista
y el **10-123** de la lista de Rolfsen)

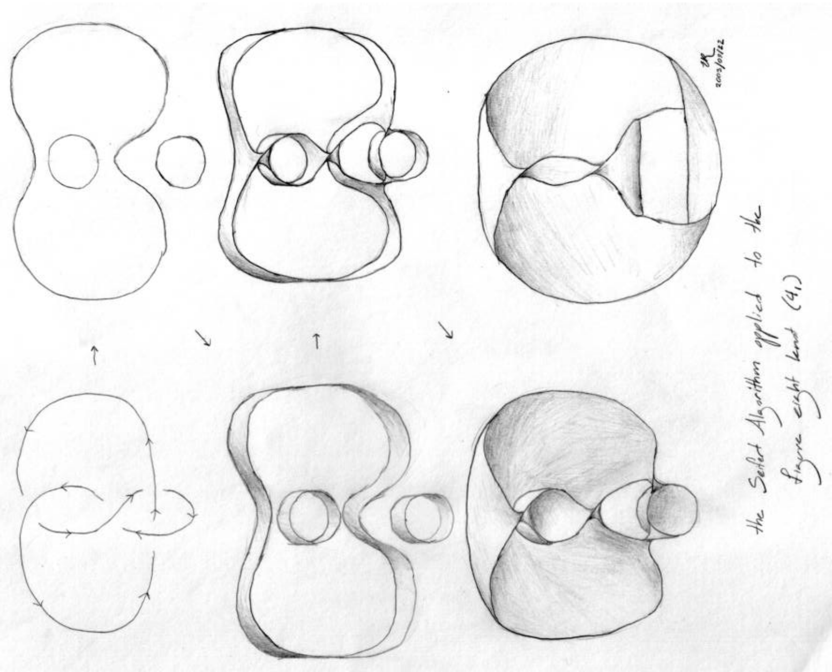








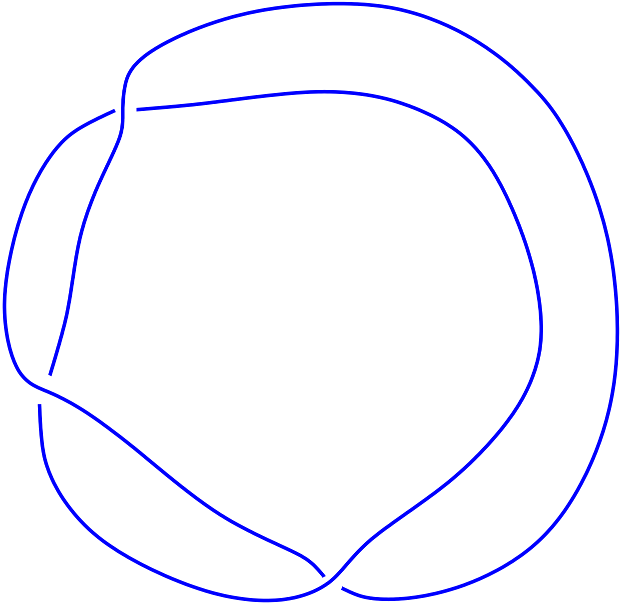
Bar Natan

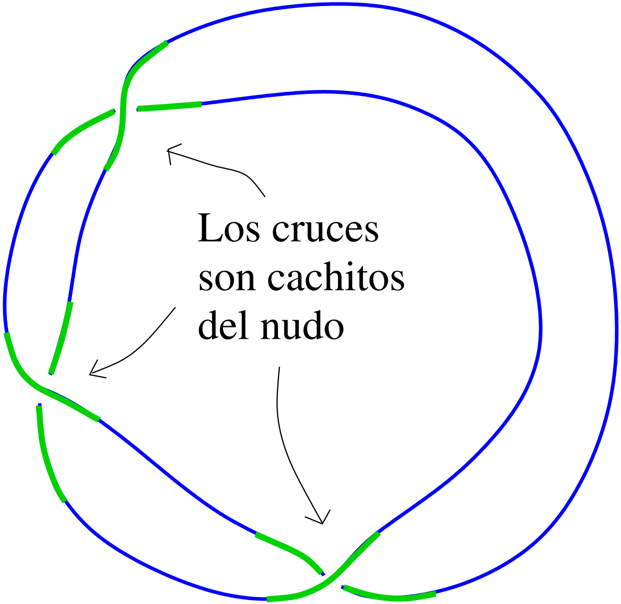

the Seifert Algorithm applied to the
figure eight knot (4_1)
ZK
2003/01/22

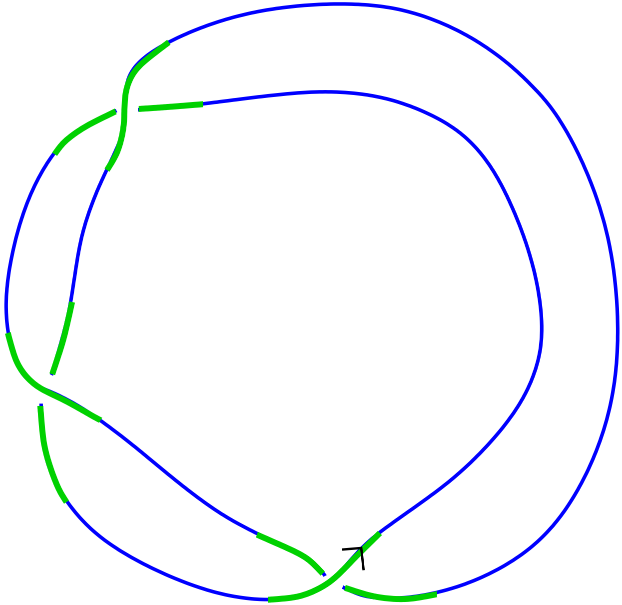
Se puede ver que la superficie que sale del algoritmo de Seifert para un nudo dado, resulta orientable

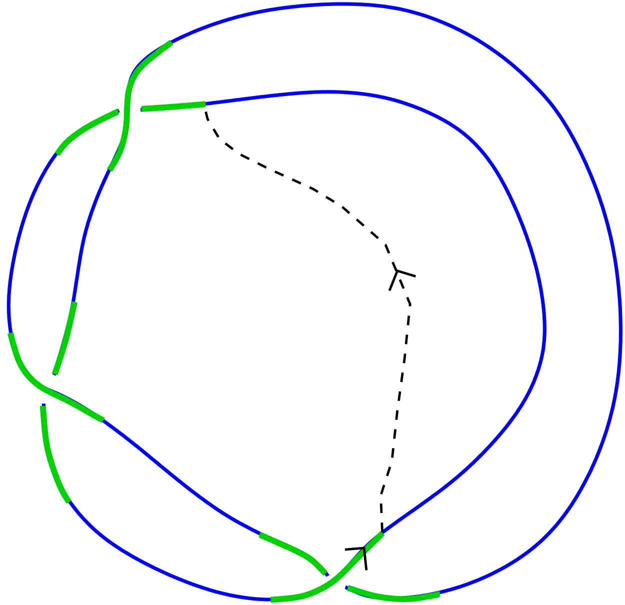
Supongamos que no

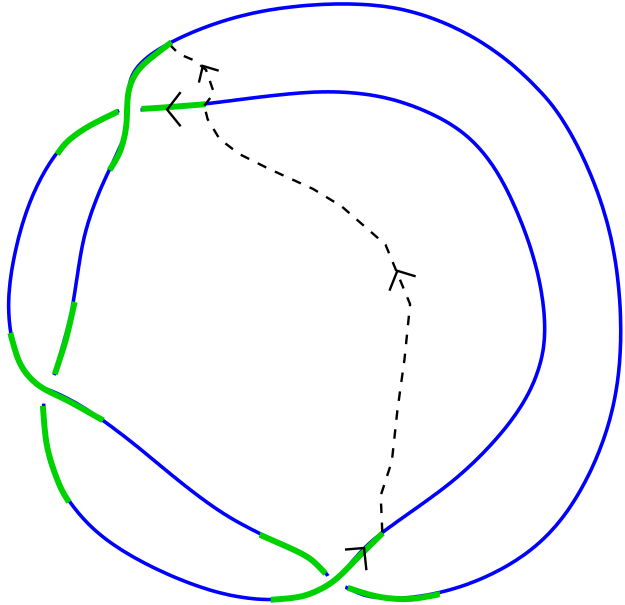
(supongamos que la superficie que salió del algoritmo de Seifert para un nudo fue no orientable)

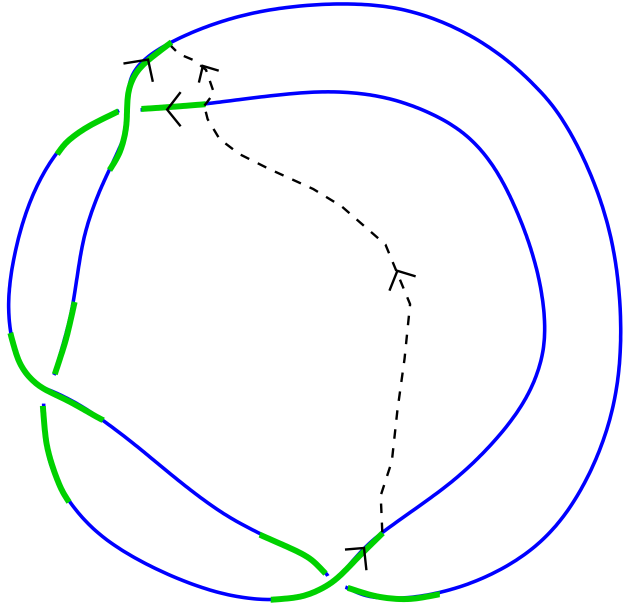


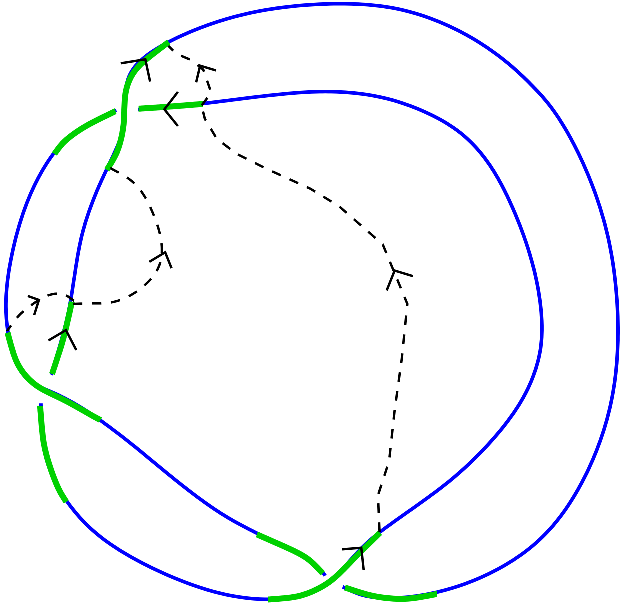

Los cruces
son cachitos
del nudo

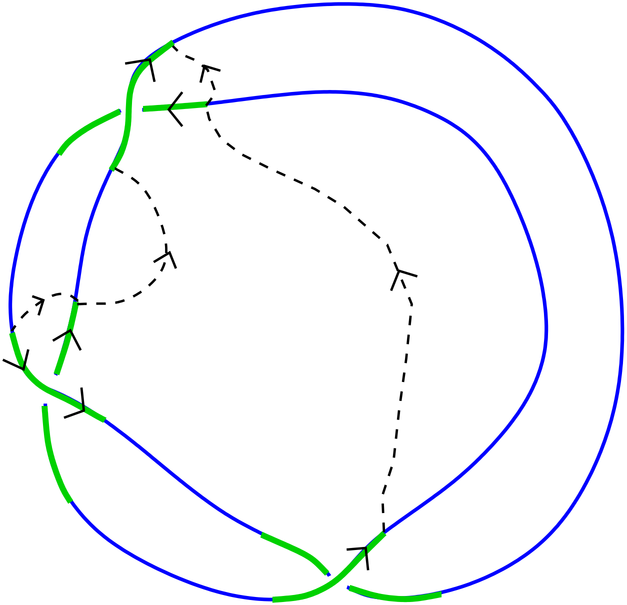


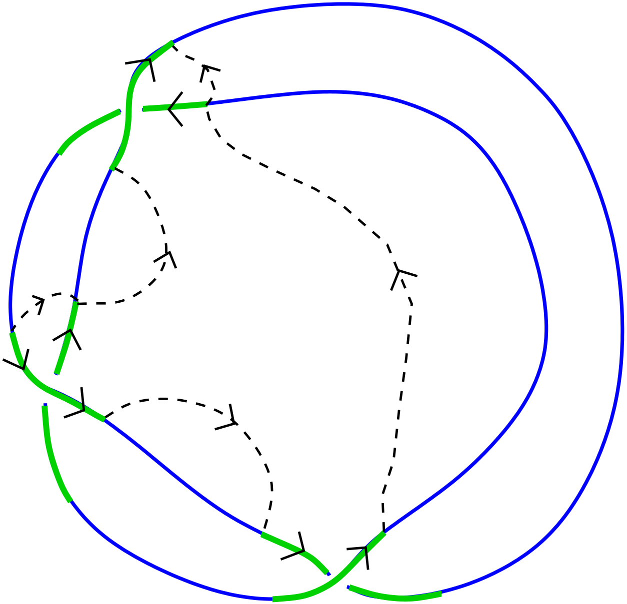


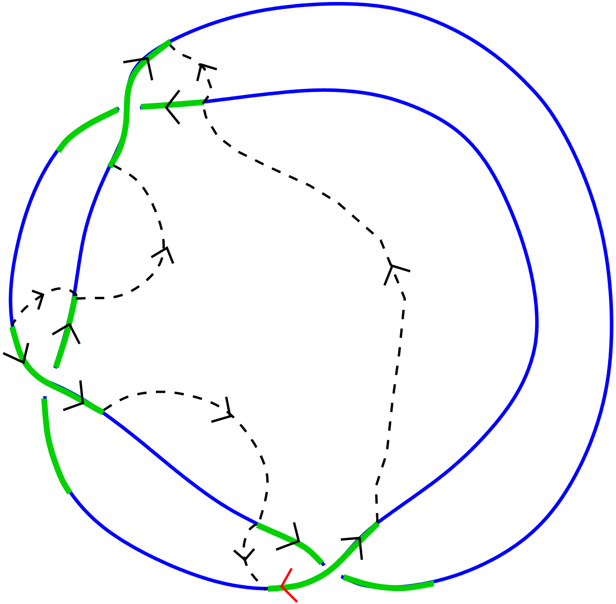


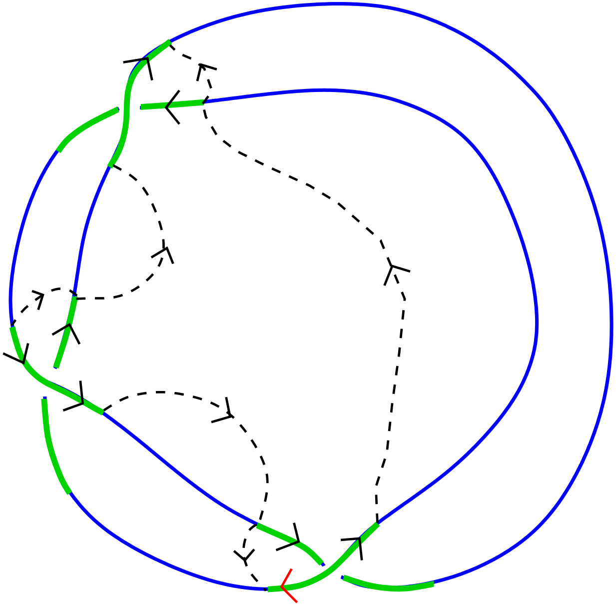












!!!

Dibuja tu nudo preferido y hazle su superficie

Pero nosotros ya sabemos mucho

¡Vamos al knotinfo!