FDIS 2025, schedule and abstracts

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Index of abstracts

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A Parallel Iterative Method for Hamiltonian Mechanics and Discrete Optimal Control Theory

Zamurat A. Adegboye[1] David Martín de Diego[2] Sebastián J. Ferraro[3]
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Recently, we have introduced a parallel iterative method for discrete Lagrangian mechanics that allow us to numerically compute trajectories of the continuous system for boundary value problems \cite{FeMaSaIFAC, our-paper-in-progress}. In this work, we will study the extension to Hamiltonian systems and specially in systems that appears in optimal control theory.

Beltrami problem and Nijenhuis operators

D. Akpan (joint work with A.Bolsinov)

Two (pseudo)-Riemannian metrics are called geodesically equivalent if their geodesics coincide as unparameterized curves. In 1865, E. Beltrami explicitly posed the problem of describing all such metrics. This problem was solved by U. Dini in 1869 for the two-dimensional Riemannian case near almost every point, and in 1896, T. Levi-Civita extended the solution to all dimensions. The pseudo-Riemannian case, near almost every point and in any dimension, was resolved much later, by A. Bolsinov and V. Matveev in 2015 (with the two- and three-dimensional cases addressed earlier by A.Z. Petrov in 1949 and by Bolsinov-Matveev-Pucacco in 2009).

The new results of my talk, in collaboration with A. Bolsinov, provide a solution to the two-dimensional Beltrami problem at the remaining points, where the (1,1)-tensor connecting the geodesically equivalent metrics undergoes a change in its Segre characteristics.

One of the methods used in the proof arises from the relationship between geodesically equivalent metrics and quadratic integrals of the geodesic flow. Another group of methods stems from Nijenhuis geometry, a novel research direction that provides fresh insights into differential geometry and mathematical physics.

If time permits, I will comment on the multidimensional case, emphasizing the connection between geodesic equivalence, integrable geodesic flows, and Nijenhuis geometry.

Acknowledgment. This research was supported by the Ministry of Science and Higher Education of the Republic of Kazakhstan (grant No. AP23483476) and by the DFG 529233771.

Existence and confinement of periodic magnetic orbits at low energy

Valerio Assenza

February 19, 2025

Abstract

Magnetic systems form a standard class of classical Hamiltonian systems that describe the motion of a charged particle moving on a Riemannian manifold under the influence of a magnetic force. At energy levels above the Mañé critical value, the dynamics closely resemble that of geodesic flow and have been extensively investigated. Despite several remarkable results obtained in recent decades, the magnetic dynamics at low energy levels remain incompletely understood. For instance, the existence of a periodic orbit with energy below the Mañé critical value is still unknown. Using recent techniques developed in magnetic geometry, we prove the existence of contractible closed orbits at every energy level close to zero, confined to regions with high magnetic intensity. This work is a collaboration with Gabriele Benedetti and Leonardo Macarini.

On the geometric formulation of the notions of particular integral and particular integrability in Classical Mechanics

Rafael Azuaje
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Abstract

In this talk it is presented the notions of particular integral and particular integrability, within the geometric framework of Classical Hamiltonian Mechanics on symplectic and contact manifolds. In general terms, a particular integral is a quantity that is conserved possibly only on a dynamical invariant submanifold of the phase-space; the notion of particular integrability allows the integrability by quadratures of the equations of motion in certain region of the phase-space where a sufficient amount of conserved quantities (particular integrals) are found. In the special case of contact Hamiltonian systems, which describe dissipative systems, the so called dissipated quantities are particular integrals, they allow a reduction of the equations of motion, and in the particular case of the so called "good" contact Hamiltonian systems, we could have particular integrability.

Relative Equilibria in the Restricted Curved Three-Body Problem

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Abstract. We consider the circular restricted three-body problem in 2D spaces of constant curvature. Our work investigates existence and stability of relative equilibria (RE) which generalise the Lagrange libration points $L_1, \ldots, L_5$ to the case of nonzero curvature. In our approach, we fix the Riemannian distance between the primaries and study the behaviour of the RE in terms of the mass ratio of the primaries and the curvature. This approach clarifies the underlying geometry and allows us to recover and extend previous results obtained by Martinez and Simò (2017) in simpler terms. We first show how the compactness of the 2D sphere leads to the existence of new RE for positive curvature. We then show how positively curving the space yields larger ranges of stability for L_4 and L_5 and may also stabilise other RE. This is work in progress in collaboration with Luis C. García-Naranjo.

Exact quasi-periodic solutions to the KdV, sine(sinh)-Gordon, and ±mKdV equations

Julia Bernatska

Abstract. Exact quasi-periodic solutions of the KdV, sine(sinh)-Gordon, and mKdV (focusing and defocusing) equations are expressed via multiply periodic ℘-functions. These solutions are obtained by algebraic integration of hamiltonian systems from the corresponding hierarchies. The KdV, sine(sinh)-Gordon, and ±mKdV hierarchies all possess spectral curves which are hyperelliptic, with one branch point at infinity. Such a spectral curve of an arbitrary genus g is uniformized by the associated ℘-functions, which generalize the Weierstrass ℘-function. The ℘-functions are used to express quasi-periodic solutions to g-gap systems. Solutions to the KdV, and sine(sinh)-Gordon hierarchies were known before, and solutions to the mKdV hierarchies (focusing and defocusing) are new. The problem of reality conditions for all mentioned hierarchies is revised based on the knowledge of behavior of ℘-functions, and an accurate solution is obtained, which is also a new result. The proposed solutions are illustrated by plots in genera 2 and 3.

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Multicomponent generalisations of KdV and Camassa-Holm equations and their finite-dimensional reductions .

Alexei Bolsinov
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Abstract

We suggest a geometric construction leading to a new series of multi-component integrable PDE systems that contains as particular examples (with appropriately chosen parameters)many famous integrable systems including KdV, coupled KdV, Harry Dym, coupled Harry Dym, Camassa-Holm, multicomponent Camassa-Holm, Dullin-Gottwald-Holm and Kaup-Boussinesq systems.

We suggest a methodology for constructing a series of solutions for all systems of this type. The crux of the approach lies in reducing this system to a dispersionless integrable system which is a special case of linearly degenerate quasilinear systems actively explored since the1990s and recently studied in the framework of Nijenhuis geometry. These infinite-dimensional integrable systems are closely connected to certain explicit finite-dimensional integrable systems of Stäckel-Benenti type. We provide a link between solutions of our multicomponent PDE systems and solutions of this finite-dimensional system, and use it to construct animations of multi-component analogous of soliton and cnoidal solutions .

INTEGRABLE SUB-RIEMANNIAN GEODESIC FLOW IN THE ENGEL TYPE GROUP.

ALEJANDRO BRAVO-DODDOLI

The Engel-type group, denoted as $\mathrm{Eng}(n)$, is a $(2n+2)$-dimensional Carnot group equipped with a non-integrable distribution of rank $(n+1)$. Every Carnot group $\mathbb{G}$ admits the structure of a subRiemannian manifold. Given a Carnot group with a subRiemannian structure, there exists a Hamiltonian function $H_{sR} : T^*\mathbb{G} \to \mathbb{R}$ such that its solutions, when projected to $\mathbb{G}$, correspond to geodesics. When $n = 1$, $\mathrm{Eng}(n)$ reduces to the classical Engel group, whose sub-Riemannian geodesic flow is integrable when endowed with left-invariant subRiemannian metric. In joint work with Enrico Le Donne and Nicola Paddeu, we extend this setting by endowing $\mathrm{Eng}(n)$ with a left-invariant sub-Riemannian metric. We define $\mathbb{A}$ as the maximal abelian subgroup of $\mathrm{Eng}(n)$. The action of $\mathbb{A}$ on $\mathrm{Eng}(n)$ gives rise to $(n + 2)$ integrals of motion. Together with the Hamiltonian function H_{sR}, this yields $(n+3)$ constants of motion in involution. Next, we define a Hamiltonian group action of the special orthogonal group $\mathrm{SO}(n)$ on $T^*\mathrm{Eng}(n)$ that preserves the Hamiltonian H_{sR}. This symmetry leads to additional, non-commutative constants of motion, and these integrals are quadratic in the momenta. Notably, the Hamiltonian group action of $\mathrm{SO}(n)$ on $T^*\mathrm{Eng}(n)$ does not arise from a co-lift of an action of $\mathrm{SO}(n)$ on $\mathrm{Eng}(n)$. By exploiting these symmetries, we derive $(n - 1)$ new integrals of motion in involution, thereby ensuring that the sub-Riemannian geodesic flow is Liouville integrable. This construction thus provides:

- A new example of an integrable sub-Riemannian geodesic flow on a Carnot group with arbitrary rank distribution.
- A novel Hamiltonian group action that does not correspond to the co-lift of a group action.

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The affine Bonnet problem

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Abstract

The Bonnet problem in Euclidean surface theory is well-known: Given a metric g on an oriented surface M^2 and a function H, when (and in how many ways) can (M, g) be isometrically immersed in $\mathbb{R}^3$ with mean curvature H? For generic data (g, H), such an isometric immersion is impossible and, in the case that it does exist, the immersion is unique. Bonnet showed that, aside from the famous case of surfaces of constant mean curvature, there is a finite dimensional moduli space of (g, H) for which the space of such *Bonnet immersions* has positive dimension.

The corresponding problem in affine theory is still not completely solved. After reviewing the Euclidean results on this problem by O. Bonnet, J. Radon, É. Cartan, and A. Bobenko, I will give a report on affine analogs of these results. In particular, I will consider the classification of the data (g, H) for which the space of *affine Bonnet immersions* has positive dimension, showing a surprising connection with integrable systems in the case of data with the highest possible dimension of solutions.

From the Lagrange Triangle to the Figure Eight Choreography: Proof of a Conjecture of C. Marchal

Renato C. Calleja Castillo

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Abstract

In the context of the three-body problem with equal masses, Marchal conjectured in 1999 that the most symmetric continuation class of Lagrange's equilateral triangle solution, known as the P12 family of Marchal, includes the remarkable figure-eight choreography discovered by Moore in 1993 and proven to exist by Chenciner and Montgomery in 2000. In this talk, I will present a framework for verifying the existence of the P12 family as a zero-finding problem. Additionally, we will explore the relation between this verification and Marchal's conjecture. This work is a collaboration with Carlos García-Azpeitia, Olivier Hénot, Jean-Philippe Lessard, and Jason Mireles James.

Vladimir Dragović, The University of Texas at Dallas

Integrable Magnetic Flows on Spheres and Nonholonomic Mechanics

Abstract:

We introduce and study the Chaplygin systems with gyroscopic forces. We put a special emphasis on the important subclass of such systems with magnetic forces. In a reduction, we construct Hamiltonian magnetic systems on spheres S^n. We prove the integrability of the latter systems for n = 2, 3, 4, and 5. We conjecture the integrability of those systems for all n. This is based on joint work with Borislav Gajic and Bozidar Jovanovic and the following papers:

[1] Dragović, V., Gajić, B., Jovanović, B., Demchenko's nonholonomic case of a gyroscopic ball rolling without sliding over a sphere after his 1923 Belgrade doctoral thesis, Theor. Appl. Mech. (2020)

[2] Dragović, V., Gajić, B., Jovanović, B., Gyroscopic Chaplygin systems and integrable magnetic flows on s pheres, J. Nonlinear Sci. (2023)

[3] Dragović V., Gajić, B., Jovanović, B., Integrability of homogeneous exact magnetic flows on spheres, arXiv: 2504.20515

Integrable systems related to separation of variables on symmetric spaces of rank 1

Holger Dullin

July 10, 2025

Separation of variables gives a way to construct families of integrable systems. We study such (quantum) integrable systems on spheres and other symmetric spaces of rank 1. For example choosing a separating coordinate system for the geodesic flow on S^3 induces a Liouville integrable system, which after symplectic quotient by the S^1 action induced by the geodesic flow induces an integrable system on the Grassmannian $Gr(2,4)$ which is $S^2 \times S^2$. We show that in this family there is only one such integrable system on $Gr(2,n+1)$ that is toric. The image of the classical momentum map on S^3 is a cone over a square, containing the joint spectrum of the corresponding quantum integrable system. Repeating the construction for $\mathbb{R}P(3) = SO(3)$ also gives a cone, but with half the volume and the square rotated by 45 degrees. The arrangement of the joint spectra is different, but of course such that Weyl's law holds in the semi-classical limit with half the number of states for $SO(3)$. Generalisations to $\mathbb{C}P(n)$ and using a recent result with Bolsinov, Matveev, Nikolayevsky are discussed. This is joint work with Damien McLeod.

Conformal, projective, and Weyl geometries

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Abstract

Mostly, this talk will be a review of known constructions and especially the links between them. The formulae governing conformal and projective differential geometry already reveal striking similarities, really just the tip of the iceberg. Both geometries also exhibit some intriguing patterns of invariant differential operators, some of which can be used to formulate and sometimes answer questions on the existence of preferred metrics or connections. A specific question along these lines concerns the Weyl metrisability of projective structures and, if time permits, I'll provide some answers obtained in recent joint work with Omid Makhmali. But I'll start from scratch so I might not get that far.

On the four-body limaçon trisectrix choreography: maximal superintegrability and choreographic fragmentation

Adrian M. Escobar-Ruiz[1]

(in collaboration with Manuel Fernández-Guasti)

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Abstract:

In this talk, we explore the dynamical and symmetry properties of a four-body system exhibiting choreographic motion along a limaçon trisectrix, building on recent results [Celest. Mech. Dyn. Astron. 137, 4 (2025)]. We demonstrate that the reduced Hamiltonian governing the motion in six-dimensional relative space is *maximally superintegrable*, admitting a complete set of eleven independent integrals of motion, which we indicate explicitly. We show that the system allows a complete separation of variables. The emergence of the observed choreography is not solely due to superintegrability. Instead, it arises from a subtler mechanism: the presence of *particular involution* and existence of *particular integrals*. Additionally, we describe how a more general four-body choreography can fragment into two distinct two-body choreographies, revealing a rich structure in the solution space.

Affine nonholonomic rolling on the plane

Luis Garcia Naranjo
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Abstract

We introduce a class of examples which provide an affine generalization of the nonholonomic problem of a convex body that rolls without slipping on the plane. These examples are constructed by taking as given two vector fields, one on the surface of the body and another on the plane, which specify the velocity of the contact point. We investigate dynamical aspects of the system such as existence of first integrals, smooth invariant measure, integrability and chaotic behavior, giving special attention to special shapes of the convex body and specific choices of the vector fields for which the affine nonholonomic constraints may be physically realized. We also discuss some remarkable behavior occurring when the body is a homogeneous sphere and the vector fields have discontinuities with specific symmetries. This is joint work with Mariana Costa-Villegas.

A Geometric and Dynamical Review of the Spin-Orbit Problem

Brayan Guerra

June 3, 2025

Abstract

The spin-orbit problem arises in celestial mechanics as a model for the rotational dynamics of a satellite orbiting a central body, particularly when the satellite has an asymmetric mass distribution. Historically motivated by the synchronous rotation of the Moon, the model provides a rich framework for understanding resonance phenomena in finite-dimensional dynamical systems. This poster offers a review of the origin and mathematical formulation of the conservative spin-orbit model, emphasizing its Hamiltonian structure and its relation to symplectic geometry. We also examine the dissipative version of the model, where energy loss due to internal friction or tidal forces leads to the emergence of attractors and modifies the qualitative dynamics. By comparing these two settings, we highlight how the spin-orbit problem serves as a foundational example in the study of geometric and dynamical structures in celestial mechanics.

Global Structure of Completely Integrable Systems in Dimension 4

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Abstract

When studying conservative physical phenomena within the framework of classical mechanics, we often encounter completely integrable systems. Such systems admit a particularly rich topological and geometric structure on phase space, allowing for local action-angle coordinates that linearize the dynamics on invariant tori and define a torus bundle structure. However, Hamiltonian monodromy appears as a topological obstruction to the global existence of such coordinates, and thus to the triviality of the associated fibration.

In this talk, I will revisit geometrically these concepts in the setting of integrable systems on $\mathbb{R}^4$. Then, using spectral Lax pairs, I will show how one can associate a Riemann surface to the system with the property that the computation of Hamiltonian monodromy reduces to the calculation of the residue at infinity of a meromorphic 1-form on this surface. I will conclude with an overview of some perspectives and possible extensions of this work.

Degenerate singularities in $(1 : \pm m)$-oscillators

Tobias Våge Henriksen
University of Antwerp, Belgium

Abstract

Kalashnikov conjectured that there are six generic degenerate singularities that may appear in 2-degree-of-freedom integrable systems in resonance. One of them is the well-known cusp, whilst two more are known in systems with $(1 : \pm 2)$ resonance, but are not encountered frequently. Degenerate singularities in resonance $(1 : \pm 3)$ or higher has not been discovered before. Here we provide a family of integrable systems displaying all six generic degenerate singularities from Kalashnikov's list.

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Recent developments and advances around hypersemitoric systems

Sonja Hohloch
University of Antwerp

Hypersemitoric systems are two degree of freedom integrable Hamiltonian systems on 4-dimensional compact symplectic manifolds with possibly mild degeneracies where one of the integrals gives rise to an effective Hamiltonian S^1-action.

We will give an overview on the progress towards a symplectic classification and nearby developments like the definition of the 'polygon invariant', progress around the Taylor series invariant for hyperbolic-regular points associated with flaps and swallowtails, and bifurcation behaviour arising from nontrivial isotropy resp. resonances. For details, we refer to the posters and/or talks of collaborators also presented at this conference.

Non-compactness effects in integrable systems: examples and results

Vladislav Kibkalo

(assist. prof., Lomonosov MSU, Moscow, Russia)

The topological classification of integrable Hamiltonian systems, constructed and developed by A.T. Fomenko, his scientific school and co-authors, was applied to a wide class of mechanical and physical problems. An important assumption of many statements of the theory is the compactness of the fibers of the Liouville foliation.

In systems with non-compact foliations, new effects arise: e.g. the flows of Hamiltonian vector fields can become incomplete, and the preimage of the bifurcation value does not necessarily contain critical points (in particular, it can be empty). We will briefly discuss several well-known systems with non-compact foliations, as well as the results obtained by various authors on their topological analysis and the problem of classification. We note here the review by A.T.Fomenko and D.A.Fedoseev on systems with non-compact foliations (2020, J. Math. Sc.).

Pseudo-Euclidean analogues of integrable mechanical systems (see A.~Borisov, I.~Mamaev, 2016) turn out to be an important class of such systems. New results on topology of Liouville foliations of pseudo-Euclidean Euler, Lagrange and Kovalevskaya tops, Zhukovsky and Klebsch systems will be presented. Both compact and non-compact fibers, their bifurcations (including non-critical one) appear in such systems. Bifurcations and Liouville foliations bases (analogs of Fomenko graphs) are also determined. Several results on non-compactness effects in billiard systems also will be discussed

Results are supported by the Russian Science Foundation, project 24-71-10100.

Generalized separation of variables and NIjenhuis geometry

Andrey Yu. Konyaev

April 2025

Let M^n be a manifold, equipped with metric g and potential function U. The corresponding geodesic flow is given on $T^*\mathrm{M}^n$ via Hamiltonian

$$\frac{1}{2}g^{ij}p_ip_j + U.$$

. It is a well-known fact that if such flow admits an orthogonal separation of variables, then the separation coordinates are given by a certain Nijenhuis operator with real simple spectrum.

It turns out that this relation with Nijenhuis geometry is much deeper — one can construct an analog integrable system not only for the aforementioned operators, but for operators containing Jordan blocks. This is done via a simple algebraic construction.

This relation not only provides integrable systems (which are, in some sense, close neighbors of well-known separable systems), but also allows one to solve some fundamental problems in the theory of projectively equivalent metrics.

Thick Arnold tongues and cellular flows.

Mark Levi
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Abstract

This problem was originally motivated by an observation that the rotation of Earth creates a very small but also very steady force upon icebergs, suggesting the question: what is the effect of a small steady force on dynamics of a particle carried by a fluid flow? An interesting effect arises in the model problem: the flow is Hamiltonian in $\mathbb{R}^2$, with a cellular structure, i.e. periodic in both variables, with the particle subjected to a steady force acting in a given direction. It is clear that the drift depends on the direction of applied force, but how? Numerical experiments show somewhat surprising dependence. I will describe end explain this unexpected phenomenon and show how Arnold tongues arise. Unlike the Arnold tongues for circle maps, these occupy the set of full measure. The phenomenon is robust. This is joint work with Alexey Okunev.

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Autonomous and non-autonomous restrictions of soliton hierarchies as a source of Stäckel and Painlevé-type systems

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In [1]-[3] the authors presented a systematic way of deforming, using appropriate Killing vectors, Stäckel separable systems $\xi_{t_r} = Y_r(\xi) = \pi dH_r(\xi)$, $r = 1, \ldots, n$ on a $2n$-dimensional Poisson manifold (M, π) to non-autonomous integrable systems $\xi_{t_r} = Y_r(\xi, t) = \pi dH_r(\xi, t_1, \ldots, t_n)$ that are Frobenius integrable so that $\frac{\partial Y_r}{\partial t_s} - \frac{\partial Y_s}{\partial t_r} + [Y_s, Y_r] = 0$, $r, s = 1, \ldots, n$. Recently, in [7], we have developed an analogous theory for soliton hierarchies. We have demonstrated how to deform, using an appropriate master symmetry, an autonomous soliton hierarchy $u_{t_n} = K_n[u]$, $n = 1, 2, \ldots$ into a Frobenius integrable non-autonomous soliton hierarchy $u_{t_n} = K_n([u], x, t_1, t_2, \ldots)$, $n = 1, 2, \ldots$ so that the Frobenius integrability condition $\frac{\partial K_r}{\partial t_s} - \frac{\partial K_s}{\partial t_r} + [K_r, K_s] = 0$ holds.

Since the classical works of Bogoyavlensky, Novikov, Mokhov and others there has been a tremendous amount of research devoted to connections between soliton hierarchies $u_{t_n} = K_n[u]$, $n = 1, 2, \ldots$and their integrable finite-dimensional reductions; a substantial portion of this research focused on stationary flows. In [4]-[6] we have revisited this idea in a novel, systematic way by investigating not only a particular stationary flow $K_{n+1}[u] = 0$ of a soliton hierarchies but rather it's so called stationary system, that is the finite dimensional system defined by the stationary flow $K_{n+1}[u] = 0$ itself together with all the lower flows from the hierarchy:

$$u_{t_1} = K_1[u], \ldots, u_{t_n} = K_n[u], \qquad K_{n+1}[u] = 0$$

In result, we were able to show, by presenting explicit Miura-type maps between the jet variables $(u, u_x, u_{xx}, \ldots)$ and the Viéte variables (q, p), that in the case of particular soliton hierarchies (KdV, cKdV and AKNS) the related stationary systems can be represented as classical Stäckel separable systems.

In an ongoing research project we generalize the above concept of stationary systems to what we call *non-autonomous restrictions of soliton hierarchies*. These restrictions are defined through invariant time-dependent constraints that are appropriate deformations of the stationary flow $K_{n+1}[u] = 0$ by times-dependent linear combinations of a master symmetry and lower flows, the idea which is based on results in [7]. It turns out that this class of time-dependent restrictions of soliton hierarchies is represented by non-autonomous Hamiltonian finite-dimensional dynamical systems of Painlevé type (in the sense that they possess an isomonodromic rather than isospectral representation). The original Painlevé equations are non-autonomous nonlinear ODE's that, at the beginning of the 20th century, allowed for defining new special functions. Our hope it that our approach will yield a *systematic* way of producing new Painlevé-type systems.

In this talk I will present the above ideas, with focus on autonomous and non-autonomous restrictions of soliton hierarchies. This talk is a result of a joint work with **Maciej Błaszak** and **Błażej M. Szablikowski**, Poznań University, Poland.

References

[1] Błaszak M., Marciniak K., Sergyeyev A., *Deforming Lie algebras to Frobenius integrable non-autonomous Hamiltonian systems*, Rep. Math, Phys. **87** (2021), No. 2, pp. 249-263

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$D+\alpha$ operator and related problems in physics.

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The three-dimensional Moisil-Teodorescu differential operator D is defined by

$$D = e_1\partial_1 + e_2\partial_2 + e_3\partial_3,$$

where $\partial_k = \partial/\partial x_k$, $k = 1, 2, 3$. It is known that the action of the operator D to any differentiable function $w = w_0 + \vec{w}$ can be written as

$$Dw = -\operatorname{div}\vec{w} + \operatorname{grad} w_0 + \operatorname{curl}\vec{w},$$

With this we are able to find different representations to classical equations

- A Beltrami Field is a function such that

  $$\operatorname{curl}\vec{w} + \lambda\vec{w} = 0, \quad \text{in } \Omega. \tag{1}$$

  This problems is equivalent to solve

  $$(D+\lambda)w = 0 \tag{2}$$

  which in [1] find explicit solutions.

- The Helmholtz equation $\Delta + \lambda^2$ can be decomposed as follows (see [2])

  $$\Delta + \lambda^2 = -(D+\lambda)(D-\lambda) \tag{3}$$

- Maxwell's equations for time-harmonic electromagnetic fields in a chiral medium have the form

  $$\begin{aligned}&\operatorname{div}\widetilde{E}(x) = \operatorname{div}\widetilde{H}(x) = 0, \;\; \operatorname{curl}\widetilde{E}(x) = i\omega\widetilde{B}(x)\\ &\operatorname{curl}\widetilde{H}(x) = -i\omega\widetilde{D}(x),\end{aligned}$$

  with the constitutive relations (see [2]) and using the change of variables

  $$\vec{\zeta}(x) = \vec{E}(x) + i\vec{H}(x), \;\; \vec{\eta}(x) = \vec{E}(x) - i\vec{H}(x).$$

  Is equivalent to

  $$\left(D + \frac{\alpha}{1+\alpha\beta}\right)\vec{\zeta}(x) = 0, \left(D - \frac{\alpha}{1-\alpha\beta}\right)\vec{\eta}(x) = 0. \tag{4}$$

  We constructed an invertible quaternionic integral operator which transforms solutions of the operator D into solutions these equations.

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THE AUBRY SET OF SYMBOLIC DYNAMICS WITH UNCOUNTABLE SYMBOL: FROM THE VIEW POINTS OF THE WEAK KAM APPROACH AND VARIATIONAL METHODS

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Abstract. We consider the Aubry set for the XY model, symbolic dynamics $([0,1]^{\mathbb{N}_0}, \sigma)$ with the uncountable symbol $[0,1]$, and study its action-optimizing properties. Moreover, for a potential function that depends on the first two coordinates we obtain an explicit expression of the set of optimal periodic measures and a detailed description of the Aubry set. We also show the typicality of periodic optimization for 2-locally constant potentials with the twist condition. Our approach combines the weak KAM method for symbolic dynamics and variational techniques for twist maps. This is joint work with Yuika Kajihara (Kyoto University) and Mao Shinoda (Ochanomizu University).

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Quadratic Killing tensors on symmetric spaces

An Ky Duy Nguyen
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Abstract

We will present some recent results on the structure of the algebra of Killing tensors on Riemannian symmetric spaces. The fundamental question (Bolsinov -Matveev-Miranda-Tabachnikov, 2018), is whether any Killing tensor field on a Riemannian symmetric space is decomposable, that is, is a polynomial in Killing vector fields. For spaces of constant curvature, the answer is in the positive, as has been known for quite some time. The same is true for the complex projective space (Eastwood, 2023). For other rank one symmetric spaces, the answer is almost always in the negative (Matveev-Nikolayevsky, 2024). We show that for several classes of symmetric spaces, all quadratic Killing tensors are decomposable. This is a joint project with my PhD supervisors Y.Nikolayevsky (La Trobe University, Australia) and V.Matveev (University of Jena, Germany).

On classification of regular semisimple algebraic Nijenhuis operators

Andrey Oshemkov

Abstract

Algebraic Nijenhuis operators naturally appear in the study of integrable systems. They can be described as linear operators L on a Lie algebra $\mathfrak{g}$ satisfying the identity

$$L[L\xi, \eta] + L[\xi, L\eta] - [L\xi, L\eta] - L^2[\xi, \eta] = 0$$

for arbitrary $\xi, \eta \in \mathfrak{g}$, where $[\,,]$ is the commutator in $\mathfrak{g}$.

For the case when such an algebraic Nijenhuis operator on a Lie algebra $\mathfrak{g}$ is regular semisimple, i.e., it has an eigenbasis with pairwise different eigenvalues, there is an exellent statement due to Y. Kosmann-Schwarzbach and F. Magri which says that each pair of eigenvectors from this basis generates a two-dimensional subalgebra in $\mathfrak{g}$. Moreover, the inverse procedure also takes place, namely, the presence of a basis in a Lie algebra $\mathfrak{g}$ with such properties makes it possible to construct a regular semisimple algebraic Nijenhuis operator on $\mathfrak{g}$. Such bases in Lie algebras are called Nijenhuis eigenbases.

There are examples of Lie algebras possessing Nijenhuis eigenbases, but it is not clear how to describe all Lie algebras with such bases. Besides that, different bases can generate equivalent Nijenhuis operators. The aim of the present talk is to propose some approach to description of Nijenhuis bases themselves, without thinking about which Lie algebras they define. In particular, it will be demonstrated how one can obtain all (non-equivalent) algebraic Nijenhuis operators in small dimensions by using the proposed approach.

From complexity-one spaces to integrable systems

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Abstract

A complexity-one space is a $2n$-dimensional symplectic manifold M equipped with an effective Hamiltonian action of an $n-1$ dimensional torus. The moment map for such an action can be identified with $n-1$ independent and Poisson commuting real valued functions on the manifold, call them $f_1, \ldots, f_{n-1}$. On the other hand, an integrable system on M is given by n such functions. Thus, the functions induced by a complexity-one space are only one function short of an integrable system. This leads to the following class of questions: under what conditions on a complexity-one space does there exist one more function f_n so that $f_1, \ldots, f_n$ form an integrable system of a desired type? There are many versions of this question, corresponding to what type of integrable system is desired. We will review some cases in which this question is well understood, and introduce joint work with Tolman and Liu about some new cases.

Atlas of Bifurcation Diagrams for the Constrained Problem of Three Vortices

Gleb Palshin

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A multiparameter family of completely integrable Hamiltonian systems is in question. Its motion equations

$$\Gamma_\alpha \dot{x}_\alpha = \frac{\partial H}{\partial y_\alpha}, \quad \Gamma_\alpha \dot{y}_\alpha = -\frac{\partial H}{\partial x_\alpha}, \quad \alpha = 1, 2, \tag{1}$$
$$H = \frac{\Gamma_1}{\lambda_1} \ln |r_1| + \frac{\Gamma_2}{\lambda_2} \ln |r_2| + \frac{\Gamma_1 \Gamma_2}{\lambda_1 \lambda_2} \ln |r_1 - r_2|,$$

govern the dynamics of vortices at positions $r_\alpha = (x_\alpha, y_\alpha)$ with relative intensities $\Gamma_\alpha \in \mathbb{R}\backslash\{0\}$ and relative polarities $\lambda_\alpha = \pm 1$ in the presence of a third vortex fixed at the origin (see [1–3]). In addition to H, the system admits a functionally independent integral $F = \Gamma_1 r_1^2 + \Gamma_2 r_2^2$, which makes it a completely Liouville-integrable system with two degrees of freedom.

For arbitrary admissible relative polarities, it corresponds to the model of vortices in a ferromagnetic medium (see [4]) with a fixed irregularity. For $\lambda_1 = \lambda_2$, system (1) also coincides with the model of parallel hydrodynamic vortex filaments in an unbounded perfect fluid in the presence of a topographic irregularity (see, e. g., [5]).

We reduce system (1) to a Hamiltonian system with one degree of freedom, find the explicit form of its bifurcation diagram of the integral mapping, and construct the Fomenko invariants of the Liouville foliation (rough molecules; see [6]). The results are presented as an atlas of augmented bifurcation diagrams, which assigns a specific type of bifurcation diagram (equipped with topological invariants of the Liouville foliation) to each set of parameter values.

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A Generalization of Weinstein's Morphism

Andrés Pedroza

In 1989, A. Weinstein introduced a morphism

$$\mathcal{A}\colon \pi_1(\mathrm{Ham}(M,\omega)) \to \mathbb{R}/\mathcal{P}_2(M,\omega),$$

defined via the action functional of Hamiltonian diffeomorphisms. Here, $\mathrm{Ham}(M,\omega)$ denotes the group of Hamiltonian diffeomorphisms of the symplectic manifold (M,ω).

In this talk, we present a generalization of Weinstein's morphism to higher homotopy groups:

$$\mathcal{A}\colon \pi_{2k-1}(\mathrm{Ham}(M,\omega)) \to \mathbb{R}/\mathcal{P}_{2k}(M,\omega),$$

for all $1 \leq k \leq n$. As an application, we show that the groups $\pi_{2k-1}(\mathrm{Ham}(\widetilde{\mathbb{C}P}^n, \widetilde{\omega}_\rho))$ contain an element of infinite order.

Non-integrability of the n-body problem

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We consider the classical planar n body problem. In the center of mass frame. It admits the energy and the angular momentum first integrals. The integrability of the system restricted to a common level of these first integrals is investigated. We show that if $n > 2$, the system is not integrable on all but one level corresponding to the zero value of the energy and angular. In our proof we use differential Galois methods. We also discuss the applicability of our approach to study the integrability of charged n body problem.

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Camassa-Holm hierarchy and N-dimensional integrable systems

Zhijun George Qiao

In this talk, we will present one of the CH developments, namely, the Camassa-Holm hierarchy and its integrable structure etc. We will see how the CH hierarchy is related to finite-dimensional integrable systems, and furthermore algebro-geometric solutions of the CH hierarchy are shown on a symplectic submanifold. Other similar peakons models, including the DP, the b-family, and cubic equations (MOCH, FORQ/MCH, Novikov etc) will be mentioned as well. Some results are selected from my 2003 CMP paper,

Finite gap solutions for BKM-Systems

Abstract

BKM systems were introduced in [Bolsinov et al., 2022] by Bolsinov, Konyaev and Matveev as a consequence of the program on Nijenhuis geometry. They are integrable evolutionary PDEs (with differential constraints, depending on the type I-IV of the BKM system) including many famous examples like KdV, Camassa-Holmes etc. In [Bolsinov et al., 2024] so-called finite gap type solutions were constructed using solutions of some ODEs corresponding to geodesically equivalent metrics. This allows one to find particular solutions of the PDEs by means of solving (numerically or analytically) some ODEs. We will explain this construction, show some simulations and -if time allows- derive some explicit analytical solutions to some example BKM systems.

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Title: **The geometric structure that describes monopole dynamics**

Speaker: Cristina Sardón

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Abstract:

For a particle in the magnetic field of a cloud of monopoles, the naturally associated 2-form on phase space is not closed, and so the corresponding bracket operation on functions does not satisfy the Jacobi identity. Thus, it is not a Poisson bracket; however, it is twisted Poisson in the sense that the Jacobiator comes from a closed 3-form. The space $\mathcal{D}$ of densities on phase space is the state space of a plasma. The twisted Poisson bracket on phase-space functions gives rise to a bracket on functions on $\mathcal{D}$. In the absence of monopoles, this is again a Poisson bracket. It has been shown by Heninger and Morrison that this bracket is not Poisson when monopoles are present. In this note, we give an example where it is not even twisted Poisson. Therefore, in general, we can claim that such twisted Poisson structure does not exist, but: are there specific configurations of clouds of monopoles that we can actually endow with such structure? I will disclose this secret in this seminar!.

Main Reference: Plasma in a monopole background does not have a twisted Poisson structure. M. Lainz, C. Sardón, A. Weinstein. Phys. Rev. D **100** (2019), and references within the bibliography given in this article.

2D superintegrable systems and their algebraic structure

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Abstract

Superintegrable systems in dimension 2 admit three functionally independent integrals of motion. In this talk, I will focus on systems with quadratic integrals and discuss how maximally superintegrable systems exhibit an additional algebraic structure. This structure can be used to find new examples of superintegrable systems.

VARIATIONALITY OF CONFORMAL GEODESICS IN DIMENSION 3

BORIS KRUGLIKOV, VLADIMIR MATVEEV, WIJNAND STENEKER

Abstract. Conformal geodesics form an invariantly defined family of unparametrized curves in a conformal manifold generalizing unparametrized geodesics/paths of projective connections. The equation describing them is of third order, and it was an open problem whether they are given by an Euler–Lagrange equation. In dimension 3 (the simplest, but most important from the viewpoint of physical applications) we demonstrate that the equation for unparametrized conformal geodesics is variational.

Second-Order Superintegrable Systems: Geometric Insights via Affine Hypersurfaces

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Abstract: Second-order (maximally) superintegrable systems are a classical area of study in mathematical physics, and beyond. A famous example is the Kepler-Coulomb system, which models both the two-body problem in celestial mechanics and the Hydrogen atom in quantum physics. The classification of second-order superintegrable systems is an open problem, with a complete classification existing only in dimension two (partial results exist in dimension three).

In the first part of the talk, I will introduce the geometric framework, developed jointly with J. Kress and K. Schöbel [1,2,C]. Unlike other techniques, our method is manageable in any dimension. It encodes the systems via "structure tensors". An alternative formulation involves affine connections, offering further insight into the geometric properties of the system [1,3].

The second part of the talk will focus on novel results and applications of the framework. Specifically, I will show how a significant subclass of second-order superintegrable systems (so-called abundant systems) can be realised as affine hypersurfaces, establishing a correspondence between these systems and a certain subclass of affine hypersurface normalisations, which respects conformal rescalings (in preparation with V. Cortés).

Affine hypersurfaces play a key role in Hessian geometry. A Hessian metric is a (Riemannian) metric that can be expressed as the Hessian of a (convex) function with respect to a flat connection. Many non-degenerate second-order superintegrable systems are underpinned by Hessian metrics (with J. Armstrong [B]).

In the final part, I will explore abundant superintegrable systems on flat spaces. Their associated Hessian geometry defines a commutative and associative product structure via the Amari–Chentsov tensor. As a result, these systems can be realised as Manin-Frobenius manifolds, subject to certain simple compatibility conditions [A].

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Reduction of Elementary Integrability of Polynomial Vector Fields

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Abstract

Prelle and Singer showed in 1983 that if a system of ordinary differential equations defined on a differential field K has a first integral in an elementrary field extension L of K, then it must have a first integral consisting of algebraic elements over K via their constant powers and logarithms. Based on this result they further proved that an elementary integrable planar polynomial differential system has an integrating factor which is a fractional power of a rational function. Here we extend their results and prove that any n dimensional elementary integrable polynomial vector field has $n-1$ functionally independent first integrals being composed of algebraic elements over K. Furthermore, using the Galois theory we prove that the vector field has a rational Jacobian multiplier. This talk is based on the papers in the references.

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