Multicomponent generalisations of KdV and Camassa-Holm equations and their finite-dimensional reductions.

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Abstract

We suggest a geometric construction leading to a new series of multi-component integrable PDE systems that contains as particular examples (with appropriately chosen parameters)many famous integrable systems including KdV, coupled KdV, Harry Dym, coupled Harry Dym, Camassa-Holm, multicomponent Camassa-Holm, Dullin-Gottwald-Holm and Kaup-Boussinesq systems.

We suggest a methodology for constructing a series of solutions for all systems of this type. The crux of the approach lies in reducing this system to a dispersionless integrable system which is a special case of linearly degenerate quasilinear systems actively explored since the 1990s and recently studied in the framework of Nijenhuis geometry. These infinite-dimensional integrable systems are closely connected to certain explicit finite-dimensional integrable systems of Stäckel-Benenti type. We provide a link between solutions of our multicomponent PDE systems and solutions of this finite-dimensional system, and use it to construct animations of multi-component analogous of soliton and cnoidal solutions .