The affine Bonnet problem

Robert L Bryant
Duke University , Durham, North Carolina, USA
bryant@math.duke.edu

Abstract

The Bonnet problem in Euclidean surface theory is well-known: Given a metric g on an oriented surface M^2 and a function H, when (and in how many ways) can (M, g) be isometrically immersed in $\mathbb{R}^3$ with mean curvature H? For generic data (g, H), such an isometric immersion is impossible and, in the case that it does exist, the immersion is unique. Bonnet showed that, aside from the famous case of surfaces of constant mean curvature, there is a finite dimensional moduli space of (g, H) for which the space of such *Bonnet immersions* has positive dimension.

The corresponding problem in affine theory is still not completely solved. After reviewing the Euclidean results on this problem by O. Bonnet, J. Radon, É. Cartan, and A. Bobenko, I will give a report on affine analogs of these results. In particular, I will consider the classification of the data (g, H) for which the space of *affine Bonnet immersions* has positive dimension, showing a surprising connection with integrable systems in the case of data with the highest possible dimension of solutions.