From complexity-one spaces to integrable systems

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Abstract

A complexity-one space is a $2n$-dimensional symplectic manifold M equipped with an effective Hamiltonian action of an $n-1$ dimensional torus. The moment map for such an action can be identified with $n-1$ independent and Poisson commuting real valued functions on the manifold, call them $f_1, \dots, f_{n-1}$. On the other hand, an integrable system on M is given by n such functions. Thus, the functions induced by a complexity-one space are only one function short of an integrable system. This leads to the following class of questions: under what conditions on a complexity-one space does there exist one more function f_n so that $f_1, \dots, f_n$ form an integrable system of a desired type? There are many versions of this question, corresponding to what type of integrable system is desired. We will review some cases in which this question is well understood, and introduce joint work with Tolman and Liu about some new cases.