Reduction of Elementary Integrability of Polynomial Vector Fields

Xiang ZHANG

Shanghai Jiao Tong University, Shanghai 200240, P.R. China

Abstract

Prelle and Singer showed in 1983 that if a system of ordinary differential equations defined on a differential field K has a first integral in an elementrary field extension L of K, then it must have a first integral consisting of algebraic elements over K via their constant powers and logarithms. Based on this result they further proved that an elementary integrable planar polynomial differential system has an integrating factor which is a fractional power of a rational function. Here we extend their results and prove that any n dimensional elementary integrable polynomial vector field has $n-1$ functionally independent first integrals being composed of algebraic elements over K. Furthermore, using the Galois theory we prove that the vector field has a rational Jacobian multiplier. This talk is based on the papers in the references.

References

- Wenyong Huang, Xiang Zhang. Reduction of Elementary Integrability of Polynomial Vector Fields. Preprint, 2024. https://arxiv.org/abs/2412.04750

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