Top. def.

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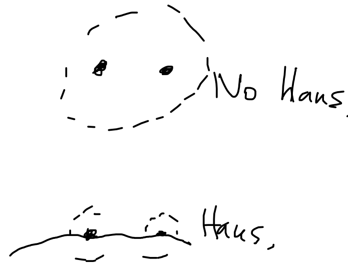
Var. — intrínseco (wikipedia) — Hausdorff, 2ndo contable(?)



extrínseco (GeP, Milnor) — $\hookrightarrow Y^k \subset \mathbb{R}^n$.

***7.** (*Stack of Records Theorem.*) Suppose that y is a regular value of $f: X \to Y$, where X is compact and has the same dimension as Y. Show that $f^{-1}(y)$ is a finite set $\{x_1, \ldots, x_N\}$. Prove there exists a neighborhood U of y in Y such that $f^{-1}(U)$ is a disjoint union $V_1 \cup \cdots \cup V_N$, where V_i is an open neighborhood of x_i and f maps each V_i diffeomorphically onto U. [HINT: Pick disjoint neighborhoods W_i of x_i that are mapped diffeomorphically. Show that $f(X - \cup W_i)$ is compact and does not contain y.] See Figure 1-13.

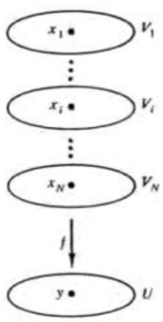


Figure 1-13

$$X \to Y$$

~~X~~ lemma: $\cancel{f: X \xrightarrow{\text{cont.}} Y}$ $\Rightarrow f^{-1}(y)$ es finito.
(pura top.?) cpt.

$$C = f^{-1}(y) \subset X$$

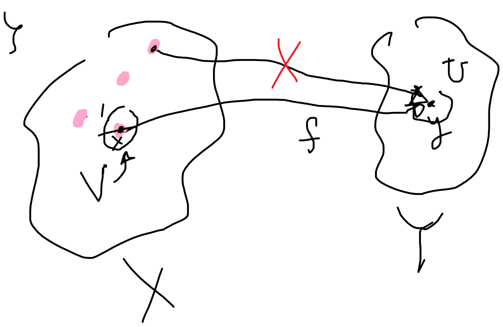
- $\{y\} \subset Y$ es cerrado. (rel. Y) ✓
- $\Rightarrow C$ es cerrado (rel. X) $\Rightarrow$ compacto
- T.F.Inv: $\forall x \in C$ $T_x X \xrightarrow{df_x} T_y Y$ es iso. lin. $\Rightarrow x \in V \underset{ab.}{\subseteq} X$, $y \in U \underset{ab.}{\subseteq} Y$

$$f|_V : V \xrightarrow{\text{difeo}} U \ni y$$

$$\Rightarrow \underline{C \cap V = \{x\}} \leftarrow ?$$

$X = \{\frac{1}{n} \mid n = 1, 2, 3 \ldots\} \subset \mathbb{R}$

es var ? SI!



Def. $\forall x \in X$, $\exists$ un difeo loc. entre una vec. de x (en X) y $\mathbb{R}^0 = \{0\}$.

pero $X \cup \{0\} \subset \mathbb{R}$ no lo es! ← ej.

8. Let

$$p(z) = z^m + a_1 z^{m-1} + \cdots + a_m$$

be a polynomial with complex coefficients, and consider the associated map $z \dashrightarrow p(z)$ of the complex plane $\mathbf{C} \longrightarrow \mathbf{C}$. Prove that this is a submersion except at finitely many points.

$f : \mathbb{C} \to \mathbb{C}$ holom. (analítica ~~an~~ compleja)

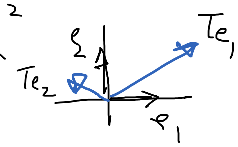
$z \in \mathbb{C}$ es regular ssi $f'(z) \neq 0$ "$a+ib$"

$\Updownarrow$

"$df_z : \mathbb{R}^2 \to \mathbb{R}^2$ es $\underline{\underline{\text{iso}}}$ lin

$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

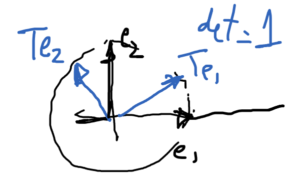
$T : \mathbb{R}^2 \to \mathbb{R}^2$



$TT^t = I$

$(\det)^2 = 1$

$\left\{ \begin{array}{l} \text{ortogonal} \\ \det = \pm 1 \end{array} \right.$



$\lambda = Re^{i\Theta}$

$z = re^{i\theta}$

$\lambda z = Rre^{i(\theta+\Theta)}$

$e_1 \to 1$

$e_2 \to i$

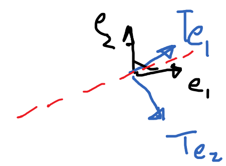
$T\, i = i\lambda = -b + ia$

$\mathbb{R}^2 \quad \mathbb{R}^2$

$T : \mathbb{C} \to \mathbb{C}$

$Tz = \lambda z$ t. l. compleja

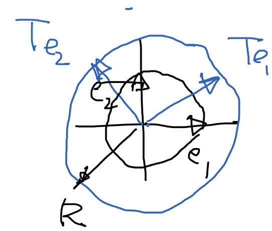
$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ $\lambda = a + ib$



$\det = -1$

Def (C R): f es holo si $Df_z : \mathbb{C} \to \mathbb{C}$ es compleja

$\Rightarrow Df$ es iso $\Leftrightarrow f'(z) = 0$



"conforme"