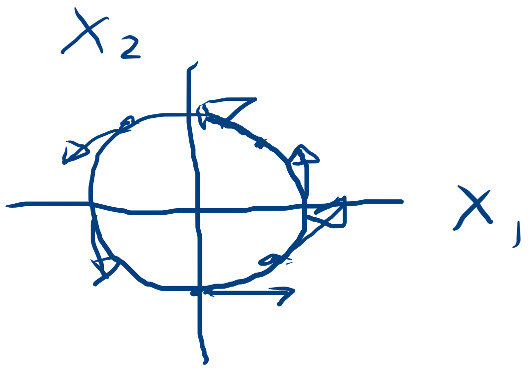
***7.** A point $x \in X$ is a *zero* of the vector field $\vec{v}$ if $\vec{v}(x) = 0$. Show that if k is odd, there exists a vector field $\vec{v}$ on S^k having no zeros. [HINT: For $k = 1$, use $(x_1, x_2) \longrightarrow (-x_2, x_1)$.] It is a rather deep topological fact that nonvanishing vector fields do not exist on the even spheres. We will see why in Chapter 3.



$$v(x) = (-x_2, x_1, -x_4, x_3, \ldots, -x_{2n}, x_{2n-1})$$

$$k = 2n - 1$$

$$\underbrace{x = (x_1, \ldots, x_{2n})}_{\cancel{\text{im}}\text{par}} \in \mathbb{R}^{2n}, \quad \|x\| = 1.$$

$$\mathbb{R}^{2n} = \mathbb{R}^2 \times \mathbb{R}^2 \times \cdots \times \mathbb{R}^2$$

$$((x_1, x_2), (x_3, x_4), \ldots)$$

<u>P.D.</u> $v(x) \in T_x S^k = x^\perp$.

***8.** Prove that if S^k has a nonvanishing vector field, then its antipodal map is homotopic to the identity (Compare Section 6, Exercise 7.) [HINT: Show that you may take $|\vec{v}(x)| = 1$ everywhere. Now rotate x to $-x$ in the direction indicated by $\vec{v}(x)$.]

$v : S^k \to TS^k$, $v(x) \in T_x S^k$ $(\iff p \circ v = id_{S^k})$

$v(x) \neq 0$, $\forall x \in S^k$.

normalizamos, $\hat{v} = v / \|v\|$ suave.

$$\begin{tikzcd} & TS^k \arrow[d, shift left, "p"] \\ S^k \arrow[ur, "v"] \arrow[r, "id"] & S^k \arrow[u, shift left, "v"] \end{tikzcd}$$

ej② con $g(x) = \frac{1}{\|v(x)\|}$

(fibración)

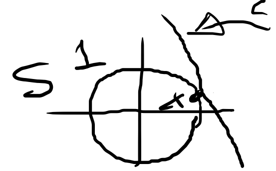
"haz vectorial"

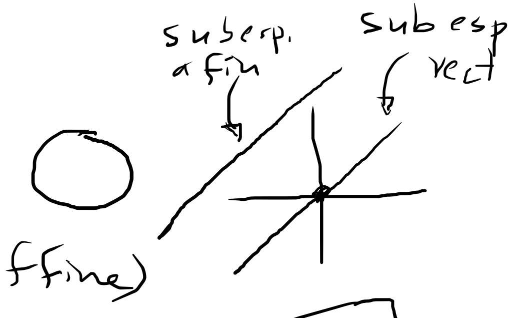
$H(x,t) = x\cos(\pi t) + \hat{v}(x)\sin(\pi t)$.

$\|H(x,t)\|^2 = \|x\|^2 \cos^2 + \|\hat{v}\|^2 \operatorname{sen}^2 = 1$.

$H(x,0) = x$, $H(x,1) = -x$.

¿es tang. a S^1 en x?! NO

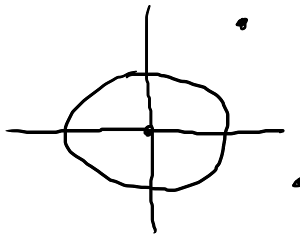




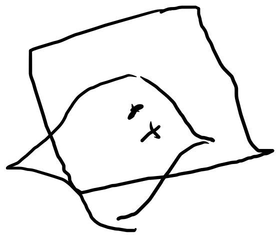
punto en $\mathbb{R}^n$

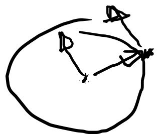
vector en $\mathbb{R}^n$

el mismo vector



$v = \gamma'(0)$





$X \xrightarrow{f} Y$ suave es <u>local</u>, i.e.

si $\forall x, \exists U \ni x$, t.q. $f|_U$ es suave.

$X \supset U \xrightarrow{f|_U} Y \ni y = f(x)$ $\quad f|_U = \underbrace{(f|_U \circ \varphi^{-1})}_{\text{suave}} \circ \underbrace{\varphi}_{\text{suave}}$

$\cong$ by φ $\qquad$ $\cup W'$

$V \subset \mathbb{R}^k$ $\qquad V' \quad \mathbb{R}^n \quad \mathbb{R}^N$

ej 2

$g : X \to \mathbb{R}$, $(x, v) \mapsto (x, g(x)v)$,

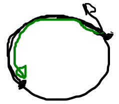
$TX \to TX$ $\quad (x, 0)$

<u>homotopía</u>:

$f_0, f_1 : X \to Y$

$\boxed{id, f : X \longrightarrow X}$

S^1



$H(x, t)$

<u>Usando núm complejos</u>:

$S^{2n-1} \subset \mathbb{R}^{2n} = \mathbb{C}^n$

$\langle z_1, z_2 \rangle = x_1 x_2 + y_1 y_2$

$= Re(z_1 \bar{z}_2)$

$z_1 = x_1 + i y_1$, $\quad z_2 = x_2 + i y_2$

$v(z) = iz = (iz_1, \dots, iz_n) = (i(x_1 + i y_1), \dots)$

$z = (z_1, z_2, \dots, z_n) = (x_1 + i y_1, \dots)$

$= ((-y_1 + i x_1), \dots)$

$\langle z, iz \rangle = Re\left(\sum_{j=1}^{n} z_j \overline{(i z_j)}\right) =$ $\quad \overline{i z_j} = -i \bar{z}_j$

$= Re\left(i \sum |z_j|^2\right) = 0$.

$z \cos(\pi t) + i z \operatorname{sen}(\pi t)$

$= e^{i\pi t} z$

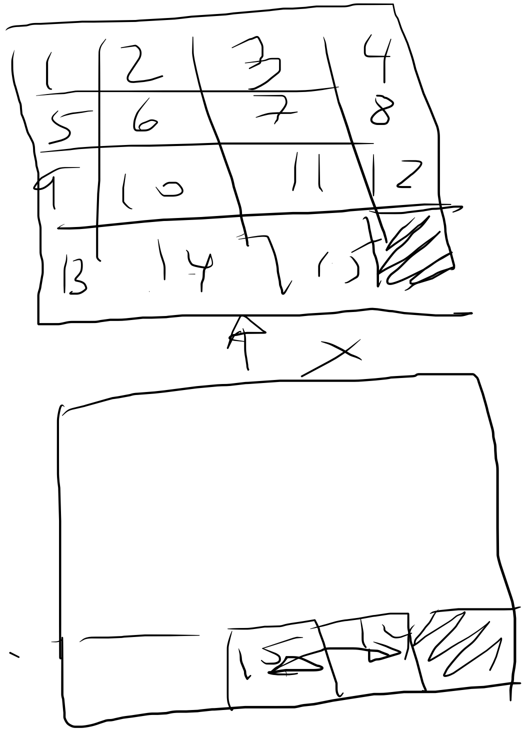
$S^3 \subset \mathbb{C}^2 \circlearrowleft S^1$

Thm: S^{2n} no admite campo vect. no nulo.

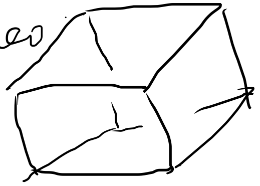
① $\chi(S^{2n}) = 2$,

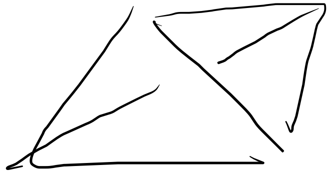
χ car. de Euler.

② Si ~~...~~ en var. M existe campo vectorial no nulo $\Rightarrow \chi(M) = 0$.



$\therefore \chi(S^{2n+1}) = 0$

$S^2 \overset{\text{homeo}}{\approx}$  $\leftarrow \chi = V - A + C = 8 - 12 + 6 = \underline{\underline{2}}$.

$\approx$  $\leftarrow \chi = 4 - 6 + 4 = \underline{\underline{2}}$

10. *The Whitney Immersion Theorem.* Prove that every k-dimensional manifold X may be immersed in $\mathbf{R}^{2k}$.

***14.** *Inverse Function Theorem Revisited.* Use a partition-of-unity technique to prove a noncompact version of Exercise 10, Section 3. Suppose that the derivative of $f: X \longrightarrow Y$ is an isomorphism whenever x lies in the submanifold $Z \subset X$, and assume that f maps Z diffeomorphically onto $f(Z)$. Prove that f maps a neighborhood of Z diffeomorphically onto a neighborhood of $f(Z)$. [*Outline:* Find local inverses $g_i: U_i \longrightarrow X$, where $\{U_i\}$ is a locally finite collection of open subsets of Y covering $f(Z)$. Define $W = \{y \in U_i : g_i(y) = g_j(y) \text{ whenever } y \in U_i \cap U_j\}$. The maps g_i "patch together" to define a smooth inverse $g: W \longrightarrow X$. Finish by proving that W contains an open neighborhood of $f(Z)$; this is where local finiteness is needed.]

Jorge

para miercoles

Amaranta