¿De dónde vienen las var. dif. y la funciones suaves?

- Restricción

$$\begin{tikzcd} \mathbb{R}^n & \mathbb{R}^m \\ U \arrow[u, phantom, "\cup" rotate=90] \arrow[r, "f"] & V \arrow[u, phantom, "\cup" rotate=90] \\ X \arrow[r] & Y \\ \mathbb{R}^n \arrow[u, phantom, "\cap"] \arrow[r] & \mathbb{R}^m \arrow[u, phantom, "\cap"] \end{tikzcd}$$

- composición

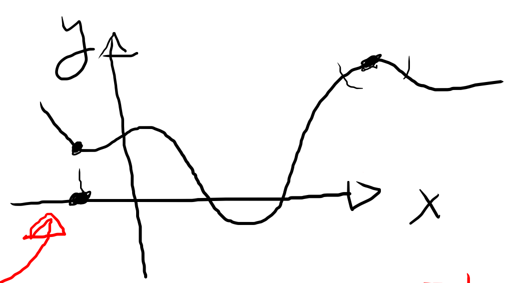
$$\begin{tikzcd} U \arrow[r, "\tilde{f}"] & V \arrow[r, "\tilde{g}"] & \mathbb{R}^l \\ X \arrow[u, phantom, "\cup"] \arrow[r, "f"] & Y \arrow[u, phantom, "\cup"] \arrow[r, "g"] & Z \\ \mathbb{R}^n \arrow[u, phantom, "\cap"] & \mathbb{R}^m \arrow[u, phantom, "\cap"] & \mathbb{R}^l \arrow[u, phantom, "\cap"] \end{tikzcd}$$

- gráficas de funciones

Receso
1:13 → 1:20

X
$\shortparallel$
$\mathrm{graph}(f) \xrightarrow{\varphi} \mathbb{R}^1$

$\tilde{\varphi}(x,y) = x, \; \varphi = \tilde{\varphi}|_X$



$y = f(x)$

$\mathbb{R}^2$

$\mathrm{graph}(f) \subset \mathbb{R}\times\mathbb{R}$
$= \{(x, f(x)) \mid x \in \mathbb{R}\}$

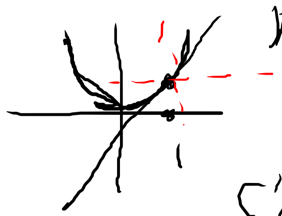
φ^{-1}

afirmación: la gráfica de $f: \mathbb{R}^1 \to \mathbb{R}^1$ es una variedad de dim 1.

$f: X \to Y$

$\mathrm{graph}(f) \subset X \times Y$

Derivada



$\mathbb{R} \xrightarrow{f} \mathbb{R}$ $\quad f(x) = x^2$

$f'(1) = 2$

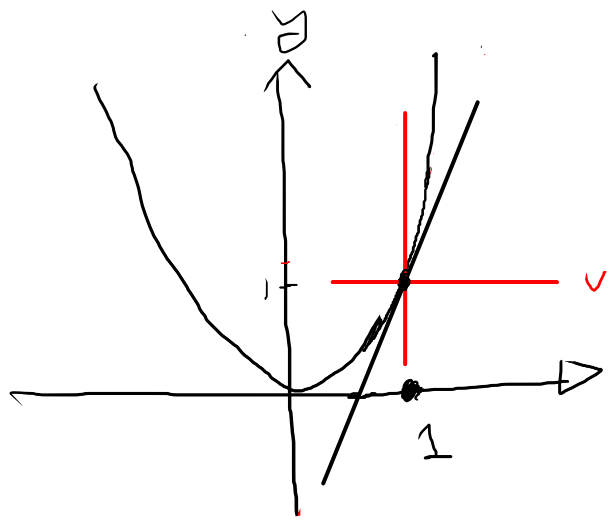
$f'(1): \mathbb{R} \to \mathbb{R}, \ v \mapsto 2v$

$\mathbb{R}^n \xrightarrow{f} \mathbb{R}^m$, $\quad df_x: \mathbb{R}^n \to \mathbb{R}^m$

$\overset{\uparrow}{x}$ $\qquad$ transf. lineal

Notación: $Df(x)$, df_x, $f'(x)$, $f_*(x)$, ...

$\qquad\uparrow$ aquí



$f: \mathbb{R} \to \mathbb{R}$

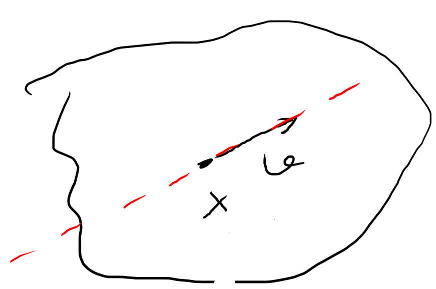
$y = f(x) = x^2$

$f'(1) = 2$?

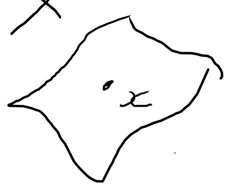
$f'(1): \mathbb{R} \to \mathbb{R}$

$f'(1)v = 2v$

$f: \mathbb{R}^2 \to \mathbb{R}$



$X^k \subset \mathbb{R}^n$



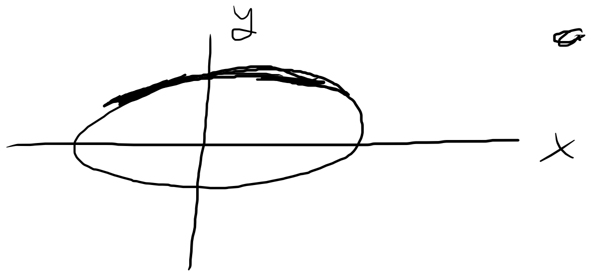
$X \xrightarrow{f} \mathbb{R}$

$df_x: T_x X \to \mathbb{R}$

$$\boxed{df_x(v) = \frac{d}{dt}\Big|_{t=0} f(x+tv)}$$

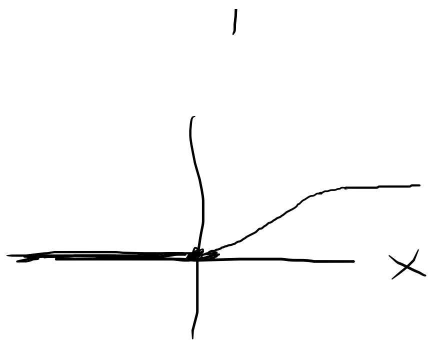
Teo: si f es C^1

$\Rightarrow df_x$ es lin. $\forall x$ $\quad$ $\boxed{X \subset \mathbb{R}^n}$, $T_x X \subset \mathbb{R}^n$ ← subesp. k-dim



$x^2+2y^2=1$

$y=\pm\sqrt{1-x^2}/2$



$$f(x)=\begin{cases} e^{-1/x^2} & x\geq 0 \\ 0 & x<0 \end{cases}$$

$f^{(n)}(0)=0$

$$e^{-\frac{1}{x^2}}=\frac{1}{e^{1/x^2}}\xrightarrow[x\to 0]{}0$$

Ej 1.5 (p. 6)

$$\begin{array}{ccc} \mathbb{R}^n & & \\ \cup & & \\ V & \xrightarrow{\varphi} & \mathbb{R}^k \end{array}$$

subesp. vect k-dim

$e_1, e_2, \dots, e_k$

$\tilde{e}_1, \dots, \tilde{e}_k, \dots, \tilde{e}_n$

$e_1=\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix},\ e_2=\begin{pmatrix}0\\1\\0\\\vdots\end{pmatrix},\ \dots$

$$\varphi^{-1}\begin{pmatrix}x_1\\\vdots\\x_k\end{pmatrix}=\sum x_i\tilde{e}_i$$

$$\varphi\left(\sum_{i=1}^{k}x_i\tilde{e}_i\right)=\sum^{k}x_i e_i$$

$$\tilde{\varphi}\left(\sum_{1}^{n}x_i\tilde{e}_i\right)= \text{— " —}$$