

Transversalidad

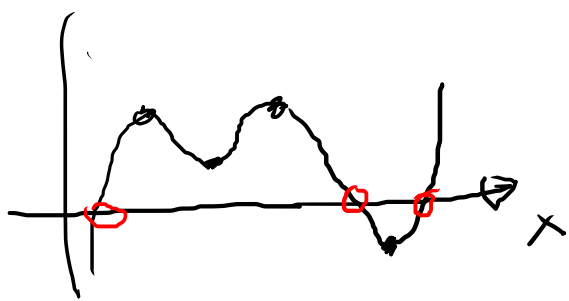
bueno

malo?

$f \cap Z$

Def: $f: X \rightarrow Y$

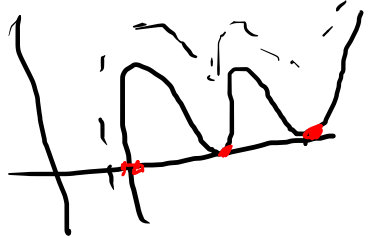
$$y = f(x) = a_0 + a_1 x + \dots + a_5 x^5$$



$$X \subset^c Y$$

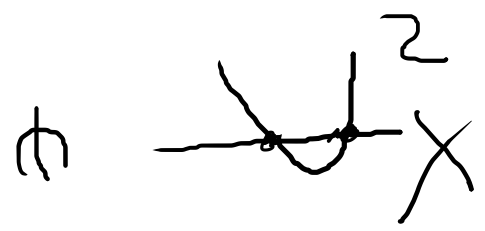
$$Z \subset Y$$

$f(x) + \epsilon$



$$i_x \cap Z \Leftrightarrow Z \cap X$$

X, Y, Z tienen una intersección transversal



~~$f \cap Z$~~



$$\begin{cases} f_1(x_1, \dots, x_n) = c_1 \\ f_2(\dots) = c_2 \\ \vdots \\ f_k(\dots) = c_k \end{cases}$$

Def: $f: X \rightarrow Y$ es un $Z \subset Y$ si
 A subvar.

$$\text{Im}(df_x) + T_y Z = T_y Y$$

$$T_y Y$$

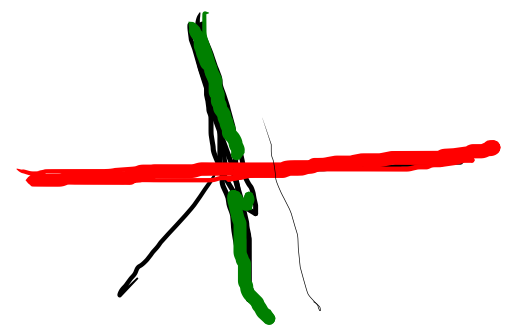
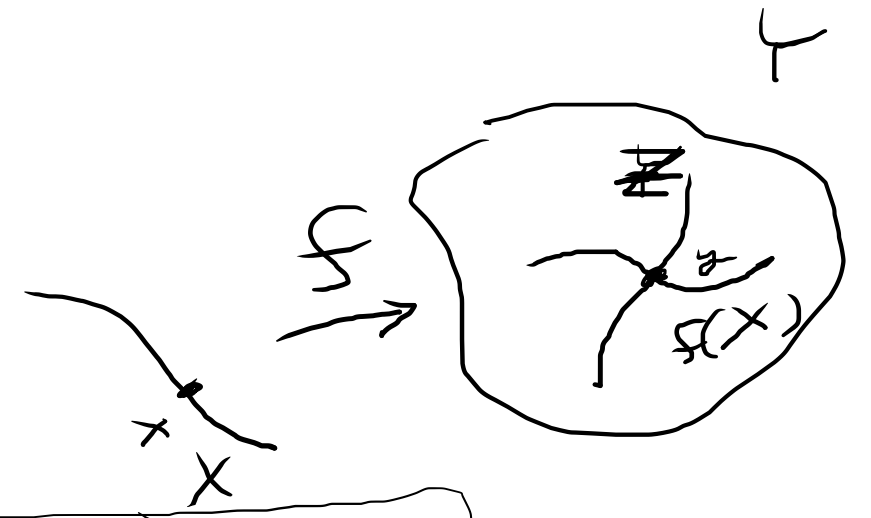
$$\forall y \in Z \cap f(x)$$

$$\exists z \in f^{-1}(y) ; \emptyset$$

$$Z \cap f(x) = \emptyset$$

$$f(x) = (x, 0, 0)$$

$$X = \mathbb{R}, Y = \mathbb{R}^3, Z = \text{el } e_j^i$$



$$f: X \rightarrow Y$$

Teo: si $X \cap Z = (f \cap Z) \cup Z$

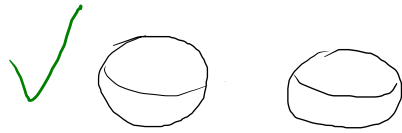
$\Rightarrow X \cap Z$ es una variedad

de $\text{codim} = \text{codim}(X) + \text{codim}(Z)$.

$$x = (x_1, \dots, x_n) \in \mathbb{R}^n$$

$$X = \begin{cases} f_1(x) = c_1 \\ f_2(x) = c_2 \end{cases} \quad X = X_1 \cap X_2$$

$$Z = \begin{cases} f_3 = c_3 \\ f_4 = c_4 \\ f_5 = c_5 \end{cases}$$



Ejemplo

$$Y = \mathbb{R}^3$$

$$f_1 = x = 0$$

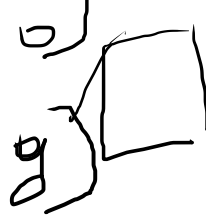
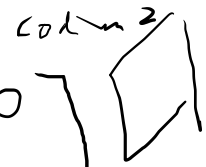
$$f_2 = y = 0$$

$$f_3 = x - y$$

$$(f_1, \dots, f_k): \mathbb{R}^n \rightarrow \mathbb{R}^k$$

Si df es sobre $k \leq n$

$\Rightarrow f^{-1}(c)$ si $\dim = k$.



$$f_3 = x - y$$

$$f = (f_1, f_2, f_3): \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

(x, y, z) es sumersión

$(x, y, x-y)$ NO es sumersión.

f_1, f_2, \dots, f_k son independientes si $\forall x \in M$,

$$df_1(x), \dots, df_k(x) \in T_x^* M$$

$$k \leq n = \dim M$$

$$f: M \rightarrow \mathbb{R}^k$$

$$f_i: M \rightarrow \mathbb{R}$$

$$df: T_x M \rightarrow T_y \mathbb{R} = \mathbb{R}$$

$$T_x(\mathbb{R}^n) = \mathbb{R}^n$$