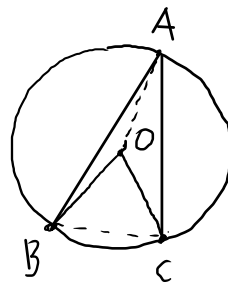


III.20. In a circle the angle at the center is double the angle at the circumference, when the rays forming the angles meet the circumference in the same two points.

En un círculo el ángulo central
 el doble de ángulo circunscrito
 apollado en el mismo arco:



$$\angle BOC = 2 \angle BAC$$

Dem: $\angle BAC = \angle BAO + \angle OAC$

(2) $OA = OB$ (radios del mismo círculo)

(3) $\angle BAO = \angle ABO$ (I.S.)

(4) $OA = OC$ (rad. del mismo círculo)

(5) $\angle OAC = \angle OCA$ (I.S.)

(6) $OB = OC$ (rad. del mismo círculo)

? (7) $\angle OBC = \angle OCB$ (I.S. + 6)

(8) $\angle BOC = 180 - (\angle OBC + \angle OCB)$

$$\begin{aligned} (9) \quad 180 &= \angle BAC + \angle ABO + \angle OBC + \angle AOC + \angle OCB \\ &= \angle BAC + \angle BAC + 180 - \angle BOC \\ &\quad - \angle BOC \end{aligned}$$

$$(10) \quad \angle BOC = 2 \angle BAC$$

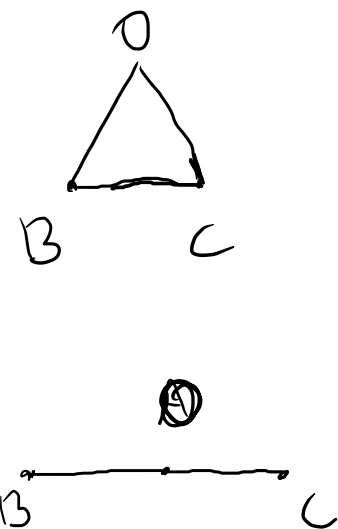
Q.E.D. (9),

$$\angle 5; \quad \angle BOC = 180^\circ$$

$$\Rightarrow \angle OBC = \angle OCB = 0^\circ$$

$$180 - \angle BOC \quad (8)$$

(la suma de áng. en un Δ es 180°)

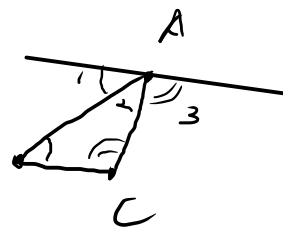


(**) Dem. que suma de áng. en Δ es = 180

(1) pasamos una recta paralela a BC por A. (Ax.)

(2) $\angle A_1 = \angle B$, $\angle A_3 = \angle C$

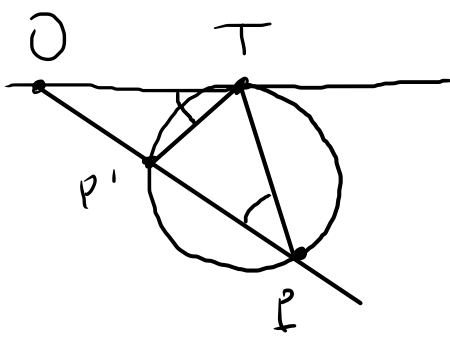
(3) $180 = \angle A_1 + \angle A_2 + \angle A_3 = \angle A_2 + \angle B + \angle C$ (áng. alt. int. entre rect. ||)



Q.E.D.

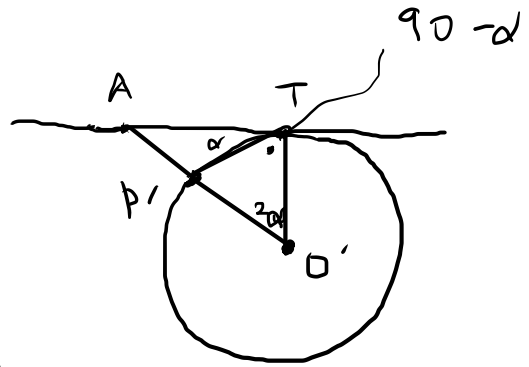
III.32. If a chord of a circle be drawn from the point of contact of a tangent, the angle made by the chord with the tangent is equal to the angle subtended by the chord at a point on that part of the circumference which lies on the far side of the chord (e.g., in Figure 1.3c, $\angle OTP' = \angle TPP'$).

Sobre la tangente en T de un círculo se escoge un punto O, y una recta por O que interseca el círculo en 2 puntos, P y P', donde P' es más cerca a O.



Ent. $\angle OTP' = \angle TPP'$.

Dem. ① $\angle TPP' = \alpha$ (def.)



Receso hasta 1:27

② O' el centro de círculo (def.)

③ $O'T = O'P'$ (rad. del circ.)

④ $\angle TP'O' = \angle P'TO'$ (3 + I.5)

⑤ $\angle TO'P' = 2\alpha$ (1 + III.20)

⑥ $2\alpha + 2\angle P'TO' = 180^\circ$ (Suma de ang. en Δ + 4 + 5)

⑦ $\angle P'TO' = 90 - \alpha$ (6)

⑧ $AT \perp O'T$ (la tang. \perp radio*)

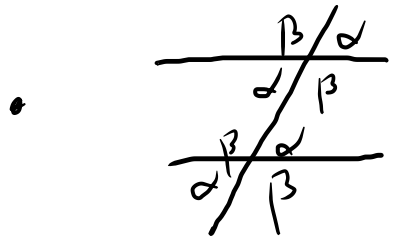
⑨ $\angle ATP' = \angle OTP' = 90 - (90 - \alpha) = \alpha$, (7 + 8),

Q.E.D.

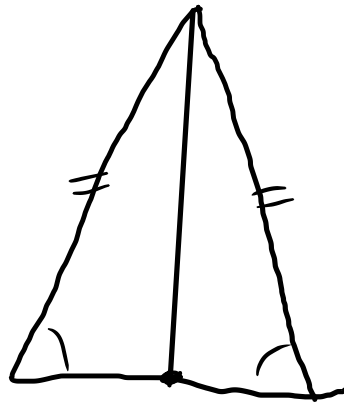
* demo. más tarde.

Axiomas / hechos sin demostrar y no obvias \rightarrow 2 pts def. 1 recta.

- LLL, LAL, ALA, ~~LAA~~



- Dado un segmento AB y un núm r , $0 \leq r \leq 1$, existe un sol punto C en AB tal que $\frac{|AC|}{|AB|} = r$



(En particular, se puede hablar del punto medio de un segmento, $r = 1/2$)